Financial reporting standards and investment efficiency in growth firms*

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Abstract

We analyze the role of financial reporting standards (backward vs. forward looking, conservative vs. aggressive) in stimulating investment efficiency in growth firms. In a dynamic investment game where the investor faces uncertainty regarding the duration that a firm is able to generate profitable growth opportunities, we show that underinvestment and overinvestment may arise. We show that perfect backward looking financial information ameliorates both the overinvestment and the underinvestment problems from a social standpoint. Perfect forward looking financial information ameliorates the underinvestment problem better than backward looking financial information does and it completely resolves the overinvestment problem. We further consider aggressive and conservative reporting in a mixed attribute financial reporting system that combines perfect backward looking financial information with noisy forward looking financial information. We find that aggressive reporting better addresses the issue of underinvestment while conservative reporting better addresses the issue of overinvestment. It suggests that counter-cyclical reporting standards address investment inefficiencies better than pro-cyclical reporting standards.

Keywords: Backward looking financial information, Forward looking financial information, Investment efficiency, Overinvestment, Underinvestment, Aggressive financial reporting, Conservative financial reporting

1 Introduction

Start-ups and younger firms generally need to rely on external capital to finance their profitable investment opportunities. Over time they go through several rounds of external financing to expand their operating activities up to a scale that either external financing is no longer needed or their profitable investment opportunities have been exhausted. In acquiring external financing, information asymmetries exist between the firm and potential investors in several respects including (1) the point in time that the set of profitable investment opportunities has been exhausted and (2) the alignment of interests between the firm's manager and investors. Importantly, these two types of information asymmetries may result in investment inefficiencies. Underinvestment arises when investors stop investing while the firm still has access to profitable investment opportunities. Overinvestment arises when investors are still investing even though the firm no longer has any profitable investment opportunities; in this case external financing only serves to generate private benefits to the firm's manager.

This paper analyzes how financial reporting can reduce both types of investment inefficiencies. It considers an infinitely repeated game between a firm's manager and an investor. Each period, the investor is endowed with capital funds and the firm's manager gains access to a new investment opportunity. To undertake this investment opportunity, the firm needs to rely on external financing that the investor can provide. The firm's manager knows whether the investment opportunity has positive net present value but the investor does not. It is assumed that the net present value of new investment opportunities weakly decreases over time and becomes negative at some point in time. The investor, however, does not know when this point in time has arrived. The investor also faces uncertainty with respect to manager's interests. The interests of a good type manager are perfectly aligned with those of the investor and undertakes an egalitarian division of the resources the firm generates. A bad type manager has selfish interests and expropriates some or all of the firm's assets once all positive NPV projects have been undertaken. At the start of each period, the investor updates his beliefs regarding manager type and the profitability of the new investment opportunity using all past financial reports and any past dividends received. The reporting standards underlying the financial reports affect the investor's beliefs and becomes the channel that drives our results.

The main findings are as follows: first, financial reporting based on backward looking information (e.g., historical cost information) reduces overinvestment and underinvestment. However, financial reporting based on forward looking financial information (e.g. fair value information) ameliorates the underinvestment problem better than backward looking financial information does and it completely resolves the overinvestment problem.

Second, aggressive reporting is more effective in reducing underinvestment than neutral reporting. The intuition for this is that in the case of underinvestment, the investor is pessimistic about the NPV of the firm's investment opportunity and only a good news financial report may induce the investor to continue investment. In other words, the financial reporting system should be such that it maximizes the likelihood of a good news financial report conditional on investors believing that the investment opportunity has positive NPV following the disclosure of the good news financial report. When for a neutral reporting system, investors believe that the investment opportunity has strictly positive NPV following the good news report, one can increase the likelihood of a good financial report by allowing some degree of over-reporting. Although this will reduce the informative value of the good financial report and investors' beliefs about the NPV of the investment opportunity, but as long as this reduction is sufficiently small, investors are still willing to invest.

Third, conservative reporting may be more effective than neutral reporting in resolving the overinvestment problem. To see this, consider the flip side of the example in the preceding paragraph. In the case of overinvestment, investors are too optimistic about the NPV of the firm's investment opportunity. Investors will discontinue their investment when the financial report contains sufficiently bad news so that investors believe the investment opportunity has negative NPV. When for a neutral reporting system, investors believe that the investment opportunity has strictly negative NPV following the bad news report, one can increase the likelihood of a bad financial report by allowing some degree of under-reporting. Although this will reduce the informative value of the bad financial report, but as long as this reduction is sufficiently small, investors will still discontinue investment.

