Non-Balanced Growth: The Role of Land*

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Abstract

The paper analyzes the role of land in explaining non-balanced growth in an economy. It develops a three-sector, closed-economy model, with agriculture, manufacturing and services sectors. The central assumption is that agriculture is most intensive in the use of land, followed by manufacturing and then services. Exogenous growth in sectoral TFP and labor serve and endogenous evolution of capital form the basis of growth. It is shown that if TFP growth rate differences are small, the ranking of sectoral output growth rates is the reverse of that of sectoral land-intensity, i.e., the growth rate is the fastest in the services sector, followed by manufacturing and then agriculture. The same growth ranking is preserved in the presence of capital accumulation if services are the most capital-intensive sector, followed by manufacturing and then agriculture. We also decompose sectoral growth differentials to analyze the strengths of the different sources of growth. We find that in short run land intensity differences play a larger role in explaining non-balanced growth and in long run the capital intensity differences have a larger explanatory power.

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1 Introduction

Non-balanced growth is a salient feature of a modern economy (Ray (2010) among others). If we disaggregate an economy into three broad sectors - agriculture, manufacturing and services - non-balanced growth is reflected in terms of differential output growth across these three sectors. EUKLEMS data records gross output volume indices of these sectors (barring those service sectors where government plays a significant role, like health and social services, public administration etc.) for member countries of the European Union as well as selected developed countries. Compounded annual growth rates based on this data are reported in Table 1.

We observe that for majority of countries in the data set, over the respective sample periods, output growth is highest in the services sector, followed by manufacturing while agriculture has posted the lowest growth rate among the three sectors.[1] We may term this as *SMA ranking*.

In Table 1, U.S. and U.K. can be noted major exceptions: over the period 1970s-2007, agriculture has outpaced manufacturing while the services sector has witnessed the fastest growth rate. However, if we consider more recent years, the SMA ranking holds for them too. Over the period 1992-2007, the annual growth rates of the services, manufacturing and agriculture sectors in the U.S. were 4.6%, 1.3% and 1.2% respectively, and in the U.K., these were 4.1%, 0.3% and -0.5% respectively over the period 1989-2007.[2]

It is generally difficult to obtain data on sectoral gross outputs for developing economies. However, sector-wise breakups of value-added, rather than gross output, are available for BRICS countries. Figure 1 depicts the evolution of sectoral GDP shares in these countries from 1970 to 2012.[3] It can be seen that in all BRIC countries the share of services value added has increased dramatically - which includes the manufacturing

[1]The countries in the EUKLEMS dataset were among the top 50 countries in the global human development index ranking, 2013, hence considered 'developed'.

[2]Other nations in the EUKLEMS dataset which are 'anomalies' to SMA ranking are mostly the Eastern European countries, which face obstacles in the growth of service activities. As noted in Sanz (2014), structural rigidities due to national regulations in service sectors and low levels of integration in the internal market, hinder the development of high value added business services. As these frictions are stronger in the more interior European countries, this explains the slower growth of services sector output in these countries. Overall, it is reasonable to state that in recent years, in the absence of any governance issues, market forces have driven output growth to be highest in the services sectors, followed by manufacturing and agriculture, in that order.

[3]Unlike the EUKLEMS data set, the World Development Indicators includes all services (both market and non-market produced) in the 'services' category.

Table 1: Compounded Annual Growth Rate (in %) for Sectoral Real Gross Output for Selected Developed Countries. *Source: EUKLEMS Database*

Country	Years	Agriculture	Manufacturing	Services
Australia	1970 – 2007	1.4	1.8	3.5
Austria	1970 – 2007	0.7	2.9	3.7
Belgium	1970 – 2007	0.8	2.4	2.7
Canada	1970 – 2004	2.8	2.9	4.5
Cyprus	1995 – 2007	1.1	0.3	4.3
Czech Republic	1995 – 2007	0.3	6.0	4.3
Denmark	1970 – 2007	1.5	1.5	3.2
Estonia	1995 – 2007	11.6	16.6	15.8
Finland	1970 – 2007	0.7	2.8	3.4
France	1970 – 2007	1.0	2.2	3.2
Germany	1970 – 2007	1.5	1.8	3.1
Greece	1970 – 2007	0.3	1.6	3.8
Hungary	1991 – 2007	-2.2	4.1	3.5
Ireland	1970 – 2007	1.9	6.1	4.7
Italy	1970 – 2007	0.5	2.6	3.1
Japan	1973 – 2006	-0.4	1.4	3.2
South Korea	1970 – 2007	2.3	9.4	7.8
Latvia	1995 – 2007	1.2	1.7	2.4
Lithuania	1995 – 2007	-0.2	6.3	6.7
Luxembourg	1970 – 2007	0.5	1.8	8.4
Malta	1995 – 2007	3.4	0.1	4.5
Netherlands	1970 – 2007	1.7	2.1	3.4
Poland	1995 – 2006	0.1	6.9	4.9
Portugal	1970 – 2006	1.2	2.9	3.8
Slovak Republic	1995 – 2007	1.8	6.7	3.0
Slovenia	1995 – 2006	-0.3	4.3	3.8
Spain	1970 – 2007	2.1	2.8	3.7
Sweden	1970 – 2007	0.8	2.1	2.7
United Kingdon	1970 – 2007	1.0	-0.1	3.5
USA	1977 – 2007	1.6	1.1	4.2

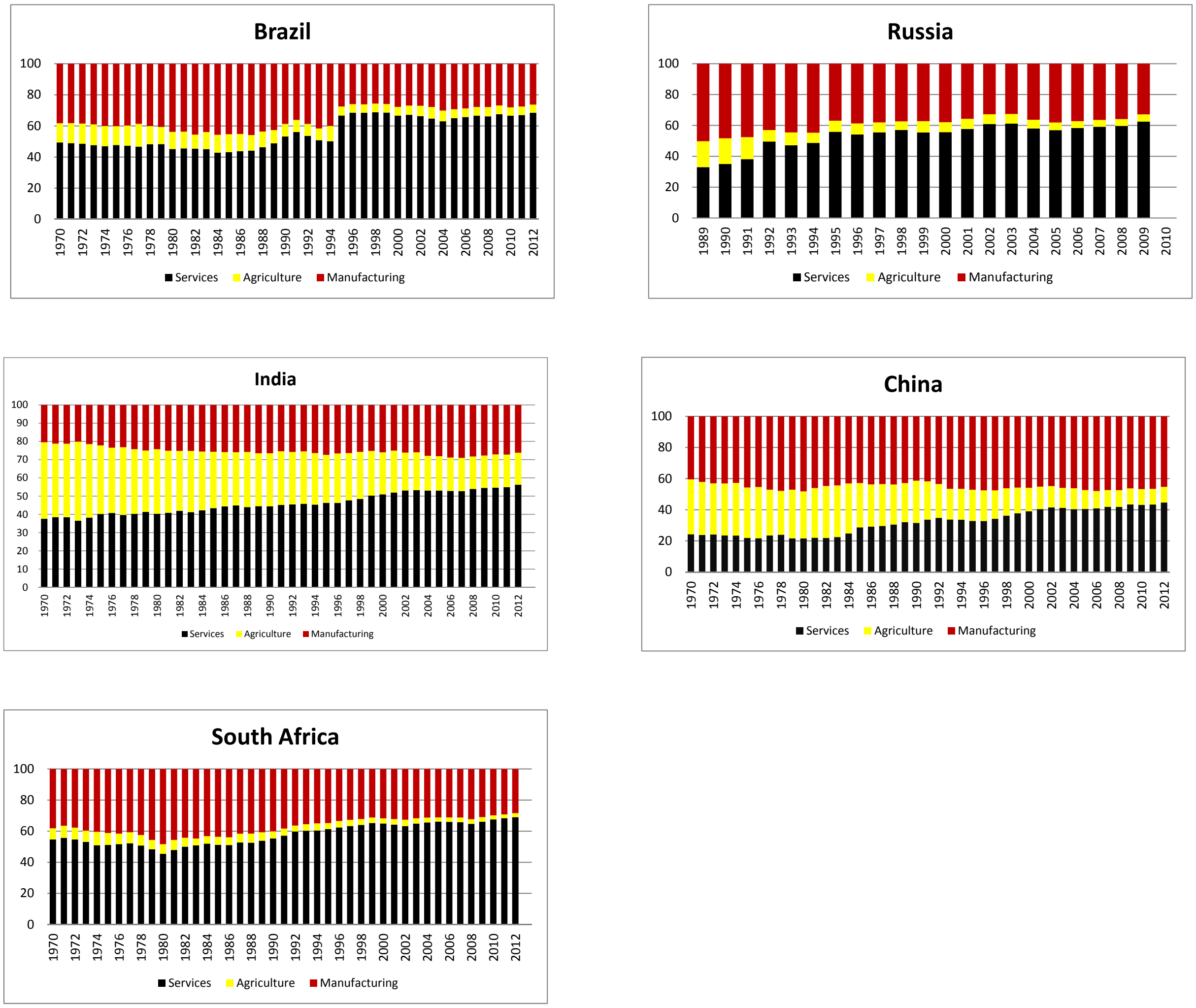

Figure 1: Sectoral Shares in Value Added for BRICS nations. *Source: World Development Indicators, World Bank*

hub of the World - China. In two of five BRICS countries, the manufacturing share has improved, but far less compared to services. The SMA ranking in terms of value-added holds for all BRICS countries except for Russia. The same ranking would be implied in terms of sectoral output unless relative price movements also have the same ranking and are sufficiently large.

Thus, if one were to seek a general pattern of inter-sectoral output, available data suggest the SMA ranking across countries. The current paper analyzes supply-side factors that may lead to such ranking in a market-oriented economy. Even for those

Table 2: Change in Sectoral Value Added Percentage Shares for BRICS Nations over the Entire Sample Period. *Source: World Development Indicators, World Bank*

Country	Years	Agriculture	Manufacturing	Services
Brazil	1970 – 2009	-14.5	-11.7	26.2
Russia	1989 – 2009	-12.1	-17.4	29.5
India	1970 – 2009	-25.1	7.4	17.7
China	1970 – 2009	-11.8	1.4	10.4
South Africa	1970 – 2009	-8.2	-6.7	14.9

countries which do not follow the stylized output growth ranking, the model is general enough to identify the sources/factors that account for such deviations from the broad pattern (such as USA, UK, South Korea over the period 1970s-2007).

Conventional explanation behind why the services sector may grow faster than manufacturing and manufacturing may grow faster than agriculture is based on demand-side reasons, e.g. Kongsamut et al. (2001), Eichengreen and Gupta (2013), among others. Here, the central idea is that income elasticity of demand for services is greater than unity, while that of the agriculture and manufacturing goods are less than unity and unity respectively. This explains that over time demand for services grows faster than demand for manufacturing and agriculture, which yields the SMA ranking.

There are supply-side models of non-balanced growth across sectors, but they do not proclaim to explain why the service sector grows faster than manufacturing. For instance, Baumol (1967) considers two sectors, both using only labor as input, wherein one sector has higher productivity growth than the other. Total labor supply is fixed. In long run, the output ratio is believed to be a constant (as neither of the two commodities tend to 'vanish' in long run). Given the technologies, it is immediate that labor in the less productive sector should grow at the expense of the more productive sector. Under the presumption that services and manufacturing are respectively less and more productive vis-a-vis each other, this would imply higher employment growth in the services sector compared to manufacturing. However, in developing countries data suggests a higher TFP growth in the services sector than in manufacturing (Ghani (2010)), which implies that productivity growth differences do not explain the trends in employment growth in these countries. Ngai and Pissarides (2007) develop a multi-sector model to examine the role of differences in TFP growth in explaining non-balanced sectoral growth in employment. All sectors share the same production function, but differ in productivity growth rates. This implies that the price of a less productive sector grows at a faster rate compared to the price of a more productive sector. Of the m goods, $m-1$ are final consumption goods and one good is consumed as well as invested in the form of capital. If the substitutability between the final goods is low, i.e. the consumption demand is too inelastic, the expenditure on a particular good is driven by its price change. Thus, the economy's share of expenditure on the least productive sector grows and along the balanced growth path employment shifts from the more productive sector to the less productive sector. This corroborates with the changes in employment shares witnessed in the US economy. Employment in the agricultural sector (that has witnessed the highest growth in TFP) has declined while that in the services sector (having lowest TFP growth rate) has risen. Manufacturing, being the capital producing sector, witnessed an initial rise and then fall in its employment share. However, both the Baumol and the Ngai-Pissarides models are at odds with the observed pattern of sectoral *output* growth ranking.

More recently, Acemoglu and Guerrieri (2008) have analyzed how sectoral intensity differences in the use of capital may lead to non-balanced growth of output as well as employment. In a two-sector model of capital accumulation with two factors of production, labor and capital, they show that the capital accumulation will be accompanied by capital deepening so that the output (employment) of the relatively capital intensive sector would grow faster (slower). While the Acemoglu-Guerrieri model falls short of asserting that their model meant to explain the higher growth rate of the service sector relative to manufacturing, they present data on capital intensities across sectors in the U.S., which indicates that the services sector as a whole is the mildly more capital intensive than manufacturing.[4] Hence, one can interpret their model as one which provides a capital deepening argument as to why the service sector output would grow faster than manufacturing, but does not explain the higher employment growth of the services sector vis-a-via manufacturing sector.

How far does the capital deepening argument apply to the differences in growth rates between manufacturing and agriculture? To answer this, we look at the non-labor shares across services, manufacturing and agriculture for OCED and some other countries. Insofar as non-labor input is representative of capital, Table 3 shows that for many countries, services are mildly more capital intensive than manufacturing,

[4]They tabulate the capital shares for selected industries within the services and manufacturing sectors. Over 1987-2005, the average capital share in manufacturing was about 0.37 and that in the selected services industries was about 0.373. According to EUKLEMS database, in the period 1970-2007, USA, UK have a slightly higher capital intensity in services as compared to manufacturing. This stems for higher use of IT capital in services sector. Also see Kutscher and Mark (1983).

Table 3: Sectoral Non-Labor Shares of Selected Countries. *Source: OECD Database (2005-10)*

Country	Agriculture	Manufacturing	Services
France	0.76	0.34	0.42
Spain	0.79	0.40	0.48
Japan	0.85	0.50	0.47
Germany	0.55	0.31	0.49
Australia	0.77	0.42	0.44
UK	0.53	0.29	0.38
USA	0.71	0.42	0.43
EU	0.65	0.33	0.41
China	0.09	0.69	0.63
Brazil	0.57	0.51	0.52

while agriculture is more capital intensive than manufacturing. Therefore, while the capital deepening argument serves to explain (in part) why the service sector would grow faster than manufacturing, its applicability toward explaining differential growth between manufacturing and agriculture is relatively weak.

The central objective of this paper is to introduce the role of land as a non-reproducible input and differences in the use of land intensity in production in the context of stylized differences in growth across services, manufacturing and agriculture. True, land does not figure prominently in the literature on growth.[5] But, as much incontrovertible is that it is required for production, transportation, consumption, waste disposal, etc. In the last decade, the demand for land has grown phenomenally, which, in turn, has led to substantial increase in land prices. Even if we set aside land demand for housing, the production sectors have been investing in land, both in the developed and developing countries. According to Land Matrix (an online database on land deals), over 48 million hectares of land has been bought and sold since 2000 till 2013. While the largest land deals took place in South East Asia (primarily India, China and Malaysia), the "land rush" in the last five years or so is seen in Africa (e.g. South

[5]There are a few studies only. Nichols (1970) is one of the early papers, where land is introduced as third input in production, besides labor and capital in a Solow economy. There is land and labor augmenting technical progress at an exogenous rate. Wealth has two components: capital and the value of land, a function of price of land. In steady state, land price and output grow at the same rate. Roe et al. (2009) have several chapters on multi-sector growth, with land as an input only in agriculture sector (not in manufacturing or services) and with the added role of land as an asset. Unlike Nichols (1970), Roe et al. (2009) use an infinite-horizon Ramsey framework, but in both papers in steady state the asset value of land grows at the same rate as the GDP of the economy.

