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Issues in Association Rule Mining and Interestingness

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Abstract

This work presents unaddressed issues in the field of Association Rule Mining (ARM). Looking at the previous literature of varied areas and applications of ARM, we identify three broad categories of ARM where the research is still in the nascent stage. We review papers in the three categories of fuzzy association rules, multilevel association rules and negative association rules to study the state-of-art research conceptually and algorithmically. As a result, we provide a compendium of gaps and unaddressed issues in these domains using our understanding of ARM and interestingness.

Keywords: Association rules; fuzziness; multilevel; negative rules; interestingness; market basket analysis
1. Introduction

Association rule mining has been applied to broadly two types of data—transaction set and quantitative attribute data. The transaction datasets comprise of items that are associated together through any event such as market basket or web log analysis. On the other hand, quantitative attribute data consist of variables that are either binary or categorical. Hence, quantitative association rules are generated by partitioning these categorical or binary variables (Srikant and Agrawal, 1996). Partitioning of quantitative attributes leads to information loss. In order to minimize this information loss, the fuzzy set concept is used.

We also contribute to the AR mining literature by addressing issues relevant to various facets of interestingness. Fuzzy interestingness, unexpectedness in multilevel rules and generation of negative rules are some of the broad themes we have investigated. These issues help us in delving new and important problems that remain unaddressed in the current literature.

Organisation of the paper: In Section 2, we describe issues related to fuzziness and ARs. We also look at the fuzzy interestingness measures and their application in market basket. In Section 3, we look at the ambiguous definition of unexpectedness for multilevel ARs. In Section 4, we review the literature of negative rules and look at some unaddressed issues in the same.

2. Fuzziness and Association Rules

Kuok, Fu and Wong (1998) argue that fuzzy set concept is better than the discrete interval method (Srikant and Agrawal, 1996) since it provides a smooth transition between member and non-member of a set while partitioning. Because of such an approach of fuzzy sets, fewer boundary elements are excluded while partitioning the quantitative attributes.

Kuok et al (1998) define fuzzy association rules of the form:

\[ X \text{ is } A \rightarrow Y \text{ is } B \]

where \( X \) and \( Y \) are quantitative attributes and \( A \) and \( B \) are fuzzy sets corresponding to \( X \) and \( Y \) respectively.

2.1 Fuzzy Interestingness measures

Two measures of interestingness are used for generating fuzzy AR. Significance factor, which is equivalent to support in positive AR, gives the number of records supporting the itemset and also their degree of support.

\[ \text{Significance } < X, A > = \frac{\text{Sum of votes satisfying } < X, A >}{\text{Total Number of records}} \]

Votes satisfying a set \( <X,A> \) implies to the degree of membership of each record having attribute \( X \) lying under the fuzzy class \( A \).

We claim this measure of significance similar to support in positive AR because of the following reasons:
Significance and support measures are valid for itemsets and not rules

Significance and support measures reflect the support for the itemset relative to the entire dataset

Significance has two-step calculation due to the nature of fuzzy sets involved, in comparison to the discrete sets where support is used

The second measure of interestingness that is used by Kuok et al (1998) is called certainty factor. They use two methods to calculate certainty factor, but do not link the information gained from both the methods. We try to address this gap by separating the two methods as two distinct objective interesting measures in congruence with positive AR.

The first way to calculate certainty factor by Kuok et al (1998) is using significance.

\[
\text{Certainty} (X, A \rightarrow Y, B) = \frac{\text{Significance of } < Z, C >}{\text{Significance of } < X, A >}
\]

where \(Z= XUY\) and \(C = AUB\)

This is similar to the confidence measure for positive AR which uses support.

\[
\text{Confidence} (X \rightarrow Y) = \frac{\text{Support of } X U Y}{\text{Support of } X}
\]

The second way of calculating uncertainty is using Pearson’s correlation coefficient. Since fuzzy rules are different from positive rules, the calculation of expectation of antecedent and consequent is a little different. The vote of a record is zero if the membership function outcome of it is less than a user specified threshold \(\omega\). We are not commenting on the logical explanation of the formula for correlation given by the authors, but on the rationale of calling it as another method of calculating certainty factor.