One implication of our findings is that when considering the optimal duration of investment, aggressive reporting and conservative reporting are desirable respectively for resolving underinvestment and overinvestment issues. This contrasts the conventional opinion of standard setters that neutral or conservative reporting is always the preferred option. In this respect it also important to observe that an upper bound on the level of aggressive reporting arises endogenously: too aggressive reporting will make the good news financial report insufficiently informative so that it will no longer induce the investor to make the investment. Opportunistic use of aggressive reporting should therefore be less of a concern for standard setters.

Our findings seem consistent with practice. Underinvestment is primarily a concern for start-up and growth firms where investors may be holding back investment because of higher uncertainty and/or risk. Such firms seem to engage in more aggressive reporting by focusing on alternative financial performance measures. During the 1990's, technology companies claimed that profit figures were understated because accounting standards did not properly reflect their investments in intangibles. Standard setters responded to this by making the accounting standards more aggressive in the sense that it would allow for the capitalization of internally developed software. More recently, internet-based companies like Google and Facebook put more emphasis on non-financial information (e.g., web-traffic, number of daily users) rather than financial information when searching for external financing.

In contrast, overinvestment is more of a concern for mature firms with little growth opportunities. Standard setters have increased the level of conservative reporting to reduce opportunistic reporting behavior. For example, lease accounting reduced the opportunities for off-balance sheet financing and following Enron, consolidation rules for special purpose entities became more stringent. Currently, standards setters are also debating the use of non-GAAP reporting practices.

Finally, observe that our findings suggest that reporting standards should be counter-cyclical in order to address over and underinvestment issues. In bull markets, when investor sentiment or optimism is high, overinvestment is more of a concern than underinvestment so that conservative reporting may be more desirable. Conversely for bear markets, when investor sentiment is low, underinvestment is more of a concern than overinvestment so that aggressive reporting may be more desirable. This highlights an important drawback of fair value based accounting standards, which tend to be pro-cyclical. Standard setters should be aware that fair value based standards may increase over and underinvestment problems in capital markets. In this respect, our paper is related to Bertomeu and Magee (2011) that shows that the demanded level of financial reporting quality varies with economic cycles. Bertomeu and Magee (2011), however, does not distinguish between backward and forward looking reporting and aggressive and conservative reporting.

Our dynamic setting draws on the dynamic reporting game of Liang, Marinovic and Varas (2017). While they have a mass of risk neutral investors, we consider a single strategic (representative) investor. Furthermore, our analysis focuses on the investment decision, i.e., the flow of capital from the investor to the firm, rather than valuation. Our characterization of good and bad type manager is analogous to their characterization of honest and dishonest manager type.

Managerial reputation building takes the form of perfect mimicking of the good type manager by the bad type manager (Kreps and Wilson 1982, Kreps, Milgrom, Roberts and Wilson 1982) in all periods preceding the last one before which the switch from increasing to decreasing returns to scale occurs. Existing literature on managerial reputation building focuses on the relation between financial reports and a manager's reputation for being informationally endowed (Einhorn and Ziv 2008) or on the relation between financial reports and a manager's reputation for being 'forthcoming' (Beyer and Dye 2012). We extend this literature by analyzing the interaction of such reputation building with reporting horizon.

The bad type manager tries to build a reputation for being a good type as in Lunawat (2013) and Lunawat (2016). However, unlike prior work, we are able to solve for an equilibrium in pure strategies because the addition of reinvestment aligns the strategies of good and bad type managers in all but one period. Veering away from mixed strategy equilibria of prior work allows a sharp focus on the role of reporting horizon. Existing literature focuses on the relation between liquidity and reporting horizon (Allen and Carletti 2008, Plantin, Sapra and Shin 2008). We contribute to this literature by analyzing the relation between investment efficiency and reporting horizon.

The rest of the paper proceeds as follows. Section 2 develops the model and Section 3 solves for the equilibrium. Section 4 analyzes the case without financial reporting. Sections 5 and 6 respectively analyze the case with backward looking and forward looking financial information. Section 7 concludes.