Africa, Tanzania and Mali).

Land contributes to different sectors of production in different ways. In agriculture, it is almost synonymous with output – food. In manufacturing, it provides an area for production – base and space, and, for many services, it is just a location. In most countries that have favorable climate and relief conditions, a large proportion of land is used in agriculture.[6] Setting up manufacturing plants requires large expanses of land. Most countries have allocated vast regions to develop manufacturing plants and townships for workers together with transportation and other infrastructures facilities. Of late, land issues have been springing up in countries like India and China. In China, the demand for industrial land is about 67,000 acres per year, but the supply is less than 40% of that (Anderson (2011). In India too, the demand for land has played a crucial role in the growth of industry. Difficulty in acquiring land is one of the primary reasons for low investment in power sector (Singh (2012)).[7] Service-sector firms or providers, typically small in size, often require a modest 'floor space' – in a multi-storey building, at a home or along the corridors of shops and other establishments.

It is almost natural to hypothesize that among the three broad sectors, agriculture is most land-intensive, manufacturing is the next and the services sector is the least land-intensive. The implication of this hypothesis towards non-balanced growth is immediate: the supply of land being inelastic, *ceteris paribus*, the services sector would tend to grow faster than manufacturing and the latter would tend to grow faster than agriculture.

However, data on inter-sectoral land-use intensity is rather meager. To our knowledge, CORINE data is the only database which classifies land-use by industry type, and it covers European countries only. Using this database, Hubacek and Giljum (2003) calculate total sectoral land area (in hectares) per unit of sectoral output (in tones) and call this measure the land appropriation coefficient (LAC). This measure quantifies the intensity of land-use in different sectors. They find that in 1999, the LAC for agriculture in EU-15 was 89.67, 0.79 for manufacturing and 0.19 for electricity, water, transport and services. This indicates that agriculture output has very high dependence on land as compared to manufacturing and services - which is supportive of our

[6] According to World Bank database, the share of arable land of total available land is 0.6 in India, 0.3 in Brazil, 0.4 in USA and 0.7 in UK.

[7] Decentralized manufacturing production is a relatively new trend. A firm manufactures its good in parts in plants across the globe. Various parts of a product are then assembled near the points of sale. Although this method has greatly reduced the manufacturing sector's dependence on vast plots of land, land continues to be an important factor for industrial growth.

hypothesis on land intensity.

It is important to note however that most measures of capital include land. If so, given our hypothesis on land intensity differences, the difference of non-labor share between services and manufacturing would underestimate that of capital share between the two sectors and that between manufacturing and services. In other words, the presumption of the hypothesis of services being more capital intensive than manufacturing and the latter being more capital intensive than agriculture is higher than what is suggested in Table 3.[8]

More specifically, the model of the paper provides a *non-balanced growth decomposition* into sectoral TFP growth rates and the parts attributable to intensity differences in terms of land use and capital use and how TFP shocks may affect non-balanced growth in the long run and in the short run.

Section 2 presents an elementary model of growth without capital accumulation, featuring land as an input in production. For analytical tractability it is assumed that agriculture and manufacturing use land and labor, while services are produced by labor alone. Growth is driven by TFP growth across sectors and growth of labor, both exogenous. The impact of growth of labor is proportional to land-intensity differences. By construction, non-balanced growth decomposition does not include differences in capital intensity. The model in section 2 serves as a prelude to our main model in Section 3, which incorporates capital accumulation. Growth decomposition includes capital intensity differences as well and we characterize that for the long run and during transition. Furthermore, we analyze the impact of TFP shocks on growth differences in the long run and during transition. Section 5 concludes.

2 An Elementary Growth Model with Land as an Input

There are three sectors: agriculture (a), manufacturing (m), the numeraire sector, and services (s). Each is produced in a perfectly competitive market with constant-returns technology and consumed by households. There are two primary inputs - labor and land. Land supply is fixed. Sectors a and m use both inputs, while sector s uses labor only. Thus, sector s is (trivially) the least land-intensive. We assume sector a is more land-intensive, relative to labor, than sector m. The three goods are differentiated on

[8]It must however be borne in mind that neither the land-intensity ranking nor the capital intensity ranking should be viewed as substitutes of each other, and neither is meant to claim itself as the most important explanation for non-balanced growth.

the basis of land intensity differences, not through differences in household's income elasticity of their demand. Land has an additional role of being an asset. Households 'accumulate' land, although in the aggregate, land accumulation is zero.

Technologies in sectors a and m are Cobb-Douglas. Let γ and α be the share land in total cost in these sectors respectively. That sector a is more land intensive than sector m is captured by

Assumption 1 $\gamma > \alpha$.

We shall express production functions in terms of unit cost functions. These are $c_a(r_{Dt}, w_t)/A_t \equiv r_{Dt}^{\gamma} w_t^{1-\gamma}/A_t$ for agriculture and $c_m(r_{Dt}, w_t)/M_t \equiv r_{Dt}^{\alpha} w_t^{1-\alpha}/M_t$ for manufacturing. We assume constant-returns technology for services too: $c_s(w_t)/S_t \equiv w_t/S_t$. Variables r_{Dt} and w_t are the land rental rate and wage rate respectively and A_t, M_t and S_t are overall productivity (TFP) parameters in sectors a, m and s respectively.

We can now write down the production side of this economy in general form in terms of the familiar zero-profit and full-employment conditions, *a la* Jones (1965).

$$\frac{c_a(r_{Dt}, w_t)}{A_t} = p_{at}; \quad \frac{c_m(r_{Dt}, w_t)}{M_t} = 1; \quad \frac{c_s(w_t)}{S_t} = p_{st} \tag{1}$$

$$\frac{1}{A_t}\frac{\partial c_a(r_{Dt}, w_t)}{\partial r_{Dt}} Q_{at} + \frac{1}{M_t}\frac{\partial c_m(r_{Dt}, w_t)}{\partial r_{Dt}} Q_{mt} = \bar{D} \tag{2}$$

$$\frac{1}{A_t}\frac{\partial c_a(r_{Dt}, w_t)}{\partial w_t} Q_{at} + \frac{1}{M_t}\frac{\partial c_m(r_{Dt}, w_t)}{\partial w_t} Q_{mt} + \frac{1}{S_t}\frac{dc_s(w_t)}{dw_t} = L_t \tag{3}$$

where p_{at} and p_{st} are prices of food and services (in terms of manufactures), $\bar{D}$ is the total, fixed endowment of land in the economy and L_t is the total labor supply. These are five equations in five variables: the two factor prices and three outputs.

In the demand side, a household's total utility at t equals $L_t U_t$, where

$$U_t = \phi_a \ln C_{at} + \phi_m \ln C_{mt} + \phi_s \ln C_{st}, \ \phi_a, \ \phi_m, \ \phi_s > 0; \ \phi_a + \phi_m + \phi_s = 1,$$

and C_{jt} denotes per capita consumption of good j. This is maximized subject to the budget:

$$L_t(p_{at}C_{at} + C_{mt} + p_{st}C_{st}) = E_t, \tag{4}$$

where E_t is the total expenditure. The demand functions are:

$$L_t C_{at} = \frac{\phi_a E_t}{p_{at}}; \quad L_t C_{mt} = \phi_m E_t; \quad L_t C_{st} = \frac{\phi_s E_t}{p_{st}}. \tag{5}$$

The static equilibrium is described by the supply-side equations, and, the following market clearing conditions:

$$L_t C_{at} = Q_{at}; \quad L_t C_{mt} = Q_{mt}; \quad L_t C_{st} = Q_{st}. \tag{6}$$

Any two of the above along with supply-side equations solve the system. Let N_{jt} denote factor N employed in sector j, where $N = D, L$ (denoting land and labor) and $j = a,\ m,\ s$.

Proposition 1

$$\begin{aligned}
D_{at} \propto \bar{D}; \quad D_{mt} \propto \bar{D}; \quad L_{at} \propto L_t; \quad L_{mt} \propto L_t; \quad L_{st} \propto L_t \\
Q_{at} \propto A_t \bar{D}^{\gamma} L_t^{1-\gamma}; \quad Q_{mt} \propto M_t \bar{D}^{\alpha} L_t^{1-\alpha}; \qquad Q_{st} \propto S_t L_t \\
r_{Dt} \propto M_t \bar{D}^{-(1-\alpha)} L_t^{1-\alpha}; \quad w_t \propto M_t \bar{D}^{\alpha} L_t^{-\alpha} \\
p_{at} \propto A_t^{-1} M_t \bar{D}^{-(\gamma-\alpha)} L_t^{\gamma-\alpha}; \quad p_{st} \propto S_t^{-1} M_t \bar{D}^{\alpha} L_t^{-\alpha} \\
E_t \propto M_t \bar{D}^{-(1-\alpha)} L_t^{1-\alpha}.
\end{aligned}$$

Proof: See Appendix A. ■

Proposition 1 means that sectoral factor employment levels are independent of productivity parameters. They vary directly with respective total factor supplies only. There is no cross dependence – total labor supply does not affect sectoral land allocations nor does total land supply affect sectoral labor allocations. Cobb-Douglas specifications imply that the ratio of sectoral land allocation, D_{at}/D_{mt}, is proportional to the ratio of value of outputs, $p_{at}Q_{at}/Q_{mt}$. In equilibrium, this is equal to the ratio of respective consumption expenditures, which, in turn, is constant under log-linear preferences. Thus, sectoral land allocation is proportional to total land supply and independent of total supply of labor. Same reasoning holds for labor allocations.

We assume that population (labor supply) and TFP parameters grow at constant rates:

$$\frac{L_{t+1}}{L_t} = g_L; \quad \frac{A_{t+1}}{A_t} = g_A; \quad \frac{M_{t+1}}{M_t} = g_M; \quad \frac{S_{t+1}}{S_t} = g_S, \tag{7}$$

where g_L, g_A, g_M and g_S are greater than unity.

The household's consumption/land-investment decisions are inter-temporal. Let ρ' (< 1) and $\rho \equiv \rho' g_L$ (< 1) be the individual and population (household) size adjusted time discount factor. The household's dynamic problem is to choose $\{E_t\}_0^\infty$, $\{D_t\}_1^\infty$

that maximize its discounted lifetime utility

$$\sum_{t=0}^{\infty}\rho^t\left(\ln E_t-\phi_a\ln p_{at}-\phi_s\ln p_{st}\right)$$

subject to $p_{Dt}(D_{t+1}-D_t)+E_t\leq w_tL_t+r_{Dt}D_t$. Here D_t is the household's land holding and p_{Dt} is the price of land (in terms of manufactures). The Euler equation and the transversality conditions are:

$$\frac{E_{t+1}}{E_t}=\rho\left(\frac{r_{Dt+1}+p_{Dt+1}}{p_{Dt}}\right). \tag{8}$$

$$\lim_{t\to\infty}\frac{\rho^t}{E_t}p_{Dt}D_{t+1}=0. \tag{9}$$

The output and employment dynamics can be summarized as

Proposition 2 *Employment in each sector grows at the (gross) rate of g_L, and, output growth rates have the expressions:*

$$g_{Qa}=g_Ag_L^{1-\gamma};\ g_{Qm}=g_Mg_L^{1-\alpha};\ g_{Qs}=g_Sg_L. \tag{10}$$

Proof: It follows immediately from Proposition 1. ■

Thus, non-balanced growth results from differential TFP growth, differential land intensity (as long as total labor supply has a positive growth rate). Let $\tilde{g}_x$ denote the net growth rate of variable x, equal to g_x-1. Expressions in (10) imply $\tilde{g}_{Qa}\simeq\tilde{g}_A+(1-\gamma)\tilde{g}_L$, $\tilde{g}_{Qm}\simeq\tilde{g}_M+(1-\alpha)\tilde{g}_L$ and $\tilde{g}_{Qs}\simeq\tilde{g}_S+\tilde{g}_L$. Hence

$$\begin{aligned}\tilde{g}_{Qm}-\tilde{g}_{Qa}&\equiv\delta_{m-a}=(\tilde{g}_M-\tilde{g}_A)+(\gamma-\alpha)\tilde{g}_L;\\ \tilde{g}_{Qs}-\tilde{g}_{Qm}&\equiv\delta_{s-m}=(\tilde{g}_S-\tilde{g}_M)+\alpha\tilde{g}_L,\end{aligned} \tag{11}$$

where δ's denote difference in (net) growth rates of two sectors. Expressions in (11) are decompositions of sectoral growth rate differentials into TFP differentials (first term in the brackets in the r.h.s.) and those due to land intensity differentials (second term in the r.h.s.). Furthermore, because land intensities differ across sectors relative to labor,

Proposition 3 *Sectoral growth differentials ascribed to land intensity differential are proportional to the growth rate of labor.*

Notice that land allocation between agriculture and manufacturing is invariant over time, implying that technologies in these sectors exhibit decreasing-returns in terms of the variable input, labor, as opposed to constant-returns in services. Thus, differences in land-intensity amounts to difference in scale with respect to variable inputs, which can explain non-balanced growth across sectors. The role of land use in non-balanced growth is brought out by

Corollary 1 *Under Assumption 1 and if TFP differences are not sufficiently large, the services sector output grows the fastest, followed by the manufacturing sector and then the agriculture sector.*

There are two general conclusions. First, besides differences in TFP growth, differences in land intensity in production explain the stylized facts on relative growth rates of services, manufacturing and agriculture. Second, the growth ranking in Corollary 1 may hold even when $g_A > g_M > g_S$. That is,

Corollary 2 *Output growth ranking may be exactly the opposite of TFP growth ranking.*

Remarks

a. For developing countries, data on TFP growth is rather scarce. The few studies on TFP growth in developing countries like India, China, Pakistan, do not show any pronounced rankings (Bosworth and Collins (2008), Bosworth and Maertens (2010)). This accords with Corollary 1.

b. However, Wachter (2001), a European Central Bank study, shows that in the U.S. and France the TFP growth of manufacturing far exceeds that of services sector. But the services sector in these countries grow faster than manufacturing. Our model conveys that land constraints may very well be an underlying reason, although an empirical investigation of the same is beyond our scope.

It also immediately follows from (10) that

Corollary 3 *An increase in the TFP growth rate in a sector leads to one-to-one increase in the growth rate of output in that sector, without any spillover effects to other sectors.*

Real GDP and Land Price Dynamics

Apart from sectoral growth rates, the model predicts the growth rate of real GDP, and, that of land price in particular. The GDP of the economy in terms of manufactures is equal to $Y_t \equiv p_{at}Q_{at} + Q_{mt} + p_{st}Q_{st}$. In equilibrium, both $p_{at}Q_{at}$ and $p_{st}Q_{st}$ are proportional to Q_{mt}. Hence GDP in terms of manufactures grow at the same rate as does Q_{mt} - which, in view of Proposition 1, equals $g_M g_L^{1-\alpha} \equiv \bar{g}$.