The two ways illustrated as the example in the paper present contrasting results for the same fuzzy rule Salary, High \(\rightarrow\) Balance, Low. The certainty factor using significance is positive (0.364) highlighting the fact that the consequent is 36.4\% significant relative to the antecedent in the entire dataset. Contrary to this, the certainty factor using correlation coefficient (-0.96) presents an entirely different picture. It shows that High Salary is strongly negatively correlated with low balance, implying that the rule should not be formed. Instead a negative rule should be generated Salary, High \(\rightarrow\) ~ Balance, Low.

\[
\text{Confidence} (X \rightarrow Y) = \frac{P(XY)}{P(X)}
\]

\[
\text{Correlation} (X \rightarrow Y) = \frac{P(XY) - P(X)P(Y)}{f(P(X)) f(P(Y))}
\]

Hence, we argue that the second method is an additional measure of interestingness just as correlation is added to the support-confidence framework by Brin et al (1997). This fuzzy correlation measure can be used to generate negative fuzzy rules. We also recommend that the thresholds used for both certainty factor using significance and fuzzy correlation should be different as they give different relationships between itemsets. Using the same thresholds, as in the case of Kuok et al. may generate misleading rules.
To summarise about the certainty factor for fuzzy rules:

i. Certainty factor using significance and certainty factor using correlation reflect different relationships between itemsets, hence they should be considered as two distinct objective interestingness measures of fuzzy rules

ii. Certainty factor using significance is similar to confidence as it is applied to a rule and measures the support of consequent relative to the antecedent

iii. Fuzzy Correlation (name given by us) measures the positive or negative relationship between the antecedent and the consequent and hence can be used to generate negative fuzzy rules based on an new threshold given by the user

2.2 Extension of Fuzziness to Market Basket

We also point out one of the extension from the work of Kuok et al (1998). As mentioned earlier, fuzzy set concept is applied to quantitative attribute data such as age, gender or salary. There has been no attempts till now to apply fuzzy concepts to transaction datasets. Hence, we wish to put up an unaddressed question:

How can one incorporate fuzzy set concept in market basket data?

The typical market basket data comprises of purchase transactions, without any mention of quantities purchased in each transaction. For example, a person buying bread and jam together vis-à-vis a person buying 2 loafs of bread and 1 bottle of jam, is quite different in interpretation.

In order to apply fuzzy set concept to market basket, we need to convert the transaction into quantitative attributes. Consider a snapshot of transaction set in Table 1 which is converted to quantitative attributes in Table 2. We define the fuzzy set for the entire market transaction data as

\[ F = \{ \text{High, Medium, Low} \} \]

where High (H), Medium (M) and Low (L) represent the quantities of items purchased by customers in each transaction. H is quantity 4-5 units, medium is 3 units and low is 1-2 units.

<table>
<thead>
<tr>
<th>T.Id</th>
<th>Bread</th>
<th>Butter</th>
<th>Jam</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>3</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>T3</td>
<td>3</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Transaction set with quantities
We need to now define a membership function for fuzzy sets H, M and L. Let us consider a fuzzy rule:

Bread, Medium $\rightarrow$ Jam, High

<table>
<thead>
<tr>
<th>Bread, Medium</th>
<th>Jam, High</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>0.9</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Membership votes for antecedent and consequent of the rule

\[
\text{Significance} = \frac{0.81 + 0.15 + 0}{3} = 0.32
\]

\[
\text{Certainty} = \frac{0.32}{2.30} = 0.14
\]

Thus the rule has 32% significance and 14% certainty.

### 3. Multilevel Association Rules

Multilevel association rules are another kind of rules that consist of items from any level of the taxonomy. Typically, if a taxonomy approach is considered, the items at the leaf nodes form part in the association rules, the rest being classes (Agrawal, Imielinski and Swami, 1993; Agrawal and Srikant, 1994; Mannila, Toivonen and Verkamo, 1994). Agrawal and Srikant (1995) gave the idea of generalised rules which involves items as well as classes. They generated these rules by extending the support-confidence framework and coming up with a measure of ‘unexpectedness’ based on deviation of expectation in support values.

The extension of interestingness measures for multilevel AR based on the position in the taxonomy was done by Shaw, Xu and Geva (2009). They defined measures of diversity and peculiarity. Diversity is defined as the distance between items within a rule, based on their positions in the hierarchy. On the other hand, peculiarity determines the distance of one AR from other AR.
3.1 Neighbour-based Unexpectedness

We have identified one issue in the logical conception of peculiarity measure that seems to be ambiguous in Shaw et al. (2009) and another paper referred by the same, Dong and Li (1998). Dong and Li introduce neighbour-based unexpectedness in terms of confidence fluctuations in the neighbourhood rules. The peculiarity (Shaw et al, 2009) or syntax-based distance (Dong and Li, 1998) comprise of three parts: 1) the symmetric difference of all items in the two rules, 2) the symmetric difference of the antecedents, 3) the symmetric difference of the consequents.

\[
\text{Dist}(R_1, R_2) = \delta_1 * |(X_1 Y_1) \ominus (X_2 Y_2)| + \delta_2 * |(X_1) \ominus (X_2)| + \delta_3 * |(Y_1) \ominus (Y_2)|
\]

where \(\delta_1 > \delta_2 > \delta_3\)

If \(\delta_2\) is greater than \(\delta_3\), this means that the unexpectedness in the antecedents is given more weight than the consequents. This notion is counter-intuitive to the established definition of unexpectedness in the literature (Padmanabhan and Tuzhilin, 1999). For unexpectedness, the antecedents of both the rules should be similar and the deviation is seen in the consequents.