2 Model

The model considers an investor ("he") and an owner manager of a firm ("she") over an infinite number of periods. The firm has access to a constant returns to scale project. The return on investment of this project in period t is uncertain and described by the random variable $\tilde{\lambda}_t$. It is assumed that $\tilde{\lambda}_t \in \left\{\overline{\lambda}, \underline{\lambda}\right\}$ where $\overline{\lambda} > 1 > \underline{\lambda}$, i.e., the return on investment is either positive or negative. Uncertainty is driven by the duration n_1 that the firm remains profitable, i.e., it is assumed that $\tilde{\lambda}_t = \overline{\lambda}$ for all $t \leq n_1$ and $\tilde{\lambda}_t = \underline{\lambda}$ for all $t > n_1$ (see Figure 1A). The probability density function of n_1 is denoted by $\phi_0(n_1)$. During the time period that the firm is profitable, the manager would like to increase the scale of the firm's operations as much as possible so as to maximize the firm's operating cash flows. The manager can do this in two ways. First, the manager can reinvest prior period's operating cash flows in the current period. Second, the manager can acquire external capital from an investor. In each period, the investor

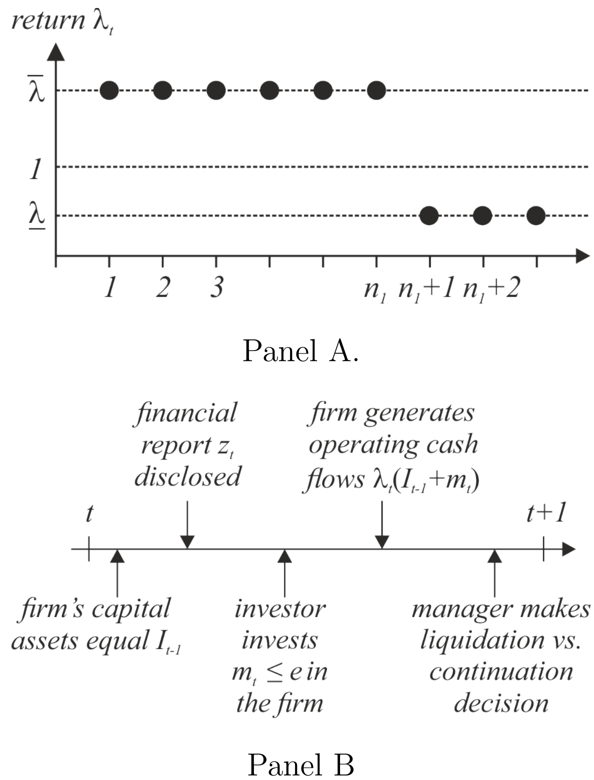

Figure 1: Timeline of the model. Panel 1A presents the distribution of profitability over time. Panel 1B presents the sequence of events in each individual period t.

has a capital endowment $e > 0$ that he can choose to invest in the firm. Let $m_t \leq e$ denote the amount of capital invested by the investor in period t.

In each period t, the sequence of events is as follows (see Figure 1B). Let I_{t-1} denote the firm's operating cash flows at the end of period $t-1$. At the start of period t, the manager discloses a financial report z_t. For now we do not further specify the characteristics of this financial report. As we analyze different types of reporting, we will specify the details of this report in later sections. Conditional on the report z_t, the investor makes his investment decision $m_t \leq e$. The manager invests $I_{t-1} + m_t$ in the firm's project yielding operating cash flows $\tilde{\lambda}_t \left(I_{t-1} + m_t\right)$ at the end of period t. Subsequently, the manager decides whether to liquidate the firm or to continue operations in the next period. When the manager liquidates the firm, she returns $\beta\tilde{\lambda}_t \left(I_{t-1} + m_t\right)$ to the investor and keeps a fraction $\left(1-\beta\right)\tilde{\lambda}_t \left(I_{t-1} + m_t\right)$ to herself and the game ends. When the manager continues operations, period $t+1$ starts with $I_t = \tilde{\lambda}_t (I_{t-1} + m_t)$. We denote the continuation/liquidation decision at the end of period t by $c_t \in \{0,1\}$ where $c_t = 1$ implies that the firm continues operations into the next period and $c_t = 0$ implies that the firm is liquidated.