Assumed preferences imply a general price index, $P_t \equiv p_{at}^{\phi_a} p_{st}^{\phi_s}$, where the price of goods are weighed by their respective weights in the household's preferences. The real GDP equals Y_t/P_t. As the value of three outputs are proportional to each other, each is proportional to real GDP. Hence

$$\frac{Y_t}{P_t} \propto \frac{(p_{at}Q_{at})^{\phi_a} Q_{mt}^{\phi_m} (p_{st}Q_{st})^{\phi_s}}{p_{at}^{\phi_a} p_{st}^{\phi_s}} \propto Q_{at}^{\phi_a} Q_{mt}^{\phi_m} Q_{st}^{\phi_s}.$$

It follows from Proposition 2 that the growth rates of real GDP and per capita real GDP are, respectively: $g_A^{\phi_a} g_M^{\phi_m} g_S^{\phi_s} g_L^{1-(\phi_a\gamma+\phi_m\alpha)}$ and $g_A^{\phi_a} g_M^{\phi_m} g_S^{\phi_s} g_L^{-(\phi_a\gamma+\phi_m\alpha)}$.

Corollary 4 *If there is no TFP growth in the three sectors, then real GDP per capita falls over time.*

This is an outcome of limited supply of land. If there is no TFP growth, then agriculture and manufacturing sector grow slower than the total labor supply while the growth rate of services sector is same as that of the total labor supply. In equilibrium per capita real GDP falls over time.

In the light of Proposition 1, E_t and r_{Dt} both grow at the rate $\bar{g}$. Using this, the land price dynamics is solved from the Euler equation (8) as a first-order difference equation:

$$p_{Dt} = \left(p_{D0} - \frac{\rho r_{D0}}{1-\rho}\right)\left(\frac{\bar{g}}{\rho}\right)^t + \frac{\rho r_{D0}}{1-\rho}\bar{g}^t. \tag{12}$$

where the initial land rental rate, r_{D0}, is derived from the static system. As $0 < \rho < 1$, $\bar{g}/\rho > g$. Hence, initially, if $p_{D0} \neq \rho r_{D0}/(1-\rho)$, it is evident from above that in the long run, the first term in the r.h.s. of (12) would dominate and thus p_{Dt} would tend to grow or decline at the rate $\bar{g}/\rho$. The transversality condition rules out this possibility however.[9] Rational agents bring to pass the initial land price being equal to $p_{D0} = \frac{\rho r_{D0}}{1-\rho}$, so that the first-term is zero and

$$p_{Dt} = \frac{\rho r_{D0}}{1-\rho}\bar{g}^t. \tag{13}$$

It follows that

Proposition 4 *Real land price grows at the same rate the real GDP of the economy.*

3 Capital Accumulation

We now introduce capital and its accumulation. Capital is assumed to be made up from the manufacturing good in one-to-one raio. We retain the land intensity ranking of Section 2. As already discussed in the Introduction, capital intensity ranking, particularly between manufacturing and agriculture, is not clear-cut across countries, and, insofar as our focus is on the role of land-use ranking, any definite ranking of capital intensity is not necessary. However, for sharpness of results, we assume that capital intensity is highest in services and least in agriculture. Further, for simplicity, we make this assumption in its extreme form: that is, capital is not used in agriculture at all while the service-sector is more capital intensive than manufacturing.[10] The end result is that the land-use intensity ranking and the opposite ranking of capital intensity both contribute toward the stylized fact of services sector growing faster than manufacturing and the latter faster than agriculture. Non-balanced growth decomposition has three elements: differences in TFP, differences in capital-intensity and differences in land-intensity relative to labor.

[9] If $p_{D0} \neq \rho r_{D0}/(1-\rho)$,

$$\lim_{t\to\infty} \frac{\rho^t}{E_t} p_{Dt} D_{t+1} = \frac{\rho^t[p_{D0} - \rho r_{D0}/(1-\rho)](\bar{g}/\rho)^t}{E_0 g^t}\bar{D} = \frac{p_{D0} - \rho r_{D0}/(1-\rho)}{E_0}\bar{D} \neq 0.$$

Hence the transversality condition (9) is not met.

[10] Roe et al. (2009) develop a three-sector growth model in the context of a small open economy. Agriculture and manufacturing are traded sectors, while services are not. Land is used only in agriculture sector. Capital and labor are used in all three sectors. In some chapters, the additional role of land as an asset has been considered. While the framework is similar to ours, the focus of their work is not on unbalanced growth. Using such a framework, they attempt to explain the dynamics of the Turkish macro economy over the last four decades.

3.1 Static Equilibrium

The production side of agriculture is same as in the elementary model. Manufacturing production requires three primary inputs: land, labor and capital. The unit cost function is given by $c_m(r_{Dt}, r_t, w_t)/M_t \equiv r_{Dt}^{\alpha} r_t^{\beta} w_t^{1-\alpha-\beta}/M_t$, α, $\beta < 1$, where r_t is rental earned by capital. The services sector uses labor and capital. Let $c_s(r_t, w_t)/S_t \equiv r_t^{\eta} w_t^{1-\eta}/S_t$, where $0 < \eta < 1$.

We impose

Assumption 2 $\gamma > \dfrac{\alpha}{1-\beta}$

Assumption 3 $\eta > \beta$.

Assumption 2 signifies that, between agriculture and manufacturing the former is more land intensive relative to labor (in the total share of land and labor in the respective sector).[11] It replaces our earlier Assumption 1. Assumption 3 reflects that capital is used more intensively in the services than in manufacturing; this is a weaker assumption than the service sector being more capital-intensive than manufacturing in the total share of capital and labor in the respective sector.

The supply side is expressed in terms of the following the zero profit and full employment conditions

$$\frac{c_a(r_{Dt}, w_t)}{A_t} = p_{at} \tag{14}$$

$$\frac{c_m(r_{Dt}, r_t, w_t)}{M_t} = 1 \tag{15}$$

$$\frac{c_s(r_t, w_t)}{S_t} = p_{st} \tag{16}$$

$$\frac{1}{A_t}\frac{\partial c_a(r_{Dt}, w_t)}{\partial r_{Dt}} Q_{at} + \frac{1}{M_t}\frac{\partial c_m(r_{Dt}, r_t, w_t)}{\partial r_{Dt}} Q_{mt} = \bar{D} \tag{17}$$

$$\frac{1}{A_t}\frac{\partial c_a(r_{Dt}, w_t)}{\partial w_t} Q_{at} + \frac{1}{M_t}\frac{\partial c_m(r_{Dt}, r_t, w_t)}{\partial w_t} Q_{mt} + \frac{1}{S_t}\frac{\partial c_s(r_t, w_t)}{\partial w_t} Q_{st} = L_t \tag{18}$$

$$\frac{1}{M_t}\frac{\partial c_m(r_{Dt}, r_t, w_t)}{\partial r_t} Q_{mt} + \frac{1}{S_t}\frac{\partial c_s(r_t, w_t)}{\partial r_t} Q_{st} = K_t, \tag{19}$$

where K_t is the aggregate stock of capital at time t.

[11] If land were present in the service sector production, the corresponding assumption would have been that manufacturing is more land intensive than service production in the total share of land and labor in the respective sector.

The demand functions for three goods are given by (5), except that E_t, total household expenditure on goods, equals total income minus savings. Static equilibrium yields sectoral levels of factor employment and output being dependent on productivity levels, factor supplies as well as total expenditure (spending). Given Cobb-Douglas technologies and log-linear preferences, such dependencies assume following forms:

Proposition 5

$$D_{at} = \bar{D} \cdot f_{Da}(\mathcal{E}_t, \mathcal{K}_t); \quad D_{mt} = \bar{D} \cdot f_{Dm}(\mathcal{E}_t, \mathcal{K}_t);$$
$$L_{at} = L_t \cdot f_{La}(\mathcal{E}_t, \mathcal{K}_t); \quad L_{mt} = L_t \cdot f_{Lm}(\mathcal{E}_t, \mathcal{K}_t); \quad L_{st} = L_t \cdot f_{Ls}(\mathcal{E}_t, \mathcal{K}_t);$$
$$K_{mt} = M_t^{\frac{1}{1-\beta}} \bar{D}^{\frac{\alpha}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}} \cdot f_{Km}(\mathcal{E}_t, \mathcal{K}_t); K_{st} = M_t^{\frac{1}{1-\beta}} \bar{D}^{\frac{\alpha}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}} \cdot f_{Ks}(\mathcal{E}_t, \mathcal{K}_t);$$
$$Q_{at} = A_t \bar{D}^{\gamma} L_t^{1-\gamma} \cdot f_{Qa}(\mathcal{E}_t, \mathcal{K}_t); \quad Q_{mt} = M_t^{\frac{1}{1-\beta}} \bar{D}^{\frac{\alpha}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}} \cdot f_{Qm}(\mathcal{E}_t, \mathcal{K}_t);$$
$$Q_{st} = S_t M_t^{\frac{\eta}{1-\beta}} \bar{D}^{\frac{\alpha\eta}{1-\beta}} L_t^{\frac{1-\alpha\eta-\beta}{1-\beta}} \cdot f_{Qs}(\mathcal{E}_t, \mathcal{K}_t);$$
$$r_{Dt} = M_t^{\frac{1}{1-\beta}} \bar{D}^{-\frac{1-\alpha-\beta}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}} \cdot f_{rD}(\mathcal{E}_t, \mathcal{K}_t); \quad w_t = M_t^{\frac{1}{1-\beta}} \bar{D}^{-\frac{\alpha}{1-\beta}} L_t^{\frac{-\alpha}{1-\beta}} \cdot f_w(\mathcal{E}_t, \mathcal{K}_t);$$
$$r_t = f_r(\mathcal{E}_t, \mathcal{K}_t);$$
$$p_{at} = A_t^{-1} M_t^{\frac{1}{1-\beta}} \bar{D}^{-\frac{(1-\beta)\gamma-\alpha}{1-\beta}} L_t^{\frac{(1-\beta)\gamma-\alpha}{1-\beta}} \cdot f_{pa}(\mathcal{E}_t, \mathcal{K}_t);$$
$$p_{st} = S_t^{-1} M_t^{\frac{1-\eta}{1-\beta}} \bar{D}^{\frac{\alpha(1-\eta)}{1-\beta}} L_t^{-\frac{\alpha(1-\eta)}{1-\beta}} \cdot f_{ps}(\mathcal{E}_t, \mathcal{K}_t).$$

where

$$\mathcal{E}_t \equiv \frac{E_t}{M_t^{\frac{1}{1-\beta}} \bar{D}^{\frac{\alpha}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}}}; \quad \mathcal{K}_t \equiv \frac{K_t}{M_t^{\frac{1}{1-\beta}} \bar{D}^{\frac{\alpha}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}}}. \tag{20}$$

Proof: See Appendix B. ■

Remarks

a. The variables $\mathcal{E}_t$ and $\mathcal{K}_t$ can be termed as 'normalized' capital stock and total household expenditure respectively.[12] Later, the dynamic system will be expressed in ($\mathcal{E}_t$, $\mathcal{K}_t$) space and in the steady state both are constant.

b. Expressions in Proposition 5 anticipate that the growth process of factor employment and outputs will be governed by *transitional effects* via endogenous evolution of $\mathcal{E}_t$ and $\mathcal{K}_t$ and *long run effects* through exogenous growth of productivities and increase in labor supply.

[12] These are counterparts of capital and consumption per effective labor in the one-sector, Solow-Ramsey-Koopman model.

c. Unlike in the previous model, the factor-employment ratios between two sectors are not time-invariant. Because a part of manufacturing output constitutes savings, the ratio of the value of outputs is *not* equal to the ratio of consumption expenditure. Therefore, for example, while D_{at}/D_{mt} is proportional to $p_{at}Q_{at}/Q_{mt}$, the latter ratio is not constant. Consequentially, land used in each sector depends on total land supply as well as total supplies of labor and capital and total household expenditure. The same holds for employment of labor and capital.

d. Productivity in manufacturing affects output of services because a part of manufacturing is converted to capital and capital is an input to the services sector. (It would have affected agricultural output if capital were used in that sector.)

3.2 Dynamics

The representative household own two assets - land and capital. They maximize the discounted sum of its welfare: $L_0 \sum_{t=0}^{\infty} \rho^t U_t$. It has two sources of income, which is used to finance purchase of goods and asset accumulation: namely, wage earnings and rental income from assets (land and capital). Its dynamic problem is

$$\text{Maximize} \sum_{t=0}^{\infty} \rho^t (\ln E_t - \phi_a \ln p_{at} - \phi_s \ln p_{st}),$$

$$\text{subject to } E_t + K_{t+1} - K_t + p_{Dt}(D_{t+1} - D_t) \leq w_t L_t + r_t K_t + r_{Dt} D_t,$$

where U_t is substituted by its indirect form. For simplicity, the rate of capital depreciation is assumed to be zero. Given L_0, D_0 and K_0, the household chooses $\{E_t\}_0^{\infty}$, $\{D_t\}_1^{\infty}$ and $\{K_t\}_1^{\infty}$. We obtain the standard Euler equation

$$\frac{E_{t+1}}{E_t} = \rho(1 + r_{t+1}). \tag{21}$$

There are two transversality conditions: (9) and

$$\lim_{t\to\infty} \frac{\rho^t}{E_t} K_{t+1} = 0. \tag{22}$$

The no-arbitrage condition between the assets is

$$1 + r_{t+1} = \frac{p_{Dt+1} + r_{Dt+1}}{p_{Dt}}. \tag{23}$$

The full employment condition for capital (19) and the service-demand function (5) together imply

$$\beta Q_{mt} = r_t K_t - \phi_s \eta E_t. \tag{24}$$

Thus, the law of motion of capital, $K_{t+1} = Q_{mt} - \phi_m E_t + K_t$, can be written as

$$K_{t+1} = \frac{r_t K_t}{\beta} - \frac{\phi_m \beta + \phi_s \eta}{\beta} E_t + K_t. \tag{25}$$

This equation, the Euler equation, the no-arbitrage condition as well as the transversality condition form the basis of the dynamic system. Using the expressions in Proposition 5 and defining $g^\circ \equiv g_M^{\frac{1}{1-\beta}} g_L^{\frac{1-\alpha-\beta}{1-\beta}}$, eqs. (36) and (39) can be expressed as

$$\begin{aligned} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} &= \frac{\rho\left[1 + f_r(\mathcal{E}_{t+1}, \mathcal{K}_{t+1})\right]}{g^\circ} \\ \mathcal{K}_{t+1} &= \frac{1}{g^\circ} \cdot \left[\frac{f_r(\mathcal{E}_t, \mathcal{K}_t)\mathcal{K}_t}{\beta} - \frac{\phi_m \beta + \phi_s \eta}{\beta}\mathcal{E}_t + \mathcal{K}_t\right]. \end{aligned} \tag{26}$$

These two equations form the core dynamic system of the economy.

For land price dynamics we rewrite the non-arbitrage equation (38) in terms of the normalized variables

$$\mathcal{P}_{Dt+1} = \frac{1 + f_r(\mathcal{E}_{t+1}, \mathcal{K}_{t+1})}{g} \mathcal{P}_{Dt} - f_{rD}(\mathcal{E}_{t+1}, \mathcal{K}_{t+1}), \tag{27}$$

where $\mathcal{P}_{Dt} \equiv p_{Dt} M_t^{-\frac{1}{1-\beta}} \bar{D}^{-\frac{\alpha}{1-\beta}} L_t^{-\frac{1-\alpha-\beta}{1-\beta}}$.