The rule \(A \rightarrow B\) is unexpected with respect to the belief \(X \rightarrow Y\) if:

1. \(A\) and \(X\) hold on a statistically large dataset
2. \(B\) and \(Y\) logically contradict each other

Thus, logically, \(\delta_3 > \delta_2\) for evaluating correct unexpectedness distance.

4. Negative Association Rules

The concept of negative association rules is still nascent in the field of data mining since researchers have not yet understood it fully, both conceptually and empirically. There have been some attempts, however, to develop algorithms for generating negative association rules. The discovery process is a difficult task since the search space for negative rules is too large. Absence of itemsets cannot be programmed and even if it is, that leads to generation of millions of negative rules that may not be of use to the manager. Hence, the objective is to find only “interesting” negative association rules that can be actioned upon by the managers. The defacto interestingness measures used for generating positive association rules are support and confidence. These measures in the Apriori algorithm prune the item sets based on the threshold for frequency count. Unlike positive association rules, negative rules cannot be generated by a simple Apriori algorithm since they involve absence of items. Thus, researchers have modified the Apriori algorithm for negative rules using different interestingness measures like correlation and expectation. There has also been attempts to use subjective interestingness measures like unexpectedness for generating negative rules.

The inception of the idea of negative implications was given by Brin, Motwani and Silverstein (1997). They extended the support-confidence framework by necessitating the use of correlation coefficient in generating interesting rules. They argued that support and confidence cannot highlight the negative relationship between two sets of items, while correlation gives the strength as well as direction of relationship. Consider an example of milk and jam. The following frequency table for been made from a hypothetical transaction dataset.
Consider the itemset: (Milk, Jam)

Now we generate rule using Apriori with minsupp = 10%, minconf = 50%

\[ Support = P(\text{Milk and Jam}) = \frac{5}{36} = 0.14 \]

\[ Confidence = \frac{P(\text{Milk and Jam})}{P(\text{Milk})} = \frac{5}{9} = 0.56 \]

Since the support and the confidence of this itemset are greater than the thresholds, the rule \( \text{Milk} \rightarrow \text{Jam} \) is generated.

However, if we calculate the correlation coefficient between milk and jam, we get altogether a different story.

\[ Correlation = \frac{P(\text{Milk and Jam})}{P(\text{Milk})P(\text{Jam})} = \frac{5}{(9)(11)} = 0.05 \]

Thus, milk and jam are indeed negatively correlated, implying the fact that people who buy milk, do not buy jam. This relationship is not captured in the support-confidence framework and the rule \( \text{Milk} \rightarrow \text{Jam} \) is misleading, in the absence of information on correlation between itemsets.

This concept of negative correlation by Brin et al (1997a) led to a stream of research in negative association rules. However, the authors do not use the original measure of correlation (Pearson, 1895) and rather use lift (Brin, Motwani, Ullman and Tsur, 1997b) as a proxy to it. One of the problems with lift is that it doesn’t consider the complement forms of itemsets. As a result of which, negative rules of the form \( A \rightarrow \neg B \) or \( \neg A \rightarrow B \) cannot be formed.

The work by Brin et al (1997a) on negative implications was extended by Antonie and Zaiane (2004) where they used Pearson’s correlation coefficient as a measure of negative association. They provide an algorithm that extends the support-confidence framework from the Apriori with a sliding correlation coefficient threshold. The algorithm checks for minimum support and confidence first, and then checks for the correlation. If the correlation is positive and greater than a threshold (t), positive association rules are generated. On the other hand, if the correlation is negative and greater than the threshold (-t) in magnitude, negative rules of the form \( A \rightarrow \neg B \) or \( \neg A \rightarrow B \) are generated.

The algorithm used by Antonie and Zaiane (2004) generates both positive and negative association rules using a single support, confidence and correlation threshold each. Although, this approach saves time and space, we are not sure if it still generates interesting negative
rules. Also, even if the algorithm is able to generate both types of rules, we fail to see the link between a positive and negative rules. Both the types are independent of each other and no common knowledge seems to evolve from them.