The investor faces information asymmetry in two respects. First, the investor does not know the time period n_1 until which the firm generates positive returns. The manager does know n_1 but cannot credibly communicate this information to the investor. The investor's prior beliefs about n_1 equal $\phi_0(n_1)$. Investor's posterior beliefs at the start of period t after having observed the disclosed financial reports of the past periods and the most recent continuation decision, are denoted by $\phi_t(n_1|H_t)$ where $H_t = \{z_1, \ldots, z_t, c_{t-1}\}$ denotes the information available to the investor.[1] Second, the investor does not know whether he is dealing with a good or bad type manager. A good type manager is a manager who liquidates the firm at the end of period n_1. The interests of a good type manager are aligned with the interests of the investors. The fraction $(1-\beta_G)$ of the liquidating cash flow that the good type manager keeps to herself can be interpreted as the compensation received for providing management services. In contrast, a bad type manager has misaligned interests. She can expropriate part of the firm's assets (e.g., private benefits from empire building) and therefore also tries to acquire capital from the investor when the firm is no longer profitable (i.e., beyond period n_1). When she decides to liquidate the firm, she therefore keeps a larger fraction $(1-\beta_B)$ to herself to reflect these private benefits. Without loss of generality, we assume $\beta_G = 1 > \beta_B$.[2] Also, when a bad type manager acquires

[1] Note that the most recent continuation decision c_{t-1} is sufficient for all continuation decisions $c_1, c_2, \ldots, c_{t-1}$ in the past periods.

[2] This implies that the investor, upon receiving a liquidating dividend, learns manager type. We assume, however, that the investor cannot fully recoup his damages through litigation because of the manager's limited liability and the manager having consumed part or all of the private benefits. Note that the ability of the bad type manager to expropriate the firm's assets also implies that the revelation principle does not apply, i.e., the investor cannot use contracting to induce truthful revelation of manager type because a bad type manager could compensate a low contract payoff by

capital from the investor beyond period n_1, we assume that he invests the capital in a risk free asset rather than the unprofitable investment opportunity so that the investment $I_{t-1} + m_t$ yields payoff $I_{t-1} + m_t$ rather than $\underline{\lambda}\,(I_{t-1} + m_t)$. Finally, let $P_t(H_t)$ denote investor's beliefs at the start of period t that the manager is a good type. The prior probability that a manager is a good type equals $\theta \in (0,1)$, i.e., $P_0 = \theta$. The prior beliefs regarding manager type and n_1 are assumed to be independent, i.e., $\phi_0(n_1|G) = \phi_0(n_1|B) = \phi_0(n_1)$ for all n_1.

3 Benchmark setting: no financial reporting

In this section, we analyze the benchmark setting when there is no financial reporting, i.e., the financial report z_t is completely uninformative about the profitability of the firm in period t. This implies that the investor can only update his beliefs based on the passage of time and the manager's liquidation/continuation decision c_t, i.e., $H_t = \{c_{t-1}\}$.

Observe that both manager types behave as automaton thereby reducing the model to a dynamic single person investment problem for the investor.[3] The optimal investment strategy consists of investment decisions $(m_1, m_2, \ldots)$ and beliefs $(\phi_1, \phi_2, \ldots)$ and $(P_1, P_2, \ldots)$ such that in each period t beliefs (ϕ_t, P_t) are rational given the history of information H_t and the investment decision $m_t \leq e$ maximizes the expected return, i.e., m_t maximizes $m_t E_{\beta,n_1}\left(\beta\overline{\lambda}^{\max(n_1-t+1,0)}\middle| H_t\right) - m_t$. The expectation is taken with respect to manager type β and the period n_1 up to which the firm is profitable. At the start of period t, $\max(n_1 - t + 1, 0)$ is the number of subsequent periods that

expropriating more assets.