3.3 Steady State

This is defined by $\mathcal{E}_t = \mathcal{E}^*$ and $\mathcal{K}_t = \mathcal{K}^*$. Eqs. (40) yield

$$\begin{aligned} r^* = f_r(\mathcal{E}^*, \mathcal{K}^*) &= \frac{g^\circ}{\rho} - 1 \\ \frac{\mathcal{E}^*}{\mathcal{K}^*} &= \frac{(g^\circ - 1)(1 - \beta\rho) + 1 - \rho}{\rho\nu}, \text{ where } \nu \equiv \phi_m \beta + \phi_s \eta < 1. \end{aligned} \tag{28}$$

The former is the modified golden rule, whereas the latter defines the trajectory where savings grow at a constant rate. Eqs. (41) implicitly solve $(\mathcal{E}^*, \mathcal{K}^*)$. In Appendix B we show that the steady state exists and it is unique.

Factor employment shares and land allocation across sectors remain invariant. Furthermore, it follows directly from Proposition 5 that

Proposition 6 *Along the steady state,*

$$g_{Qa}^{*} = g_A g_L^{1-\gamma};\ g_K^{*} = g_{Qm}^{*} = g^{\circ} = g_M^{\frac{1}{1-\beta}} g_L^{\frac{1-\alpha-\beta}{1-\beta}};\ g_{Qs}^{*} = g_S g_M^{\frac{\eta}{1-\beta}} g_L^{\frac{1-\alpha\eta-\beta}{1-\beta}}. \tag{29}$$

Since capital is not used in the agriculture sector, the growth rate of this sector is affected by the TFP growth in that sector and the exogenous growth rate of labor; it is not affected capital accumulation.

As the manufacturing good is transformed one-to-one into capital, manufacturing output and capital must grow at the same rate. Notice that TFP growth in manufacturing has *a multiplier effect* - captured by the exponent $1/(1-\beta)$ - on growth rates of manufacturing and capital. It is because it promotes capital accumulation and hence manufacturing output expands due to TFP growth rate and the induced growth of capital.

Services sector growth is affected by TFP growth in that sector as well as that in manufacturing since capital is used in the services sector. Because the growth of capital exceeds that TFP in manufacturing, TFP growth in manufacturing may exert more than one-to-one impact on the growth rate of services output. Also notice that an increase in the exogenous growth rate of labor affects the growth rates of manufacturing and services output directly as well as indirectly via enhancing the growth rate of capital.

Long-run Non-Balanced-Growth Decomposition

The current model reveals three sources of long-run sectoral output growth differences: those in TFP growth rates, land-intensity *and* capital intensity. The r.h.s. expressions of eqs. (30) below provide what we call *long-run non-balanced-growth decompositions.*

$$\begin{aligned} \delta_{m-a} &= (\tilde{g}_M - \tilde{g}_A) + \left(\gamma - \frac{\alpha}{1-\beta}\right)\tilde{g}_L + \frac{\beta}{1-\beta}\tilde{g}_M; \\ \delta_{s-m} &= (\tilde{g}_S - \tilde{g}_M) + \frac{\alpha(1-\eta)}{1-\beta}\tilde{g}_L + \frac{\eta-\beta}{1-\beta}\tilde{g}_M. \end{aligned} \tag{30}$$

The three right-hand side terms respectively express the contribution of TFP differential, land intensity differential (relative to labor) and capital intensity differential.

Note that

Proposition 7 *(a) Land and capital intensity differences affect the magnitudes of the impacts of growth rates of respectively labor and TFP in manufacturing towards non-balanced growth. (b) An increase in the growth rate of labor widens the difference in sectoral rates between manufacturing and the agriculture sector and between services and manufacturing. This is due to differences in the land-use intensities across sectors. (c) An increase in TFP growth in manufacturing leads to more than one to one widening of difference in output growth rates between manufacturing and agriculture sector and less than one to one narrowing of differences in output growth rates between services and manufacturing.*

SMA ranking is equivalent to δ_{m-a} and δ_{s-m} both being positive. However the output growth decomposition (30) per se does not necessarily imply that δ_{m-a} and δ_{s-m} need to be positive; hence it can accommodate the deviations from the general pattern of SMA ranking.

The Corollaries 1 and 2 carry over under Assumptions 2 and 3. However, Corollary 3 does not hold, i.e., there are cross sectoral effects of TFP growth.

Corollary 5 *TFP growth in agriculture or services sector affects output growth in the respective sector only, whereas that in manufacturing affect output growth in manufacturing as well as services.*

It is because capital, a manufacturing good, is used in the production of services. If capital was an input in the agriculture sector or if sectoral goods were inputs in the production of other sectors, then there would have been other cross-dependencies of sectoral TFP and output growth.

Aggregate Growth, Land Price and Kaldor Facts

As in the elementary model the real GDP growth rate equals a consumption-weighted growth rates of sectoral outputs. In the steady state, it is equal to

$$g_Y^* = g_A^{\phi_a} g_M^{\frac{\phi_m+\phi_s\eta}{1-\beta}} g_S^{\phi_s} g_L^{\frac{\phi_a(1-\beta)(1-\gamma)+\phi_m(1-\alpha-\beta)+\phi_s(1-\alpha\eta-\beta)}{1-\beta}}$$

Notice that while the weights on g_A, g_S and g_L in the growth rate of real GDP are less than one, the weight on g_M may exceed unity. If $\phi_m + \phi_s\eta > 1 - \beta$, then 1 percentage point increase in g_M raises the growth rate of real GDP by more than 1 percentage point. If the multiplier effect of g_M on manufacturing and services output growth rate is sufficiently strong, then the multiplier effect of g_M is projected on real GDP growth also.

The land-price dynamic equation (27) yields that at steady state $\mathcal{P}_{Dt} = \mathcal{P}^*_D$. Thus, land price grows at rate g° at steady state.

Lastly, as consistent with other studies on non-balanced growth such as Kongsamut et al. (2001) and Ngai and Pissarides (2007), despite non-balanced growth across sectors the model accords with Kaldor's facts on aggregate economy-wide growth.

Proposition 8 *Over the steady state, the capital-output (GDP) ratio, return on capital, factor shares in national income and the growth rate out output (real GDP) per worker are all constant.*

Proof: Output is measured by GDP of an economy. At steady state, $\mathcal{K}_t = \mathcal{K}^*$ and $\mathcal{E}_t = \mathcal{E}^*$, so it follows from Proposition 5 that GDP grows at the rate g°. Thus, the growth rate of GDP_t/L_t is constant. Further, from the Proposition 5 it is easy to see that the variables K_t/GDP_t, r_t, w_tL_t/GDP_t and r_tK_t/GDP_t are constant at steady state. ■

3.4 Dynamics off the Steady State

Imagine an economy yet to achieve steady state, or one, which was initially along the steady state and is perturbed by a shock so that $\mathcal{K}_0 \neq \mathcal{K}^*$. For simplicity, let us limit ourselves to displacement of the model economy in the local neighborhood of the steady state.

As shown in Appendix B, the steady state is saddle-path stable, and, along the saddle path,

Proposition 9 *As $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$, over time, (a) $\mathcal{K}_t$ and $\mathcal{E}_t$ increase or decrease; (b) growth rate of normalized capital falls or rises; and (c) the interest rate decreases or increases.*

This is intuitive. If, for example, the initial normalized capital stock falls short of its long run level, it builds up over time and interest rate falls. It means that the growth rate of capital exceeds its long-run rate and it falls over time towards the steady state rate of growth. This is accompanied by normalized aggregate consumption or total expenditure rising over time, i.e., the growth rate consumption being higher than its long-run rate and falling over time.

Our main objective is to understand the pattern of sectoral output growth rates. To begin with,

Proposition 10 *Sectoral factor allocations over time exhibit the following proportional relationships:*

$$\frac{D_{at}}{D_{mt}} \propto \frac{L_{at}}{L_{mt}} \propto \frac{L_{st}}{L_{mt}} \propto \frac{K_{st}}{K_{mt}} \text{ for all } t. \tag{31}$$

Proof: The first proportionality follows from the ratio of wage to land rental being same across agriculture and manufacturing. Likewise, the equality of wage to capital rental ratio across manufacturing and services implies the third proportionality relation. We have $w_t L_{at} = (1-\gamma)p_{at}Q_{at} = (1-\gamma)\phi_a E_t$, and, similarly, $w_t L_{st} = (1-\eta)\phi_s E_t$. Thus, the L_{at}/L_{st} ratio is constant over time, implying the second proportionality relation. ■

Remarks

a. In particular, the second proportionality says that labor employment growth rate is same in services and agriculture.

b. Expression (31) does not imply that the ratios remain constant over time. For instance, if L_{at}/L_{mt} rises over time, so would other ratios.

As shown in Appendix B

Proposition 11 *The ratios in (31) increase or decrease with time as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$,*

an immediate corollary of which is that

Corollary 6 $g_{Ks} \gtreqless g_{Km}$, $g_{La} = g_{Ls} \gtreqless g_L \gtreqless g_{Lm}$ *and* $g_{Da} \gtreqless 1 \gtreqless g_{Dm}$ *according as* $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$.

It says that if initially an economy is relatively capital-scarce, as capital accumulation takes place more feverishly, relatively more capital is employed in the services sector. However, this outcome does *not* depend on the services sector being more capital intensive. As capital grows at a rate higher than its long-run rate, and, capital goods are same as manufactures, relatively less manufactures are available for consumption. Insofar as it leads to a substitution in consumption towards services, there is a higher relative demand for capital in producing services, compared to manufacturing. In equilibrium, there is relatively more capital used in the services sector and relatively less capital employed in the manufacturing sector. In turn, relatively less labor and land are used in manufacturing. That is, land moves away from manufacturing to agriculture.

How do sectoral output growth rates compare with their long run growth rates?

Proposition 12 *Output growth rates in agriculture and services sectors are higher or less than the respective steady state growth rates according as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$, whereas the growth rate of manufacturing output cannot be ranked vis-a-vis its long-run growth rate.*[13]

Suppose $\mathcal{K}_0 < \mathcal{K}^*$. Corollary 6 implies that labor and capital employment in the services sector grow faster than aggregate labor and aggregate capital respectively, and, in view of Proposition 9, the growth rate of capital exceeds its long-run rate. Thus, the service output growth rate must exceed its long-run rate. Labor employment in agriculture also grows faster than aggregate labor and land employment in agriculture increases, whereas it remains constant in the steady state. It follows that output growth in agriculture exceeds its long run rate. The growth rates of labor and capital in manufacturing are lower than those of aggregate labor and capital respectively, while the growth rate of capital exceeds its long-run rate. Hence, the output growth rate of manufacturing cannot be ranked unambiguously against its long-run rate.[14] However,

Proposition 13 *The growth rate of manufacturing output is less or greater than that of aggregate capital as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$.*

It is because labor and capital employment growth rates in manufacturing fall short or exceed the respective aggregate growth rate according as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$.[15]

Non-Balanced Growth Decomposition

As seen in Section 4.2, differences in long-run sectoral growth rates have three components: TFP growth differences, differences in capital intensity and those in land intensity. To understand non-balanced growth across sectors off the steady state we rewrite the sectoral production functions as

$$Q_{at} = A_t \bar{D}^{\gamma} L_t^{1-\gamma} d_{at}^{\gamma} l_{at}^{1-\gamma};\ Q_{mt} = M_t \bar{D}^{\alpha} K_t^{\beta} L_t^{1-\alpha-\beta} d_{mt}^{\alpha} k_{mt}^{\beta} l_{mt}^{1-\alpha-\beta};\ Q_{st} = S_t K_t^{\eta} L_t^{1-\eta} k_{st}^{\eta} l_{st}^{1-\eta}, \tag{32}$$

where $d_{it} \equiv D_{it}/\bar{D}$, $l_{it} \equiv L_{it}/L_t$ and $k_{it} \equiv K_{it}/K_t$ are the respective land, labor and capital shares for sector i. The aim is to first decompose growth rate differences

[13]See Appendix B.

[14]Numerical simulations, using parameters of the USA economy and if $\mathcal{K}_0 < \mathcal{K}^*$, indeed show during the transition periods the growth rate of manufacturing output is less than its steady state growth rate.

[15]See Appendix B.

into long-run and short-run components and characterize the latter in particular. It is however not feasible to express the latter in terms of closed-form functions of the underlying exogenous sources of growth and factor intensity differentials.

Production expressions (32) make clear that short-run effects work through (a) the transitory or the short-run component of the evolution of the capital stock, and, (b) changes in sectoral factor employment shares. Needless to say, (a) and (b) both vanish in the long run.

By definition, $g_{Kt} = g_{\mathcal{K}_t} g^{\circ} = g^{\circ} + \tilde{g}_{\mathcal{K}_t} g^{\circ}$, where the first term is the long-run component of capital growth and the second its short-run component. Using this, (32) yields

$$\tilde{g}_{Qm} - \tilde{g}_{Qa} = \underbrace{[\tilde{g}_M - \tilde{g}_A] + \left(\gamma - \frac{\alpha}{1-\beta}\right)\tilde{g}_L + \frac{\beta}{1-\beta}\tilde{g}_M}_{\text{long run}} \\ + \underbrace{\beta\tilde{g}_{\mathcal{K}_t} + (\alpha\tilde{g}_{dm} - \gamma\tilde{g}_{da}) + [(1-\alpha-\beta)\tilde{g}_{lm} - (1-\gamma)\tilde{g}_{la}] + \beta\tilde{g}_{km}}_{\text{short run}} \tag{33}$$

$$\tilde{g}_{Qs} - \tilde{g}_{Qm} = \underbrace{[\tilde{g}_S - \tilde{g}_M] + \frac{\alpha(1-\eta)}{1-\beta}\tilde{g}_L + \frac{\eta-\beta}{1-\beta}\tilde{g}_M}_{\text{long run}} \\ + \underbrace{(\eta-\beta)\tilde{g}_{\mathcal{K}_t} - \alpha\tilde{g}_{dm} + [(1-\eta)\tilde{g}_{ls} - (1-\alpha-\beta)\tilde{g}_{lm}] + [\eta\tilde{g}_{ks} - \beta\tilde{g}_{km}]}_{\text{short run}}. \tag{34}$$

Note that the effect of short run growth component of aggregate capital on sectoral growth differences (through the coefficient of $\tilde{g}_{\mathcal{K}_t}$) depends on differences in capital intensity across the sectors. When $\mathcal{K}_0 < \mathcal{K}^*$, $\tilde{g}_{\mathcal{K}_t} > 0$ and thus it is a factor contributing towards growth differences between services and manufacturing and between manufacturing and agriculture.

Next consider land-use changes. We already know, when $\mathcal{K}_0 < \mathcal{K}^*$, that $\tilde{g}_{dm} < 0 < \tilde{g}_{da}$. These changes tend to narrow the difference between growth rates of manufacturing and agriculture outputs but widen that between growth rates of services and manufacturing output. Changes in labor employment shares and capital shares have similar effects. We have $\tilde{g}_{la} = \tilde{g}_{ls} > 0 > \tilde{g}_{lm}$ and $\tilde{g}_{ks} > 0 > \tilde{g}_{km}$, which tend to narrow $\tilde{g}_{Qm} - \tilde{g}_{Qa}$ and widen $\tilde{g}_{Qs} - \tilde{g}_{Qm}$. Thus, changes in labor shares and capital shares - which are short run effects - tend to reduce the output growth differences between manufacturing and agriculture, and raise the growth differences between services and manufacturing.

Opposite effects hold if $\mathcal{K}_0 > \mathcal{K}^*$.