Another issue with this algorithm is that it is restricted to just two items, one as an antecedent and other as a consequent. This assumption makes the problem quite naïve. If we consider more than two items in the itemset for candidate sets, we need to establish the correlation between different combinations of items. For example, consider (milk, jam, butter) as a candidate itemset. For a rule to be generated from this itemset, we need to have a positive correlation between items that fall on either side together.

If the negative rule is \( \text{Milk, Butter} \rightarrow \neg \text{Jam} \)

Milk and Butter should have a high positive correlation and (Milk, Butter) should have a high negative correlation with Jam. Considering this case of correlation among and across itemsets, one single value for correlation threshold might not suffice.

4.1 Lift vs. Pearson’s Correlation

So far, negative rules have been generated using two objective measures: lift and Pearson’s correlation coefficient. As stated earlier, Brin et al (1997a) used lift as a proxy for correlation. The reason for this substitution has not been cited by the authors but considering the complexity of Pearson’s coefficient, one can understand the conceptual notion the paper tried to bring in. However, we should realize the difference between both the measures in accessing negative relationships. Also, one can look at other better measures which can be used for identifying negative relationships between items.

\[
Lift = \frac{P(AB)}{P(A)P(B)}
\]

\[
\text{Pearson's Correlation } \rho = \frac{\text{Covariance}(A,B)}{sd(A) \cdot sd(B)}
\]

On simplification,

\[
\rho = \frac{P(AB) - P(A)P(B)}{\sqrt{P(A)P(B)P(A)P(B)}}
\]

Focusing on numerators, Lift only takes into account the frequency of occurrence of both items together, while Pearson’s correlation calculates the difference between co-occurrence and independent occurrence. Thus, Pearson’s correlation coefficient gives a proper measure of negative relationships.

Since we are aware that Pearson’s correlation coefficient is complex in calculations because of the denominator, we provide alternative measure of objective interestingness that captures the negative relationship similar to Pearson’s coefficient. Change of Support (CS) is a measure of
interestingness that was formulated by Yao and Zhong (1999) but has not been focused much by researchers in association rule mining.

\[
CS = P(B/A) - P(B) = \frac{P(AB) - P(A)P(B)}{P(A)}
\]

Clearly, CS is similar to \( \rho \) in terms of difference between co-occurrence and independence condition. Thus, change of support is a better objective measure to use for discovering negative rules in terms of complexity and intuition.

4.2 Unexpectedness for Negative Rules

Another approach of generating negative rules is by using the taxonomy of the dataset. Savasere, Omiecinski and Navathe (1998) use “unexpectedness” as an objective measure of interestingness. A rule is interesting if it deviates from the manager’s expectation based on previous belief. The previous belief is usually stated in terms of the a priori probabilities based on knowledge of the problem domain (Savasere et al, 1998). The major assumption in this paper, which is based on the taxonomy of the data, is called the uniformity assumption. It states that the items that belong to the same parent in a taxonomy are expected to have similar types of associations with other items. In other words, siblings in a taxonomy are substitutable. For example, if Chips are bought with Pepsi, one expects that Chips are bought with Coke as well. If the actual support of Chips and Coke deviates from the support of Chips and Pepsi, then Chips and Coke generate a negative association rule.

Savasere et al (1998) define negative rules as consisting of items a customer is not likely to buy given a set of certain items. One of the conceptual questions we would like to ask here is the difference between the notion of not buying item A given item B vis-à-vis the notion of buying item B decreases the likelihood of buying item A. Although both the notions look the same through the first glance, there is a deeper meaning attached to the latter. The first definition can be applied to any set of two unrelated items A and B; however the second definition restricts to two related items. For instance,

Form 1: Pen \( \rightarrow \) \( \neg \) Milk
Form 2: Tea \( \rightarrow \) \( \neg \) Coffee

According to form 1, a person is not likely to buy milk when he buys pen. Here, the rule makes perfect sense but pen and milk are quite unrelated. On the contrary, form 2 reads that because a person is buying tea, he is less likely to buy coffee. The second form has a notion of causality as well as substitution. Although, this paper talks about form 1 with the knowledge of taxonomy so there are less chances of rule generation with unrelated products, but one must be careful before giving a generic definition for negative rules.

The uniformity assumption made by Savasere et al (1998) that taxonomy consists of siblings that are substitutable, needs further probing. Two fundamental questions arise: 1) What do we mean by substitution here? and 2) What is the level of granularity of the taxonomy based on the application studied.
Savasere et al (1998) do not define the meaning of substitution in the context of the paper. The concept of substitution should be linked to a function that specified the level of information reusability. Substitution can be at the level of brand (Colgate vs. Pepsodent Toothpaste), application-specific (flowers vs. chocolates), seasonality-driven (ice-cream vs. hot chocolate) or at a more abstract level (desktop vs. laptop). Thus, restricting to siblings can lead to over or under representation of negative rules. The authors do not restrict the uniformity assumption only across siblings, but the cross-relation of sibling, parent and children in three ways.