[3] One could extend a manager's action space with a periodic payout decision, i.e., at the end of each period the manager decides how much of the firm's cash assets are paid out as dividends and how much is reinvested in the firm's operating activities. This would not affect any of our results as the good type manager will reinvest all cash assets up to period n_1 and the bad type manager perfectly mimics the pay out policy of the good type manager up to period n_1. For ease of exposition, we exogenously assume this equilibrium behavior for both manager types.

the firm is profitable. Hence, an investment m_t in period t yields the gross payoff $m_t \beta \overline{\lambda}^{\max(n_1-t+1,0)}$.[4]

Before we present the optimal investment strategy, we first discuss how the investor updates his beliefs based on the manager's continuation decision c_t. When the firm pays a dividend at the end of period t, the investor learns that $n_1 = t$ and that the manager is a good type. When the firm continues its operations into period t, the investor first updates his beliefs as follows:

Lemma 1 *For each period t and each history H_t with $c_{t-1} = 1$ it holds that*

$$\phi_t(n_1|H_t) = \frac{\phi_{t-1}(n_1|H_{t-1},G)P_{t-1}(H_{t-1}) + \phi_{t-1}(n_1|H_{t-1},B)(1-P_{t-1}(H_{t-1}))}{1-\phi_{t-1}(t-1|H_{t-1},G)P_{t-1}(H_{t-1})} \quad (1)$$

$$P_t(H_t) = \frac{P_{t-1}(H_{t-1}) - \phi_{t-1}(t-1|H_{t-1},G)\,P_{t-1}(H_{t-1})}{1-\phi_{t-1}(t-1|H_{t-1},G)\,P_{t-1}(H_{t-1})}, \quad (2)$$

where

$$\phi_t(n_1|H_t,G) = 1_{n_1 \geq t}\frac{\phi_{t-1}(n_1|H_{t-1},G)}{1-\phi_{t-1}(t-1|H_{t-1},G)}, \quad (3)$$

$$\phi_t(n_1|H_t,B) = \phi_{t-1}(n_1|H_{t-1},B). \quad (4)$$

Observe that the investor first updates his beliefs about n_1 conditional on manager type (cf. expressions (3) and (4)). When the investor presumes that he is dealing with a good type manager, continuation implies that $n_1 \geq t$; for if $n_1 = t-1$, a good type manager would have liquidated the firm. When the investor presumes that he is dealing with a bad type manager, he learns nothing about n_1. The reason is that a bad type manager will always try to acquire capital from the investor irrespective of whether the firm is profitable ($t \leq n_1$) or unprofitable ($t > n_1$). The investor uses this information to update his beliefs (ϕ_t, P_t). Observe that updated beliefs $\phi_t(n_1|H_t)$ depend on both $\phi_{t-1}(n_1|H_{t-1})$ and $P_{t-1}(H_{t-1})$; similarly for $P_t(H_t)$.

[4] Recall that when $n_1 < t$, a bad type manager invests any acquired capital in the risk free asset generating a rate of return equal to one, i.e., the payoff of investment m_t equals $m_t\beta_B\overline{\lambda}^{\max(n_1-t+1,0)} = m_t\beta_B\overline{\lambda}^0 = m_t\beta_B$.

Corollary 1 *When the firm continues its operations at the end of period $t-1$ then*

(i) it is more likely that the manager is a bad type, i.e., $P_t(H_t) < P_{t-1}(H_{t-1}) < \theta$.

(ii) $\phi_t(n_1|H_t) > \phi_{t-1}(n_1|H_{t-1})$ *if and only if* $n_1 \geq t$.

The intuitive explanation for (i) is that a bad type manager is more likely to continue the operations of the firm than a good type manager. Recall that a bad type manager always continues the operations of the firm when he expects the investor is still willing to invest in the subsequent period. In contrast, a good type manager only continues the operations of the firm when the firm is still profitable, i.e., $t < n_1$. Statement (ii) implies that updated beliefs assign higher likelihood to the states $n_1 \geq t$. In other words, the investor becomes more optimistic about the firm still being profitable in the current period t. The reason for this is the positive likelihood that the investor still assigns to dealing with a good type manager *and* because a good type manager continues the operations of the firm only when the firm is profitable in the current period t.

For deriving the optimal investment strategy, we make the following two regularity assumptions:

(A1) $P_0 = \theta \geq \left(\sum_{n_1=1}^{\infty} \phi_0(n_1)\overline{\lambda}^{n_1-\tau+1}\right)^{-1}$.

(A2) $m_t E_{\beta,n_1}\left(\beta\overline{\lambda}^{\max(n_1-t+1,0)}\middle| H_t\right) - m_t$ is decreasing in t and negative for sufficiently large values of t.