The overall implication is that when $\mathcal{K}_0 <$ (resp. $>$) $\mathcal{K}^*$, the short-run movements tend to unambiguously enhance (resp. reduce) growth difference between services and manufacturing, whereas the effect on growth differential between manufacturing and agriculture is likely to be relatively small, possibly negative (resp. positive). Another way to put it is that in a capital poor economy, the services sector grows faster than manufacturing sector in all periods, whereas the output growth ranking between manufacturing and agriculture is ambiguous. It is possible that the sum of contributions of changes in land-use, labor shares and capital shares may outweigh long-run effects and in the short run agriculture grows faster than manufacturing over some interval of time.

3.5 A Simulation Analysis

Analytically we were able to compare growth rates of service and agricultural sectors to their steady state growth rates, but such comparison for the manufacturing sector was not possible. The dynamics of sectoral growth rates could not be characterized. We now undertake a simulation analysis to further our understanding of the evolution of sectoral growth rates as well as obtain a quantitative assessment of non-balanced growth decompositions. The main findings are:

a. The differences in sectoral TFP growth rates have the largest contribution towards long-run output growth differences.

b. Land intensity differences play a much more important role in manufacturing-agriculture output growth gap than in services-manufacturing output growth gap.

c. In transition periods, the manufacturing output growth rate is less than its long run growth rate.

d. The aggregate capital changes and capital use changes together explain a signification fraction of short run output growth differences.

e. Land use changes, even in short run, are more crucial to the manufacturing-agriculture output growth gap as compared to services-manufacturing output growth gap.

f. We perform robustness test with respect to α, the parameter which could not be precisely estimated owing to lack of data availability. We find that an increase in manufacturing land intensity, α, has very little effect on transition growth rates, but significantly reduces manufacturing-agriculture output growth difference and significantly increases services-manufacturing output growth gap.

We choose parameter values and initial conditions in reference to the USA over the period 1977-2010. We have already noted in Table 1 that USA is an exception to the stylized sectoral output growth ranking witnessed across the countries. In USA while the services sector is the fastest growing sector, the growth rate is slowest in the manufacturing sector (not agriculture sector). On the basis of eq. (30), the anomaly can be explained by the fact that the TFP growth in agriculture sector must have been much higher than that in manufacturing. In this simulation exercise we aim to quantify the contribution of other factors.

Choice of Parameters

From the EUKLEMS dataset, we interpret the agriculture sector as the 'Agriculture, forestry, fishing, and hunting' industry and the manufacturing sector is the 'Manufacturing' industry. The services sector includes 'Wholesale and Retail Trade', 'Transportation and Storage', 'Accommodation and Food Services', 'Information and Communication', 'Financial and Insurance', 'Information and Communication', 'Financial and Insurance', 'Real Estate', 'Professional' and 'Community' services. For labor intensity in production, we take the average of the wage compensation to gross output ratio for the period 1977-2010. It gives $\gamma = 0.77$, $\alpha + \beta = 0.78$ and $\eta = 0.59$ which are the respective non-labor shares in the agriculture, manufacturing and services sectors. We do not have any information that distinguishes land and capital intensities of manufacturing sector. Based on our earlier discussion where we hypothesize that the capital intensity differences between manufacturing and services sector would most likely be small, we assume $\beta = 0.55$ (reasonably close to $\eta = 0.59$). This implies $\alpha = 0.23$. Note that the values chosen for factor intensities satisfy Assumptions (2) and (3).

Total labor force is captured by the 'full-time and part-time employees' data from the Bureau of Labor Statistics. We calculate the compounded annual growth rate of total employment for 1977-2010 and choose it as the population growth rate, leading to $g_L = 1.012$, i.e. an average annual growth rate of 1.2%. The standard methodology to calculate sectoral TFP growth rates are such that they should match the sectoral output growth rates (Acemoglu and Guerrieri (2008)). We find that in the latter years of the period 1977-2010, the services sector grew the fastest, followed by agriculture

Table 4: Parameter Values for Numerical Simulation

α	β	γ	η	ϕ_a	ϕ_m	ϕ_s	ρ	g_A	g_M	g_S	g_L	$\bar{D}$
0.23	0.55	0.77	0.59	0.01	0.15	0.84	0.98	1.0117	1.0035	1.015	1.012	10

and manufacturing output (EUKLEMS dataset: $g_{Qa} = 1.0145$, $g_{Qm} = 1.0138$ and $g_{Qs} = 1.0283$). As noted previously, unlike the stylized tend, in the US the agriculture sector grows faster than manufacturing. This is primarily driven by the growing energy demand for ethanol production from corn and is, so far, a specific feature of the US economy. These output growth rates yield the total factor productivity growth rates as $g_A = 1.0117$, $g_M = 1.0035$ and $g_S = 1.0150$.[16,17] Note, the manufacturing TFP growth rate is lower than the growth rate of agriculture TFP.

The preference parameters, the ϕ's, are taken from the average sectoral shares in gross output: $\phi_a = 0.01$, $\phi_m = 0.15$ and $\phi_s = 0.84$.

The standard parameter value for annual discount rate is adopted, giving $\rho = 0.98$ (Acemoglu and Guerrieri (2008)). Total US land area is 10 million square km, so we choose $\bar{D} = 10$. The parameter values are summarized in Table 4.

Choice of Initial Values

As we characterize the transitional dynamics *near* the steady state, the value of initial normalized capital stock is taken to be near the steady state. In this capital poor economy, we assume $\mathcal{K}_0 = 0.8\mathcal{K}^*$. Note, $\mathcal{K}^*$ depends on parameter values, which we already know. The total labor force in 1977 was about 92 million workers, so we take $L_0 = 92$. The TFP parameters are calculated so that they match the output levels, $A_0 = 836$, $M_0 = 2.33$ and $S_0 = 2.35$.The initial normalized capital stock and the initial manufacturing productivity level together yield $K_0 = 1.57 * 10^4$. The initial values are tabulated in Table 5.

Overall Trends

Consistent with Proposition 9 and Corollary 6, with low initial capital stock, simulations show that normalize capital and normalized expenditure grow over time,

[16]Ngai and Pissarides (2008) calculate the sectoral TFP growth ranking from change in relative prices. They find that for USA in the period 1930-2004, TFP growth is highest in agriculture, followed by manufacturing and services. It is known that the measurement of TFP growth depends on what factors are employed in the production of good. Like most existing estimates of sectoral TFP growth, Ngai and Pissarides (2008) also do not incorporate the differential use of land across different sectors. Hence their measures of TFP growth does not perfectly match our estimates. We resort to first principles to calculate the sectoral TFP growth in accordance with our model.

[17]From the data on output growth differences across sectors and the computed TFP growth rates, it is clear that output growth rate differences and TFP growth rate differences are not in sync.

Table 5: Initial Values for Numerical Simulation

L_0	K_0	A_0	M_0	S_0
92	$1.57 * 10^4$	836	2.33	2.35

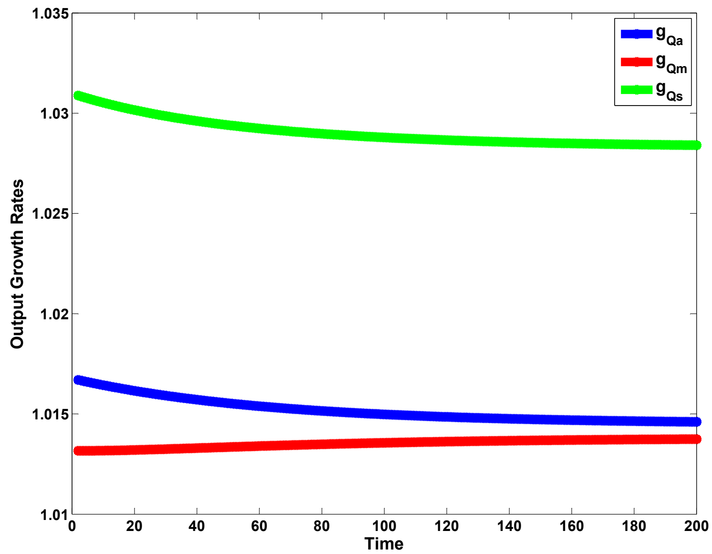

Figure 2: Sectoral Output Growth Rates over Time

interest rate falls, the factor proportions grow over time and become skewed towards the manufacturing sector. Moreover, in the light of Proposition 12, the initial growth rates of agriculture and services outputs are higher than the respective steady state growth rates.

In addition to theoretical predictions, simulations show that output growth rates of agriculture and services decline over time to their respective steady state growth rates, and, the manufacturing output growth rate rises over time. These are illustrated in Figure 2.

Non-Balanced Growth

Figure 2 also indicates that service sector's growth rate remains highest, followed by agriculture and manufacturing. This is consistent with our discussion of long-run and short-run elements of non-balanced growth decomposition. The computed TFP growth rates are such that $g_S > g_M$. Thus, following our discussion on services-manufacturing growth decomposition based on eq. (34) in the case of capital poor economy, we find that all long-run and short-run factors of output growth differences favour services sector as compared to the manufacturing sector. This explains why the growth rate of services sector is higher than that of manufacturing in both short-run and long-run.

Simulations show that agriculture grows faster than manufacturing. The long-run TFP growth rankings ($g_A > g_M$) as well as short-run growth favour higher growth in agriculture, based on eq. (33). In this case, it appears that these effects are so strong that agriculture sector grows faster than manufacturing in all periods.

Analyzing Non-balanced Growth

Table 6: Non-Balanced Growth Decomposition in the Long Run

Output Growth Differences (%)	Share of TFP Differences (%)	Share of Land-Intensity Differences (%)	Share of Capital-Intensity Differences (%)
Manu. over Agri.	[-] 52.3	19.9	27.8
Serv. over Manu.	80.2	17.6	2.2

We now investigate to find out the strengths of the different sources of non-balanced sectoral growth in our simulated economy. We present the magnitudes of the long-run sources of sectoral output growth gaps in Table 6. The three sources of long-run output growth are (a) differences in sectoral TFP growth rates, (b) joint effect of capital intensity difference and TFP growth in manufacturing, and (c) joint effect of land intensity difference and labor growth. Of these different sources of long-run output growth differences between manufacturing and agriculture sector, we find that differences in TFP growth differences have the largest contribution, albeit negative. This explains how agriculture may grow faster than manufacturing even though the factor intensity differences drive growth in opposite direction. The growth decomposition is slightly different for the services-manufacturing output growth gap. Given the direction of productivity growth ranking, and factor intensity differences between services and manufacturing, it follows from eq. (34) that all the sources favour higher growth in services as compared to manufacturing. Here also we find that the TFP growth differences have the largest contribution to the services-manufacturing output growth gap.

In Table (7) we depict the four sources of short run output growth differentials – transitory changes in aggregate capital, changes in land-use, changes in labor employment and changes in capital. As seen in eqs. (33) and (34), land-use changes affect with land intensity, capital changes affect in conjunction with capital intensity and changes in labor employment affect together with labor intensity. The direction of the different effects are same as discussed in the analytical discussion of growth decomposition. Through simulations we find the relative strengths of the different sources of short run growth differentials. We find that in the manufacturing-agriculture growth gap, changes in aggregate capital have the largest impact while in the services-manufacturing output growth differences, the effect of changes in capital use is largest. Land-use changes are more important within the manufacturing-agriculture growth ranking, but possibly not so much so in the services-manufacturing growth differences.

Table 7: Non-Balanced Growth Decomposition in the Short Run (in %) for time period $t = 1$

Output Growth Differences	Aggregate Capital Changes	Land Use Changes	Labor Use Changes	Capital Use Changes
Manu. over Agri.	45.6	[-]31.7	[-]9.6	[-]13.1
Serv. over Manu.	16.8	1.4	26.1	55.7

Robustness

How sensitive are the results to the parameter values and initial conditions? Of the thirteen parameters which fully specify the model (see Table 4), the values of all except α are taken from actual data. Due to lack of data on land-use α could not be determined. We run sensitivity checks with respect to α in a way such that a change in α keeps $\alpha + \beta$ (whose value is derived from data) unchanged. This ensures that returns to scale in manufacturing production remains unity. The initial values of employment and productivity parameters were taken from data (see Table 5), but the initial capital stock was considered to be near its steady state value. We have already observed that the initial capital stock does not influence the near-the-steady-state dynamics. Hence, we conduct robustness checks only with respect to α.

We find that a change in α by $\pm 10\%$, changes the convergence rate (i.e., $1/\mu_2$) by ∓ 0.01 percentage points. Thus, the convergence rate is fairly robust to parameter changes as well as changes in initial conditions.

The long run sectoral output growth gaps are sensitive to changes in α. We show this in Table 8. An increase in manufacturing land intensity by 10% decreases manufacturing-agriculture output growth gap by about 0.07 percentage points and

Table 8: Sensitivity Analysis

Parameter (% Change)		Change in Convergence Rate (in percentage points)	Change in steady state output growth gaps (in percentage points)[18]	
			Manu. - Agri.	Serv. - Manu.
α	+10%	-0.012	-0.067	0.027
	-10%	0.015	0.074	-0.030

it increases services-manufacturing output growth differentials by about 0.03 percentage points. Given that the initial manufacturing-agriculture output growth gap was -0.08% and services-manufacturing output growth gap was about 1.4%, the α-driven growth-gap changes are significant. Notice as the land intensity differences are smaller between manufacturing and agriculture output as compared to that services and manufacturing outputs, the effect of change in α on the former output growth gap is also lesser. This brings to focus on how small differences in α across countries would have significant effects on their inter-sectoral growth dynamics.

4 Land Transactions Restrictions

There are restrictions on conversion of agriculture land for industrial use. For example, land acquisition laws in India, Nigeria and other developing countries is fairly stringent and demands significant compensation.

For the model, it implies that there is no sectoral land transactions. Land deals occur within sectors where ownership of agriculture or manufacturing land changes hands.

4.1 Static Equilibrium

The unit cost functions for agriculture and manufacturing sectors change: $c_a(r_{at}, w_t)/A_t = r_{at}^{\gamma} w_t^{1-\gamma}/A_t$ and $c_m(r_{mt}, r_t, w_t)/M_t = r_{mt}^{\alpha} r_t^{\beta} w_t^{1-\alpha-\beta}/M_t$, where r_{at} and r_{mt} are rental rates for agriculture and manufacturing lands respectively.

Assumptions 2 and 3 hold. The zero profit conditions remain (14)-(16), except now

[18] The baseline steady state output growth gap for manufacturing - agriculture was -0.08% and for services - manufacturing was 1.45%.

land rental rates differ across sectors. The land market clearing condition changes to

$$\frac{1}{A_t}\frac{\partial c_a(r_{at},w_t)}{\partial r_{at}}Q_{at}=\bar{D}_a, \qquad \frac{1}{M_t}\frac{\partial c_m(r_{mt},r_t,w_t)}{\partial r_{mt}}Q_{mt}=\bar{D}_m, \tag{35}$$

where $\bar{D}_a$ and $\bar{D}_m$ are exogenously given agricultural and manufacturing land restrictions. Further, $\bar{D}_a+\bar{D}_m=\bar{D}$.