Consider this sample taxonomy that is used in the paper,

Let \{C,G\} be a large candidate whose support is greater than minimum support.

Now there can be three types of candidates formed from \{C,G\}:

1. Combination of immediate children- \{D,J\}, \{D,K\}, \{E,J\}, \{E,K\}
2. Combination of item and other’s children- \{C,J\}, \{C,K\}, \{G,D\}, \{G,E\}
3. Combination of item and other’s siblings- \{CH\}, \{GB\}

These three combinations take into account the granularity of specification of the taxonomy. Still, the expectation that support of two siblings is same may be misleading and depends greatly on the user application and the design of the taxonomy. Although this approach is said to use objective interestingness measure of unexpectedness, it largely depends on the domain knowledge on which the taxonomy shall be based. Thus, this approach involves subjective inspection.

A similar approach to Savasere et al (1998) has been adopted by Buckles, Yuan and Zhang (2002). They also use the concept of locality of similarity in defining siblings rules from the taxonomy. Sibling rules are a pair of positive association rules where both the siblings are expected to be related to the same consequent. For example, if \textit{Pepsi} \rightarrow \textit{Chips} is a rule that is generated through Apriori, then \textit{Coke} \rightarrow \textit{Chips} should also be generated. If the confidence measures of \textit{Coke} \rightarrow \textit{Chips} is less than the expected confidence (equal to \textit{Pepsi} \rightarrow \textit{Chips}), then a negative rule \textit{Coke} \rightarrow \neg \textit{Chips} is generated.

The domain knowledge is brought out in the taxonomy, hence this is also a subjective approach to generate negative rules. Buckles et al (2002) assert that the criterion for a negative rule utility is its relationship to a valid positive rule. This idea of linking positive and negative rules through taxonomy is indeed useful for managers for making strategic decisions. However, the same question arises here also that what level of substitution are we seeking for. As one moves to greater abstract level up the hierarchy, every item is substitutable with the other. Hence, there needs to be a function defined for substitution of items that appear in negative rules. This
paper is an attempt to extend the definition of substitution and use it for linking positive and negative association rules.

Table 5 provides a summary of the unaddressed issues associated with negative rules.

<table>
<thead>
<tr>
<th>Article</th>
<th>Broad idea</th>
<th>Measure of interest</th>
<th>Definition of substitution</th>
<th>Linking positive and negative rules</th>
<th>Gaps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brin, Motwani and Silverstein (1997)</td>
<td>Initiated the idea of negative relationships using correlation</td>
<td>Objective measure - Lift</td>
<td>None</td>
<td>No common knowledge, distinguished only based on correlation thresholds</td>
<td>Lift not a perfect proxy for correlation measure</td>
</tr>
<tr>
<td>Antonie and Zaiane (2004)</td>
<td>Used correlation for generation of negative rules</td>
<td>Objective measure- Pearson’s correlation coefficient</td>
<td>None</td>
<td>No common knowledge, distinguished only based on correlation thresholds</td>
<td>Correlation between itemsets not considered</td>
</tr>
<tr>
<td>Savasere, Omiesinski and Navathe (1998)</td>
<td>Used Unexpectedness based on support of substitute items to generates negative rules</td>
<td>Subjective measure- unexpectedness of support</td>
<td>Items are substitutable based on their positions in the taxonomy</td>
<td>No common knowledge</td>
<td>Narrow definition of Substitution</td>
</tr>
<tr>
<td>Buckles, Yuan and Zhang (2002)</td>
<td>Used expected value of confidence based on sibling substitution</td>
<td>Subjective measure- unexpectedness of confidence</td>
<td>Siblings are expected to be related to other items in a similar fashion</td>
<td>Linking is done only with respect to sibling substitution</td>
<td>User goals not considered for substitutio n</td>
</tr>
</tbody>
</table>

Table 5: Unaddressed issues associated with negative rules

5. Conclusions

We present some unaddressed issues in the field of ARM. We identify three broad categories of ARM where the research is still in the nascent stage, namely fuzziness, multilevel rules and negative rules. We then review papers in the three categories to study the state-of-art research. Our issues comprise of lacunae in conceptualisation of concepts, algorithmic nuances and better statistical validation. As a result, we provide a compendium of gaps and unaddressed issues in these domains using our understanding of ARM and interestingness. We plan to take these issues in our future works.

References