Assumption (A1) implies that the investor's prior beliefs are such that at the start of the game, investing in the firm has positive NPV. Without this assumption, the problem is trivial as the investor would never invest and the firm would not be able to undertake any investment projects. Assumption (A2) implies that as time progresses, the investor's beliefs about the return on investment in the firm becomes worse and reduce to zero. In other words, (A2) implies that in the long run the firm becomes unprofitable. Without this assumption, the problem would also be trivial as investment in the firm would always be optimal.

Proposition 1 *With uninformative financial reports, the investor invests up to period $\overline{n}_1$, where*

$$\overline{n}_1 = max \left\{ t \middle| P_t(H_t) e \sum_{n_1=t}^{\infty} \phi_t(n_1|H_t, G) \overline{\lambda}^{n_1-t+1} \right.$$
$$\left. +(1 - P_t(H_t))\beta_B e \sum_{n_1=1}^{\infty} \phi_0(n_1) \overline{\lambda}^{\max(n_1-t+1,0)} > e \right\}. \quad (5)$$

More specifically, for each period t, the optimal investment decision equals

$$m_t = \begin{cases} e & \text{if } t \leq \overline{n}_1 \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

The intuition for Proposition 1 follows from the second regularity assumption (A2) that the investor's posterior beliefs are decreasing over time. This implies that at some point in time, i.e., period $\overline{n}_1$, investment is no longer perceived as profitable so that the investor stops investing in the firm. The driving force for stopping investment can be (i) that the investor assigns too high a probability to dealing with a bad type manager, or (ii) that the investor assigns too high a probability to $n_1 = t - 1$, i.e., the previous period was the last profitable period, or (iii) a combination of both (i) and (ii). Observe that when (A2) does not hold, the investment decision is no longer characterized by $\overline{n}_1$. In that case, periods during which investment occurs alternate with periods during which investment does not occur.

3.1 Social loss

A regulator who cares about economic output, would like investment to occur for n_1 periods as this is the number of periods that the firm's investment project features positive NPV. A social loss then arises when the investor invests for too little or too many periods. Underinvestment arises when $n_1 > \overline{n}_1$, i.e., when the actual number of periods that the investment is profitable exceeds the number of periods that the investor is willing to invest. To determine the ex-ante expected loss of underinvestment, first observe that the expected return on investment when investing for n_1 periods equals

$$\sum_{t=1}^{n_1} e\left(\overline{\lambda}^{n_1-t+1}-1\right)$$

The expected return on investment when investing for $\overline{n}_1$ periods equals

$$\sum_{t=1}^{\overline{n}_1} e\left(\overline{\lambda}^{n_1-t+1}-1\right)$$

Hence, the social loss of underinvestment for given n_1 equals

$$\underline{SL} = \sum_{t=1}^{n_1} e\left(\overline{\lambda}^{n_1-t+1}-1\right) - \sum_{t=1}^{\overline{n}_1} e\left(\overline{\lambda}^{n_1-t+1}-1\right) = \sum_{t=\overline{n}_1+1}^{n_1} e\left(\overline{\lambda}^{n_1-t+1}-1\right). \quad (7)$$

Overinvestment never arises from a social standpoint. To see this, observe that overinvestment cannot arise with a good type manager as a good type manager liquidates the firm at the end of period n_1. Overinvestment also does not arise when $n_1 < \overline{n}_1$, i.e., when the actual number of periods that the investor is willing to invest exceeds the number of periods that the investment is profitable and the manager is a bad type. This is so because a bad type manager simply invests in a risk free asset after period n_1.

3.2 Investor's loss

A regulator who cares about the investor's welfare also takes into account that an investor should not invest in a bad type manager as a bad type manager expropriates all of the firm's cash flows at the expense of the investor. With uninformative reporting, an investor's loss may arise because the investor invests for too little or too many periods or because he invests in the bad type manager. Underinvestment arises when $n_1 > \overline{n}_1$, i.e., when the actual number of periods that the investment is profitable exceeds the number of periods that the investor is willing to invest. To determine the ex-ante expected loss of underinvestment, first observe that the expected return on investment for given n_1 equals

$$\sum_{t=1}^{n_1} e\theta\left(\overline{\lambda}^{n_1-t+1}-1\right) + \sum_{t=1}^{n_1} e(1-\theta)\left(\beta_B\overline{\lambda}^{n_1-t+1}-1\right)$$