The household consumption demand functions are stated in eq. (5). The static equilibrium is

Proposition 14

$$L_{at}=L_t\cdot h_{La}(\mathcal{E}_t,\mathcal{K}_t);\quad L_{mt}=L_t\cdot h_{Lm}(\mathcal{E}_t,\mathcal{K}_t);\quad L_{st}=L_t\cdot h_{Ls}(\mathcal{E}_t,\mathcal{K}_t);$$
$$K_{mt}=M_t^{\frac{1}{1-\beta}}\bar{D}^{\frac{\alpha}{1-\beta}}L_t^{\frac{1-\alpha-\beta}{1-\beta}}\cdot h_{Km}(\mathcal{E}_t,\mathcal{K}_t);K_{st}=M_t^{\frac{1}{1-\beta}}\bar{D}^{\frac{\alpha}{1-\beta}}L_t^{\frac{1-\alpha-\beta}{1-\beta}}\cdot h_{Ks}(\mathcal{E}_t,\mathcal{K}_t);$$
$$Q_{at}=A_t\bar{D}^{\gamma}L_t^{1-\gamma}\cdot h_{Qa}(\mathcal{E}_t,\mathcal{K}_t);\quad Q_{mt}=M_t^{\frac{1}{1-\beta}}\bar{D}^{-\frac{1-\alpha-\beta}{1-\beta}}L_t^{\frac{1-\alpha-\beta}{1-\beta}}\cdot h_{Qm}(\mathcal{E}_t,\mathcal{K}_t);$$
$$Q_{st}=S_tM_t^{\frac{\eta}{1-\beta}}\bar{D}^{\frac{\alpha\eta}{1-\beta}}L_t^{\frac{1-\alpha\eta-\beta}{1-\beta}}\cdot h_{Qs}(\mathcal{E}_t,\mathcal{K}_t);$$
$$r_{at}=M_t^{\frac{1}{1-\beta}}\bar{D}^{1\frac{1-\alpha-\beta}{1-\beta}}L_t^{\frac{1-\alpha-\beta}{1-\beta}}\cdot h_{ra}(\mathcal{E}_t,\mathcal{K}_t);\quad r_{mt}=M_t^{\frac{1}{1-\beta}}\bar{D}^{\frac{\alpha}{1-\beta}}L_t^{\frac{1-\alpha-\beta}{1-\beta}}\cdot h_{rm}(\mathcal{E}_t,\mathcal{K}_t);$$
$$w_t=M_t^{\frac{1}{1-\beta}}\bar{D}^{\frac{\alpha}{1-\beta}}L_t^{\frac{-\alpha}{1-\beta}}\cdot h_w(\mathcal{E}_t,\mathcal{K}_t);\quad r_t=h_r(\mathcal{E}_t,\mathcal{K}_t);$$
$$p_{at}=A_t^{-1}M_t^{\frac{1}{1-\beta}}\bar{D}^{-\frac{(1-\beta)\gamma-\alpha}{1-\beta}}L_t^{\frac{(1-\beta)\gamma-\alpha}{1-\beta}}\cdot h_{pa}(\mathcal{E}_t,\mathcal{K}_t);$$
$$p_{st}=S_t^{-1}M_t^{\frac{1-\eta}{1-\beta}}\bar{D}^{\frac{\alpha(1-\eta)}{1-\beta}}L_t^{-\frac{\alpha(1-\eta)}{1-\beta}}\cdot h_{ps}(\mathcal{E}_t,\mathcal{K}_t).$$

where $\mathcal{E}$ and $\mathcal{K}$ are normalized expenditure and capital respectively.

Household's dynamic optimization problem is unchanged, except now the intertemporal budget allows for within sector land transactions, not unrestricted land transactions.

$$\text{Maximize}\sum_{t=0}^{\infty}\rho^t(\ln E_t-\phi_a\ln p_{at}-\phi_s\ln p_{st}),$$

subject to $E_t+K_{t+1}-K_t+p^D_{at}(D_{at+1}-D_{at})+p^D_{mt}(D_{mt+1}-D_{mt})\leq w_tL_t+r_tK_t+r_{at}D_{at}+r_{mt}D_{mt}$,

where U_t is substituted by its indirect form. Given L_0, D_{m0} and K_0, the household chooses $\{E_t\}_0^\infty$, $\{D_{at}\}_1^\infty$, $\{D_{mt}\}_1^\infty$ and $\{K_t\}_1^\infty$. We obtain the standard Euler equation

$$\frac{E_{t+1}}{E_t}=\rho(1+r_{t+1}). \tag{36}$$

There are three transversality conditions: (22) and

$$\lim_{t\to\infty} \frac{\rho^t p_{at}^D D_{at+1}}{E_t} = 0, \qquad \lim_{t\to\infty} \frac{\rho^t p_{mt}^D D_{mt+1}}{E_t} = 0, . \tag{37}$$

The no-arbitrage condition between the assets are FILL IN

$$\tag{38}$$

The law of motion of capital is unchanged.

$$K_{t+1} = \frac{r_t K_t}{\beta} - \frac{\phi_m \beta + \phi_s \eta}{\beta} E_t + K_t. \tag{39}$$

This equation, the Euler equation, the no-arbitrage condition as well as the transversality condition form the basis of the dynamic system. Using the expressions in Proposition 14 and defining $g^\circ \equiv g_M^{\frac{1}{1-\beta}} g_L^{\frac{1-\alpha-\beta}{1-\beta}}$, eqs. (36) and (39) can be expressed as

$$\begin{aligned} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} &= \frac{\rho\left[1 + h_r(\mathcal{E}_{t+1}, \mathcal{K}_{t+1})\right]}{g^\circ} \\ \mathcal{K}_{t+1} &= \frac{1}{g^\circ} \cdot \left[\frac{h_r(\mathcal{E}_t, \mathcal{K}_t)\mathcal{K}_t}{\beta} - \frac{\phi_m \beta + \phi_s \eta}{\beta} \mathcal{E}_t + \mathcal{K}_t\right]. \end{aligned} \tag{40}$$

These two equations form the core dynamic system of the economy.

4.2 Steady State

This is defined by $\mathcal{E}_t = \mathcal{E}^*$ and $\mathcal{K}_t = \mathcal{K}^*$. Eqs. (40) yield

$$\begin{aligned} r^* = h_r(\mathcal{E}^*, \mathcal{K}^*) &= \frac{g^\circ}{\rho} - 1 \\ \frac{\mathcal{E}^*}{\mathcal{K}^*} &= \frac{(g^\circ - 1)(1 - \beta\rho) + 1 - \rho}{\rho\nu}, \text{ where } \nu \equiv \phi_m \beta + \phi_s \eta < 1. \end{aligned} \tag{41}$$

The former is the modified golden rule, whereas the latter defines the trajectory where savings grow at a constant rate. Eqs. (41) implicitly solve $(\mathcal{E}^*, \mathsf{K}^*)$. In Appendix B we show that the steady state exists and it is unique.

Proposition 15 *Along the steady state,*

$$g_{Qa}^* = g_A g_L^{1-\gamma};\ g_K^* = g_{Qm}^* = g^\circ = g_M^{\frac{1}{1-\beta}} g_L^{\frac{1-\alpha-\beta}{1-\beta}};\ g_{Qs}^* = g_S g_M^{\frac{\eta}{1-\beta}} g_L^{\frac{1-\alpha\eta-\beta}{1-\beta}}. \tag{42}$$

Steady State Points – comparisons with model B

- rental rate on capital unchanged
- As $\bar{D}_m < D_m^*$, it implies $\mathcal{R}_m > \mathcal{R}_D$, $\mathcal{R}_D > \mathcal{R}_a$, $\mathcal{K}$ and $\mathcal{E}$ are lower in model C. Manufacturing and services output are lower, while agriculture output is higher in model C. Wage rate, price of agriculture goods and services all are lower. Opposite effects when $\bar{D}_m > D_m^*$. Labor allocation is unaffected by land trade restrictions.

5 Concluding Remarks

In recent decades, the services sector has recorded highest output and employment growth in almost all countries. Sectoral growth is lead by services sector and followed by manufacturing and agriculture, in that order. This phenomenon is mainly attributed to demand-side factors like non-homothetic preferences. There are a few supply-side explanations for the three sector growth ranking, but they are based on sectoral TFP growth rankings. As the ranking of sectoral TFP growth rates is not uniform across developing and developed countries, these explanations are applicable to mostly developed countries. In this paper, we propose a supply-side phenomenon which explains the sectoral output growth ranking and is not country-specific. We regard that differences in factor intensities in goods production explain non-balanced growth. In particular, we postulate that given limited supply of land, the differences in land intensity across sectors manifest into differences in sectoral growth – highest growth of services (the least land intensive) sector followed by manufacturing and agriculture (the most land intensive sector).

Our analysis began with a three-sector model with only land and labor as inputs. Labor and sectoral TFP grow over time at exogenous rates. We showed that differences in growth rates of sectoral outputs are due to differences in sectoral TFP growth rate as well as due to differences in sectoral land intensity. If TFP growth differences are not large, then land intensity differences determine the inter-sectoral growth ranking. Further, it is possible that output growth ranking may be exactly opposite of TFP growth ranking.

We also extended this basic model by including capital in the production of manufacturing and services goods and incorporating endogenous accumulation of capital. Labor growth and sectoral TFP growth continue to be the sources of long run growth. Now, capital intensity differences in addition to differences in sectoral TFP growth rates and land intensity differences, all three contribute to the sectoral output growth differences. In transitional periods, the factor movements are entwined and it is not possible to characterize the exact trajectories of the economy. We simulate the model to analyze short-run trends. We find that if the initial capital stock is low, capital grows at a rate higher than its long-run rate and, as capital goods are same as manufactures, relatively less manufactures are available for consumption. Insofar as it leads to a substitution in consumption towards agriculture and services, there is a lower relative demand for capital, land and labor in producing manufacturing, as compared to other goods.

We also decompose sectoral growth differentials to analyze the strengths of the different sources of growth. We find that TFP growth differences play a significant role in explaining long-run output growth differences. In short-run, aggregate capital changes and land-use changes are the two largest contributors towards the manufacturing-agriculture output growth gap. In the short-run services-manufacturing output growth differences, capital-use changes and then the labor-use changes play the largest roles.

An interesting question ahead would be to explore the role of housing in sectoral non-balanced growth. In the extended model, the normalized capital and normalized expenditure dynamics was independent of changes in land price. However, if we bring in housing consumption, it may make the dynamic system more involved and make land a more important feature of a growing economy.

Appendix A Proof of Proposition 1

We use Jones's "hat" calculus, where equations are log-differentiated and proportionate change variables are indicated by a '$\hat{}$.' Zero-profit conditions imply

$$\begin{aligned}\gamma\hat{r}_{Dt}+(1-\gamma)\hat{w}_t &= \hat{p}_{at}+\hat{A}_t\\ \alpha\hat{r}_{Dt}+(1-\alpha)\hat{w}_t &= \hat{M}_t\\ \hat{w}_t &= \hat{p}_{st}+\hat{S}_t.\end{aligned}\tag{A.1}$$

Log-differentiating full-employment conditions,

$$\begin{aligned}\lambda_{Da}\left[\hat{Q}_{at}-(1-\gamma)(\hat{r}_{Dt}-\hat{w}_t)\right]+\lambda_{Dm}\left[\hat{Q}_{mt}-(1-\alpha)(\hat{r}_{Dt}-\hat{w}_t)\right]\\ =\hat{\bar{D}}+\lambda_{Da}\hat{A}_t+\lambda_{Dm}\hat{M}_t\\ \lambda_{La}\left[\hat{Q}_{at}+\gamma(\hat{r}_{Dt}-\hat{w}_t)\right]+\lambda_{Lm}\left[\hat{Q}_{mt}+\alpha(\hat{r}_{Dt}-\hat{w}_t)\right]+\lambda_{Ls}\hat{Q}_{st}\\ =\bar{L}_t+\lambda_{La}\hat{A}_t+\lambda_{Lm}\hat{M}_t+\lambda_{Ls}\hat{S}_t,\end{aligned}\tag{A.2}$$

where λ_{Nj} is share of factor N employed in sector j.

Market-clearing conditions imply

$$\begin{aligned}\hat{Q}_{mt}-\hat{Q}_{at} &= \hat{p}_{at}\\ \hat{Q}_{mt}-\hat{Q}_{st} &= \hat{p}_{st}.\end{aligned}\tag{A.3}$$

Eqs. (A.1) and (A.3) imply

$$\begin{aligned}(\gamma-\alpha)(\hat{r}_{Dt}-\hat{w}_t) &= \hat{Q}_{mt}-\hat{Q}_{at}+\hat{A}_t-\hat{M}_t\\ \alpha(\hat{r}_{Dt}-\hat{w}_t) &= \hat{Q}_{st}-\hat{Q}_{mt}+\hat{M}_t-\hat{S}_t.\end{aligned}\tag{A.4}$$

Substituting the above into (A.2), we solve $\hat{Q}_{at}$, $\hat{Q}_{mt}$ and $\hat{Q}_{st}$:

$$\begin{aligned}\hat{Q}_{at} &= \hat{A}_t+\gamma\hat{\bar{D}}+(1-\gamma)\hat{L}_t\\ \hat{Q}_{mt} &= \hat{M}_t+\alpha\hat{\bar{D}}+(1-\alpha)\hat{L}_t\\ \hat{Q}_{st} &= \hat{S}_t+\hat{L}_t.\end{aligned}\tag{A.5}$$

Eqs. (A.5) imply the proportionality relations for outputs claimed in Proposition 1. Substituting (A.5) into (A.3) yields proportionality relations for relative prices.

Proportionality relations for factor prices follow from (A.1) once we know those of relative prices. Those for output and prices implies the proportionality relation for E_t. We have

$$
\begin{aligned}
\hat{D}_{at} &= \hat{Q}_{at} - (1-\gamma)(\hat{r}_{Dt} - \hat{w}_t) - \hat{A}_t = \hat{\bar{D}} \\
\hat{D}_{mt} &= \hat{Q}_{mt} - (1-\alpha)(\hat{r}_{Dt} - \hat{w}_t) - \hat{M}_t = \hat{\bar{D}} \\
\hat{L}_{at} &= \hat{Q}_{at} + \gamma(\hat{r}_{Dt} - \hat{w}_t) - \hat{A}_t = \hat{L}_t \\
\hat{L}_{mt} &= \hat{Q}_{mt} + \alpha(\hat{r}_{Dt} - \hat{w}_t) - \hat{M}_t = \hat{L}_t \\
\hat{L}_{st} &= \hat{Q}_{st} - \hat{S}_t = \hat{L}_t,
\end{aligned}
$$

where we have made use of (A.4) and (A.5).

The above expressions imply the proportionality relations for sectoral factor employment in Proposition 1. The proportionality relations of relative prices follow from (A.3). In turn, those of input prices follow from (A.1). Finally, since, in equilibrium, $E_t \propto Q_{mt}$; hence their proportionality relations are the same.

Appendix B Capital Accumulation Model in Section 3

Proof of Proposition 5

The full employment conditions (17)-(19) yield the following expressions of value of sectoral outputs in terms of aggregate earnings of three factors: land, labor and capital.

$$
\begin{aligned}
\theta_1 p_{at} Q_{at} &= [(1-\alpha)\eta - \beta] r_{Dt} \bar{D} - \alpha\eta w_t L_t + \alpha(1-\eta) r_t K_t \\
\theta_1 p_{st} Q_{st} &= \beta(1-\gamma) r_{Dt} \bar{D} - \beta\gamma w_t L_t + [(1-\beta)\gamma - \alpha] r_t K_t \\
\theta_1 Q_{mt} &= -(1-\gamma)\eta r_{Dt} \bar{D} + \gamma\eta w_t L_t - \gamma(1-\eta) r_t K_t.
\end{aligned}
\tag{A.6}
$$

where $\theta_1 \equiv \gamma(\eta - \beta) - \alpha\eta$.

Next, the demand functions (4) along with agriculture and services goods market clearing conditions $L_t C_{at} = Q_{at}; L_t C_{st} = Q_{st}$ imply $p_{at} Q_{at} = \phi_a E_t$; $p_{st} Q_{st} = \phi_s E_t$. Substituting the above into the first two expressions of (A.6) and dividing the resulting equations by $(M_t \bar{D}^{\alpha} L_t^{1-\alpha-\beta})^{1/(1-\beta)}$ and rearranging give rise to

$$\begin{aligned} \beta \mathcal{R}_{dt} &= \alpha r_t \mathcal{K}_t + \theta_2 \mathcal{E}_t \\ \frac{\beta}{\mathcal{R}_{dt}^{\frac{\alpha}{1-\alpha-\beta}}} &= \theta_3 \mathcal{E}_t r_t^{\frac{\beta}{1-\alpha-\beta}} + (1-\alpha-\beta) r_t^{\frac{1-\alpha}{1-\alpha-\beta}} \mathcal{K}_t, \text{ where} \end{aligned} \tag{A.7}$$

$$\begin{aligned} \theta_2 &\equiv \phi_a \beta \gamma - \phi_s \alpha \eta \gtrless 0 \\ \theta_3 &\equiv \phi_a \beta (1-\gamma) - \phi_s [(1-\alpha)\eta - \beta] \gtrless 0, \end{aligned}$$

$\mathcal{K}_t$ and $\mathcal{E}_t$ are as defined in (20) and

$$\mathcal{R}_{dt} = \frac{r_{Dt}}{M_t^{\frac{1}{1-\beta}} \bar{D}^{-\frac{1-\alpha-\beta}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}}}. \tag{A.8}$$

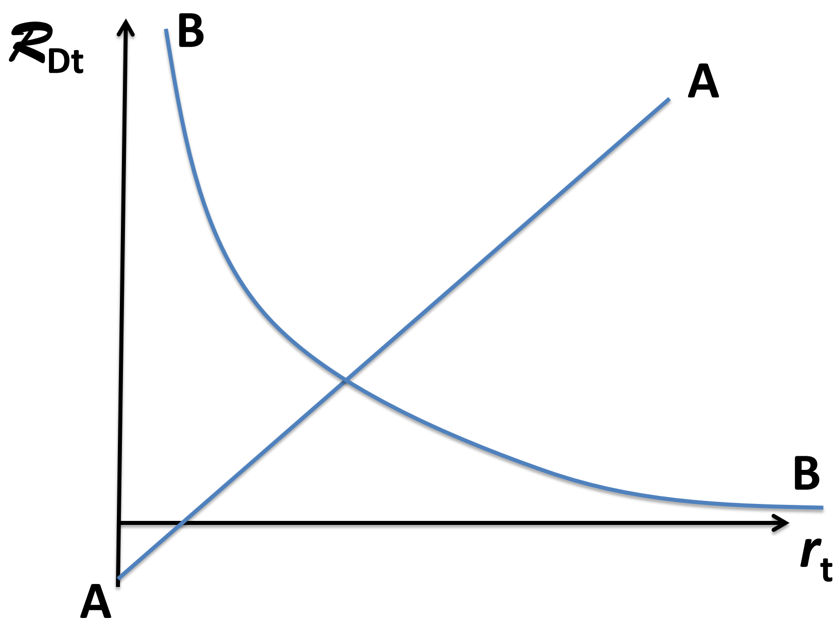

Figure 3: The Reduced Form Static System

We can call $\mathcal{R}_{dt}$ the normalized land rental, just as $\mathcal{K}_t$ and $\mathcal{E}_t$ are the normalized capital stock and total expenditure. (As discussed in the text, all normalized variables become constant or time-invariant along the steady state.) Eqs. (A.7) implicitly solve

$\mathcal{R}_{Dt}$ and r_t as functions of $\mathcal{K}_t$ and $\mathcal{E}_t$. As shown in Figure 3, the first equation in (A.7) defines a upward sloping straightline (AA) relating $\mathcal{R}_{Dt}$ and r_t, whose intercept on the vertical axis may be positive or negative as $\theta_2 \gtrless 0$. The second equation in (A.7) defines a negative locus between $\mathcal{R}_{Dt}$ and r_t (BB), which is asymptotic to the horizontal axis and asymptotic to the vertical line at $r_t = \xi$, where $\xi \geq 0$ depending on the sign and magnitude of θ_3. Hence a unique intersection between AA and BB in the first is assured. That is, solutions to $\mathcal{R}_{Dt}$ and r_t exist and they are unique. Accordingly, let

$$\mathcal{R}_{dt} = f_{rD}(\mathcal{E}_t, \mathcal{K}_t); \quad r_t = f_r(\mathcal{E}_t, \mathcal{K}_t).$$

Following the definition of $\mathcal{R}_{dt}$,

$$r_{Dt} = M_t^{\frac{1}{1-\beta}} \bar{D}^{-\frac{1-\alpha-\beta}{1-\beta}} L_t^{\frac{1-\alpha-\beta}{1-\beta}} \cdot f_{rD}(\mathcal{E}_t, \mathcal{K}_t).$$

The zero-profit condition for the manufacturing sector yields the expression of w_t in Proposition 5, where $f_w(\mathcal{E}_t, \mathcal{K}_t)$ is a function of $f_{rD}(\mathcal{E}_t, \mathcal{K}_t)$ and $f_r(\mathcal{E}_t, \mathcal{K}_t)$. In turn, the zero-profit conditions for the agriculture and service sectors imply the expressions of relative prices, where $f_{pa}(\cdot)$ and $f_{ps}(\cdot)$ are functions of $f_{rD}(\cdot)$, $f_r(\cdot)$ and $f_w(\cdot)$. Substituting factor price and product price expressions into (A.6) gives the output expressions in Proposition 5. Sectoral employment of a factor is a product of the respective factor coefficient - a function of factor prices - and output. Hence expressions of factor prices and outputs lead to the expressions of factor employments in Proposition 5. ■

Existence and Uniqueness of the Steady State

Instead of using the implicit function $f_r(\mathcal{E}_t, \mathcal{K}_t)$, we use the reduced form static system (A.7) for our purpose. Substituting these equations into the steady state relations (41), we obtain two equations in $\mathcal{R}_D^*$ and $\mathcal{K}^*$:

$$\frac{\mathcal{R}_D^*}{\mathcal{K}^*} = \frac{\phi_a \gamma [(g^\circ - 1)(1 - \beta\rho) + (1 - \rho)] + \phi_s \alpha \eta \rho (g^\circ - 1)}{\rho\nu} \tag{A.9}$$

$$\left(\frac{g^\circ - \rho}{\rho}\right)^{-\frac{\beta}{1-\alpha-\beta}} \frac{(\mathcal{R}_D^*)^{-\frac{\alpha}{1-\alpha-\beta}}}{\mathcal{K}^*} = \frac{[\phi_a(1-\gamma) + \phi_s][(g^\circ - 1)(1 - \beta\rho) + (1 - \rho)]}{\rho\nu} + \frac{\phi_s (g^\circ - 1)\rho[(1-\alpha)\eta - \beta]}{\rho\nu} + \frac{\phi_s (g^\circ - \rho)(1 - \eta)}{\rho\nu}. \tag{A.10}$$

Eq. (A.9) is a linear, positively sloped relationship between $\mathcal{R}_D^*$ and $\mathcal{K}^*$, which goes through the origin. Eq. (A.10) defines a decreasing, relationship between $\mathcal{R}_D^*$ and $\mathcal{K}^*$, asymptotic to both axes. Hence an intersection point (steady state) exists and it is unique.

Saddle-Path Stability

We first eliminate $\mathcal{R}_{dt}$ from the static system (A.7) to obtain r_t as a function of normalized expenditure ($\mathcal{E}_t$) and the ratio of capital earnings to expenditure ($r_t\mathcal{K}_t/\mathcal{E}_t \equiv \mathcal{X}_t$):

$$r_t = \frac{\beta^{\frac{1-\beta}{\beta}}}{(\alpha\mathcal{X}_t+\theta_2)^{\frac{\alpha}{\beta}}\left[(1-\alpha-\beta)\mathcal{X}_t+\theta_3\right]^{\frac{1-\alpha-\beta}{\beta}}\mathcal{E}_t^{\frac{1-\beta}{\beta}}} \equiv \Gamma(\underset{-}{\mathcal{E}_t}, \underset{-}{\mathcal{X}_t}). \tag{A.11}$$

At the steady state, both $\alpha\mathcal{X}_t + \theta_2$ and $(1-\alpha-\beta)\mathcal{X}_t + \theta_3$ are positive,[19] and, evaluated at the steady state,

$$\Gamma_{\mathcal{E}} = -\frac{(1-\beta)r^*}{\beta\mathcal{E}^*}; \quad \Gamma_{\mathcal{X}} = -\frac{r^*\Omega}{\beta}$$
$$\text{where } \Omega \equiv \frac{\alpha^2}{\alpha\mathcal{X}^*+\theta_2} + \frac{(1-\alpha-\beta)^2}{(1-\alpha-\beta)\mathcal{X}^*+\theta_3}.$$

In order to prove saddle-path stability, it seems easier to cast the dynamics in the $(\mathcal{E}_t, \mathcal{X}_t)$ space rather in terms of $\mathcal{E}_t$ and $\mathcal{X}_t$. Rearranging our original dynamic system (40) yields

$$g^\circ\mathcal{E}_{t+1} = \rho\left[1+\Gamma(\mathcal{E}_{t+1}, \mathcal{X}_{t+1})\right]\mathcal{E}_t \tag{A.12}$$

[19] We have

$$\mathcal{X}^* = \frac{(g^\circ-\rho)\nu}{(g^\circ-1)(1-\beta\rho)+1-\rho} = \frac{(g^\circ-\rho)(\phi_m\beta+\phi_s\eta)}{(g^\circ-1)(1-\beta\rho)+1-\rho}$$

$$\alpha\mathcal{X}^*+\theta_2 = \frac{\phi_m\beta[\alpha(g^\circ-1)+1-\rho]+\phi_a\beta\gamma[(g^\circ-1)(1-\beta\rho)+1-\rho]+\phi_s\eta[\alpha(g^\circ-1)\beta\rho+1-\rho]}{(g^\circ-1)(1-\beta\rho)+1-\rho} > 0.$$

If $(1-\alpha)\eta \leq \beta$, then $\theta_3 > 0$ and obviously, $(1-\alpha-\beta)\mathcal{X}^*+\theta_3 > 0$. Otherwise, if $(1-\alpha)\eta \leq \beta$

$$\begin{aligned}(1-\alpha-\beta)\mathcal{X}^*+\theta_3 &= \frac{J}{(g^\circ-1)(1-\beta\rho)+1-\rho}, \text{ where}\\ J &\equiv \phi_m\beta(1-\alpha-\beta)(g^\circ-\rho)+\phi_a\beta(1-\gamma)[(g^\circ-1)(1-\beta\rho)+1-\rho]\\ &\quad + \phi_s\{(g^\circ-\rho)\beta(1-\eta)+(g^\circ-1)\beta\rho[(1-\alpha)\eta-\beta]\}\\ &\quad > 0.\end{aligned}$$

$$\rho\left[\mathcal{X}_{t+1}+\frac{\mathcal{X}_{t+1}}{\Gamma(\mathcal{E}_{t+1},\mathcal{X}_{t+1})}\right]=\frac{\mathcal{X}_t}{\beta}+\frac{\mathcal{X}_t}{\Gamma(\mathcal{E}_t,\mathcal{X}_t)}-\frac{\nu}{\beta}. \tag{A.13}$$

Linearizing (A.12) around the steady state, using the expressions of $\Gamma_{\mathcal{E}}$ and $\Gamma_{\mathcal{X}}$ and denoting deviations from steady state by $\varepsilon_t \equiv \mathcal{E}_t - \mathcal{E}^*$ and $\chi_t \equiv \mathcal{X}_t - \mathcal{X}^*$, we have

$$\varepsilon_t = a_{11}\varepsilon_{t+1} + a_{12}\chi_{t+1}, \text{ where} \tag{A.14}$$
$$a_{11} \equiv 1+\frac{\rho(1-\beta)r^*}{\beta g^\circ} > 1; \quad a_{12} \equiv \frac{\rho\mathcal{E}^* r^*\Omega}{\beta g^\circ} > 0.$$

Similarly, eq. (A.13) yields

$$\frac{\rho(1-\beta)\mathcal{X}^*}{\beta r^*\mathcal{E}^*}\varepsilon_{t+1}+\frac{\rho\beta(1+r^*)+\rho\mathcal{X}^*\Omega}{\beta r^*}\chi_{t+1}=\frac{(1-\beta)\mathcal{X}^*}{\beta r^*\mathcal{E}^*}\varepsilon_t+\frac{\beta+r^*+\mathcal{X}^*\Omega}{\beta r^*}\chi_t. \tag{A.15}$$

Substituting the expression of ε_t into the last relation and rearranging give

$$\chi_t = a_{21}\varepsilon_{t+1} + a_{22}\chi_{t+1}. \tag{A.16}$$

where

$$a_{21} \equiv -\frac{\mathcal{X}^*(1-\beta)[\beta g^\circ(1-\rho)+(1-\beta)\rho r^*]}{\mathcal{E}^*\beta g^\circ(\beta+r^*+\mathcal{X}^*\Omega)} < 0$$
$$a_{22} \equiv \frac{\rho\beta^2 g^\circ(1+r^*)+\rho[\beta g^\circ-(1-\beta)r^*]\mathcal{X}^*\Omega}{\beta g^\circ(\beta+r^*+\mathcal{X}^*\Omega)} < 1.$$ [20]

Thus we have the following 2×2 system:

$$\begin{pmatrix}\varepsilon_t\\ \chi_t\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}\varepsilon_{t+1}\\ \chi_{t+1}\end{pmatrix}\equiv M\begin{pmatrix}\varepsilon_{t+1}\\ \chi_{t+1}\end{pmatrix}. \tag{A.17}$$

The stability of our dynamic system depends on the magnitudes of eigen roots the matrix M^{-1}. However, eigen roots of M^{-1} are inverses of those of M. We thus analyze the matrix M. Let μ_1 and μ_2 be its eigen roots. Saddle-path stability is ensured if and only if μ_1 and μ_2 are both real and only one of them has modulus greater than unity. This is proved via the following three lemmas which uses the expressions of the elements of M.

Lemma 1 $\Delta_1 \equiv (a_{11}-a_{22})^2 + 4a_{12}a_{21} > 0$.

[20] It is possible that $a_{22} < 0$.

Proof: We have

$$
\begin{aligned}
\Delta_1 &= (A+B+C)^2 - \frac{4\rho(1-\beta)r^*\mathcal{X}^*\Omega[\beta(1-\rho)g^\circ + (1-\beta)\rho r^*]}{\beta^2 g^{\circ 2}(\beta + r^* + \mathcal{X}^*\Omega)}, \text{ where} \\
&\quad A \equiv \frac{(1-\rho)\beta + (1-\rho\beta)r^* + (1-\rho)\mathcal{X}^*\Omega}{\beta + r^* + \mathcal{X}^*\Omega} > 0 \\
&\quad B \equiv \frac{\rho(1-\beta)r^*}{\beta g^\circ} > 0; \quad C \equiv \frac{\rho(1-\beta)r^*\mathcal{X}^*\Omega}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)} > 0 \\
&= (A+B+C)^2 - 4(1-\rho+B)C \\
&= (A+B+C)^2 - 4(A+B)C + 4[A-(1-\rho)] \\
&= (A+B-C)^2 + \frac{4\rho(1-\beta)r^*}{\beta + r^* + \mathcal{X}^*\Omega} \\
&> 0. \quad \blacksquare
\end{aligned}
$$

Lemma 1 implies that μ_1 and μ_2 both are real.

Lemma 2 $\Delta_2 \equiv (a_{11}-1)(a_{22}-1) - a_{12}a_{21} < 0$.

Proof: In particular, from the expression of a_{22}, we have

$$
a_{22} - 1 = -\frac{\beta g^\circ[(1-\rho)\beta + (1-\rho\beta)r^*] + [(1-\rho)\beta g^\circ + \rho(1-\beta)r^*]\mathcal{X}^*\Omega}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)}
$$

Now, using the above expression as well as those of a_{11}, a_{12} and a_{21},

$$
\begin{aligned}
\Delta_2 &= \frac{\rho(1-\beta)r^*}{\beta g^\circ}\left\{a_{22} - 1 + \frac{[(1-\rho)\beta g^\circ + \rho(1-\beta)r^*]\mathcal{X}^*\Omega}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)}\right\} \\
&= -\frac{\rho(1-\beta)r^*[(1-\rho)\beta + (1-\rho\beta)r^*]}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)} \\
&< 0. \quad \blacksquare
\end{aligned}
$$

Lemma 2 implies that $(\mu_1 - 1)(\mu_2 - 1) < 0$. Hence, if both roots are of the same sign, both must be positive, with one greater than one and the other less than one.

Lemma 3 $\Delta_3 \equiv a_{11}a_{22} - a_{12}a_{21} > 0$.

Proof:

$\Delta_3 = a_{11} - 1 + a_{22} + \Delta_2$

$$= \frac{\rho(1-\beta)r^*}{\beta g^\circ} + \frac{\rho\beta^2 g^\circ(1+r^*) + \rho[\beta g^\circ - (1-\beta)r^*]\mathcal{X}^*\Omega}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)} - \frac{\rho(1-\beta)r^*[(1-\rho)\beta + (1-\rho\beta)r^*]}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)}$$
$$= \frac{\rho(1-\beta)r^* + \beta g^\circ + \rho\mathcal{X}^*\Omega}{\beta + r^* + \mathcal{X}^*\Omega}, \text{ where we have used } \rho(1+r^*) = g^\circ$$
$$> 0. \quad \blacksquare$$

Hence, $\mu_1\mu_2 > 0$, i.e., both roots are of the same sign. In view of Lemma 2, both roots are positive, one is less than one and the other exceeds one, implying saddle-path stability.

Proof of Proposition 9

Let $0 < \mu_1 < 1 < \mu_2$. The stable root of the transition matrix M^{-1} is then $1/\mu_2$. The solution expressions are

$$\varepsilon_t = \frac{A_{\mathcal{E}}}{\mu_2^t}; \quad \chi_t = \frac{A_{\mathcal{X}}}{\mu_2^t},$$

where $(A_{\mathcal{E}}, A_{\mathcal{X}})$ is the eigen vector. To solve this vector, we note in view of the system (A.17) that

$$a_{21}A_{\mathcal{E}} + (a_{22} - \mu_2)A_{\mathcal{X}} = 0. \tag{A.18}$$

Another relationship between $A_{\mathcal{E}}$ and $A_{\mathcal{X}}$ results from that the initial condition that $\mathcal{K}_0$ is given, or, equivalently κ_0 is given, where $\kappa_t \equiv \mathcal{K}_t - \mathcal{K}^*$.

By definition

$$\mathcal{K}_t = \frac{\mathcal{X}_t\mathcal{E}_t}{r_t},$$

and linearizing it around the steady state,

$$\begin{aligned}
\kappa_t &= \left(\frac{\mathcal{X}^*}{r^*} - \frac{\mathcal{X}_t\mathcal{E}_t}{{r^*}^2}\Gamma^*_{\mathcal{E}}\right)\varepsilon_t + \left(\frac{\mathcal{E}^*}{r^*} - \frac{\mathcal{X}_t\mathcal{E}_t}{{r^*}^2}\Gamma^*_{\mathcal{X}}\right)\chi_t \\
&= \frac{\mathcal{X}^*\varepsilon_t + \mathcal{E}^*(\beta + \mathcal{X}^*\Omega)\chi_t}{\beta r^*} \\
&= \frac{\mathcal{X}^*A_{\mathcal{E}} + \mathcal{E}^*(\beta + \mathcal{X}^*\Omega)A_{\mathcal{X}}}{\beta r^*} \cdot \frac{1}{\mu_2^t}.
\end{aligned}$$

At $t = 0$, we have

$$\frac{\mathcal{X}^*A_{\mathcal{E}} + \mathcal{E}^*(\beta + \mathcal{X}^*\Omega)A_{\mathcal{X}}}{\beta r^*} = \kappa_0. \tag{A.19}$$

We can write

$$\kappa_t = \kappa_0 \frac{1}{\mu_2^t}. \tag{A.20}$$

Hence $\mathcal{K}_t$ increases or decreases over time as $\kappa_0 \lesseqgtr 0$, i.e., $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$.

Eq. (A.20) implies

$$\frac{\mathcal{K}_{t+1} - \mathcal{K}^*}{\mathcal{K}_t - \mathcal{K}^*} = \frac{1}{\mu_2} \Leftrightarrow \frac{g_K - 1}{\mathcal{K}^*/\mathcal{K}_t - 1} = 1 - \frac{1}{\mu_2},$$

where $g_K \equiv K_{t+1}/K_t$. Suppose $\mathcal{K}_0 < \mathcal{K}^*$, so that $\mathcal{K}_t < \mathcal{K}^*$. Over time $\mathcal{K}_t$ rises, which implies g_K must fall, proving part (b) of Proposition 9.

Eqs. (A.18) and (A.19) solve $A_{\mathcal{E}}$ and $A_{\mathcal{X}}$. We have

$$A_{\mathcal{E}} = \frac{\beta r^*(a_{22} - \mu_2)\kappa_0}{D}; \quad A_{\mathcal{X}} = -\frac{\beta r^* a_{21}\kappa_0}{D}, \tag{A.21}$$

where $D \equiv (a_{22} - \mu_2)\mathcal{X}^* - a_{21}\mathcal{E}^*(\beta + \mathcal{X}^*\Omega) < 0.$[21] Since $a_{22} < 1$, we have $a_{22} - \mu_2 < 0$. Given $D < 0$ it follows that $A_{\mathcal{E}} \lesseqgtr 0$ as $\kappa_0 \lesseqgtr 0$. Thus, $\mathcal{E}_t$ rises or falls over time as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$. This proves part (a) of Proposition 9.

Next, linearizing $r_t = \Gamma(\mathcal{E}_t, \mathcal{X}_t)$ around the steady state

$$\begin{aligned}
r_t - r^* &= \frac{\Gamma_{\mathcal{E}} A_{\mathcal{E}} + \Gamma_{\mathcal{X}} A_{\mathcal{X}}}{\mu_2^t} \\
&= \left[\mu_2 - 1 + \frac{(1-\rho)\beta + (1-\rho\beta)r^*}{\beta + r^* + \mathcal{X}^*\Omega}\right] \frac{(1-\beta){r^*}^2\kappa_0}{D\mathcal{E}^*\mu_2^t} \\
&\gtreqless 0 \text{ as } \kappa_0 \lesseqgtr 0.
\end{aligned}$$

Thus r_t decreases or increases with time according as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$. Part (c) of Proposition 9 is proved.

[21]Since $\mu_2 > 1$, it is enough to prove that $(a_{22} - 1)\mathcal{X}^* - a_{21}\mathcal{E}^*(\beta + \mathcal{X}^*\Omega) < 0$. We have

$$\begin{aligned}
&(a_{22} - 1)\mathcal{X}^* - a_{21}\mathcal{E}^*(\beta + \mathcal{X}^*\Omega) \\
&= -\frac{\left\{\beta^2(1-\rho)g^\circ + [(1-\rho\beta)g^\circ - \rho(1-\beta)^2]r^*\right\}\mathcal{X}^*}{\beta g^\circ(\beta + r^* + \mathcal{X}^*\Omega)} \\
&\quad - \frac{[(1-\rho)\beta g^\circ + \rho(1-\beta)r^*]{\mathcal{X}^*}^2\Omega}{g^\circ(\beta + r^* + \mathcal{X}^*\Omega)} \\
&< 0.
\end{aligned}$$

Proof of Proposition 11

From constant returns and Cobb-Douglas technologies, $L_{mt}/L_{at} \propto Q_{mt}/(p_{at}Q_{at})$. Market clearing implies $p_{at}Q_{at} = \phi_a E_t$, whereas, in view of (24), Q_{mt} is proportional to $r_t K_t - \phi_s \eta E_t$. Hence

$$\frac{L_{mt}}{L_{at}} \propto \mathcal{X}_t - \phi_s \eta.$$

In view of (A.21),

$$\mathcal{X}_t - \mathcal{X}^* = \chi_t = -\frac{\beta r^* a_{21} \kappa_0}{D \mu_2^t},$$

implying that $\mathcal{X}_t$ and thus L_{mt}/L_{at} decrease or increase over time as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$. Therefore, the ratios in (31) increase or decrease with as $\mathcal{K}_0 \lesseqgtr \mathcal{K}^*$.

Proof of Proposition 12

Suppose $\mathcal{K}_0 < \mathcal{K}^*$. Corollary 6 implies that $g_{La} = g_{Ls} > g_L$ and $g_{Ks} > g_K$. Proposition 9, part (b) says $g_K > g_K^*$. Thus, $g_{Ks} > g_K^*$, and,

$$g_{Qs} = g_S g_{Ks}^{\eta} g_{Ls}^{1-\eta} > g_S {g_K^*}^{\eta} g_L^{1-\eta} = g_{Qs}^*.$$

Proposition 11 implies that land input use rises in agriculture over time, i.e., $g_{Da} > 1$ (whereas in the long run $g_{Da}^* = 1$ as land allocation remains constant). Hence

$$g_{Qa} = g_A g_{Da}^{\gamma} g_{La}^{1-\gamma} > g_A g_L^{1-\gamma} = g_{Qa}^*.$$

Likewise, if $\mathcal{K}_0 > \mathcal{K}^*$, $g_{Qs} < g_{Qs}^*$ and $g_{Qa} < g_{Qa}^*$.

Consider manufacturing output. Suppose $\mathcal{K}_0 < \mathcal{K}^*$. Then

$$g_{Qm} = g_M g_{Dm}^{\alpha} g_{Lm}^{\beta} g_{Km}^{1-\alpha-\beta} < g_M g_L^{\beta} g_K^{1-\alpha-\beta} \lesseqgtr g_M g_L^{\beta} {g_K^*}^{1-\alpha-\beta} = g_{Qm}^* \text{ since } g_K > g_K^*.$$

Proof of Proposition 13

From (24)

$$\frac{Q_{mt}}{E_t} = \frac{\mathcal{X}_t - \phi_s \eta}{\beta}.$$

Suppose $\mathcal{K}_0 < \mathcal{K}^*$. In course of the proof of Proposition 11, it is shown that $\mathcal{X}_t$ falls over time. Therefore, $g_{Qm} < g_E$. It suffices to prove that $g_E < g_K$, i.e., $\mathcal{E}_t/\mathcal{K}_t$ decreases over time if $\kappa_0 < 0$.

Linearizing the ratio $\mathcal{E}_t/\mathcal{K}_t$ around the steady state,

$$\begin{aligned}
\frac{\mathcal{E}_t}{\mathcal{K}_t} - \frac{\mathcal{E}^*}{\mathcal{K}^*} &= \frac{1}{\mathcal{K}^*}\left(\varepsilon_t - \frac{\mathcal{E}^*}{\mathcal{K}^*}\kappa_t\right) \\
&= \frac{1}{\mathcal{K}^*\mu_2^t}\left(A_{\mathcal{E}} - \frac{\mathcal{E}^*}{\mathcal{K}^*}\kappa_0\right) \\
&= \frac{\kappa_0}{D\mathcal{K}^*\mu_2^t}\left[\beta r^*(a_{22}-\mu_2) - \frac{D\mathcal{E}^*}{\mathcal{K}^*}\right] \\
&= \frac{r^*\kappa_0}{D\mathcal{K}^*\mu_2^t}\left[-(1-\beta)(a_{22}-\mu_2) + \frac{\mathcal{E}^* a_{21}(\beta+\mathcal{X}^*\Omega)}{\mathcal{X}^*}\right] \\
&= \frac{(1-\beta)r^*\kappa_0}{D\mathcal{K}^*\mu_2^t}\left\{\mu_2 - 1 + \frac{\beta r^*[g^\circ(1-\rho\beta)-\rho(1-\beta)]}{\beta g^\circ(\beta+r^*+\mathcal{X}^*\Omega)}\right\}, \text{ where} \\
&\qquad \text{we have made use of the expressions of } a_{22}-1 \text{ and } a_{21} \\
&> 0 \text{ if } \kappa_0 < 0, \text{ since } D < 0.
\end{aligned}$$

Hence $\mathcal{E}_t/\mathcal{K}_t$ falls with time if $\kappa_0 < 0$.

Appendix C Land Restrictions

The full-employment conditions (18) and (19) yield

$$\begin{aligned}
\alpha\eta(1-\gamma)p_{at}Q_{at} &= [\beta-\eta(1-\alpha)]r_{mt}\bar{D}_m + \alpha\eta w_t L_t - \alpha(1-\eta)r_t K_t \\
\alpha\eta p_{st}Q_{st} &= -\beta e_{mt}\bar{D}_m + \alpha r_t K_t \qquad \text{(A.22)} \\
\alpha Q_{mt} &= r_{mt}\bar{D}_m
\end{aligned}$$

Substituting the zero profit condition (15), the agriculture and services market clearing conditions, $p_{at}Q_{at} = \phi_a E_t$ and $p_{st}Q_{st} = \phi_s E_t$, into the first two expressions of (A.23) and dividing the resultant equations by $(M_t\bar{D}_m L_t^{1-\alpha-\beta})^{1/(1-\beta)}$ we get:

$$\begin{aligned}
\beta\frac{\bar{D}_m}{\bar{D}}\mathcal{R}_{mt} &= \alpha r_t\mathcal{K}_t - \phi_s\alpha\eta\mathcal{E}_t; \\
\frac{\beta}{\mathcal{R}_{mt}^{\frac{\alpha}{1-\alpha-\beta}}} &= \theta_3 r_t^{\frac{\beta}{1-\alpha-\beta}}\mathcal{E}_t + (1-\alpha-\beta)r_t^{\frac{1-\alpha}{1-\alpha-\beta}}\mathcal{K}_t \qquad \text{(A.23)}
\end{aligned}$$

where

$$\mathcal{R}_{mt} = \frac{r_{mt}}{\left(M_t \bar{D}^{-(1-\alpha-\beta)} L_t^{1-\alpha-\beta}\right)^{\frac{1}{1-\beta}}}; \quad \mathcal{E}_t = \frac{E_t}{\left(M_t \bar{D}^{\alpha} L_t^{1-\alpha-\beta}\right)^{\frac{1}{1-\beta}}}; \quad \mathcal{K}_t = \frac{K_t}{\left(M_t \bar{D}^{\alpha} L_t^{1-\alpha-\beta}\right)^{\frac{1}{1-\beta}}}$$

The reduced form static system can be solved exactly as in the previous section. It follows that factor prices and outputs would be of the form stated in For sake of brevity, we skip the proof.

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