For the setting with uninformative reporting, the expected return on investment equals

$$\sum_{t=1}^{\overline{n}_1} e\theta \left(\overline{\lambda}^{n_1-t+1} - 1\right) + \sum_{t=1}^{\overline{n}_1} e(1-\theta)\left(\beta_B \overline{\lambda}^{n_1-t+1} - 1\right).$$

Hence, the investor's loss of underinvestment for given n_1 equals

$$\underline{IL} \;=\; \sum_{t=\overline{n}_1+1}^{n_1} e\theta \left(\overline{\lambda}^{n_1-t+1} - 1\right) + \sum_{t=\overline{n}_1+1}^{n_1} e(1-\theta)\left(\beta_B \overline{\lambda}^{n_1-t+1} - 1\right) \tag{8}$$

Overinvestment arises whenever the manager is a bad type and $\overline{n}_1 > n_1$. The investor's loss arises because the bad manager type expropriates a fraction $1-\beta_B$ of the total investment's payoff. To determine the ex-ante expected loss of overinvestment, first observe that the expected return on investment for given n_1 equals

$$\sum_{t=1}^{n_1} e(1-\theta)\left(\beta_B \overline{\lambda}^{n_1-t+1} - 1\right)$$

For the setting with uninformative reporting, the expected return on investment equals

$$\sum_{t=1}^{n_1} e(1-\theta)\left(\beta_B \overline{\lambda}^{n_1-t+1} - 1\right) + \sum_{t=n_1+1}^{\overline{n}_1} e(1-\theta)(\beta_B - 1).$$

Hence, the investor's loss of underinvestment for given n_1 equals

$$\overline{IL} \;=\; e(1-\theta)(1-\beta_B)(\overline{n}_1 - n_1). \tag{9}$$

The subsequent sections analyze how financial reporting can serve to address the overinvestment and underinvestment problems.

4 Backward looking financial reporting

With backward looking financial reporting, the financial report z_t reports the realized return of the preceding period $t-1$. Using that the realized return perfectly reveals λ_{t-1}, we define the backward looking financial report by $z_t = \lambda_{t-1}$ for all periods t that the firm is still in operation.

A report about $z_t = \lambda_{t-1}$ at the start of period t enables the investor to update his beliefs about n_1. When $\lambda_{t-1} = \underline{\lambda}$, he infers that $n_1 < t-1$ and stops investing. When $\lambda_{t-1} = \overline{\lambda}$, the investor infers that $n_1 \geq t-1$. He uses this inference *and* the fact that the firm has not been liquidated at the end of period $t-1$ to update his beliefs $\phi_t(n_1|H_t)$ about n_1 and his beliefs $P_t(H_t)$ that the manager is a good type:

Lemma 2 *For each period t and each history H_t with $c_{t-1} = 1$ and $z_t = \overline{\lambda}$ it holds that*

$$\phi_t^b(n_1|H_t) = \frac{\phi_{t-1}^b(n_1|H_{t-1},G)P_{t-1}(H_{t-1}) + \phi_{t-1}^b(n_1|H_{t-1},B)(1-P_{t-1}(H_{t-1}))}{1-\phi_{t-1}^b(t-1|H_{t-1},G)P_{t-1}(H_{t-1})} \tag{10}$$

$$P_t(H_t) = \frac{P_{t-1}(H_{t-1}) - \phi_{t-1}^b(t-1|H_{t-1},G)P_{t-1}(H_{t-1})}{1-\phi_{t-1}^b(t-1|H_{t-1},G)P_{t-1}(H_{t-1})} \tag{11}$$

where

$$\phi_t^b(n_1|H_t,G) = 1_{n_1 \geq t}\frac{\phi_{t-1}^b(n_1|H_{t-1},G)}{1-\phi_{t-1}^b(t-1|H_{t-1},G)}, \tag{12}$$

$$\phi_t^b(n_1|H_t,B) = 1_{n_1 \geq t-1}\frac{\phi_{t-1}^b(n_1|H_{t-1},B)}{1-\phi_{t-1}^b(t-2|H_{t-1},B)}. \tag{13}$$

Observe that the posterior $\phi_t^b(n_1|H_t,G)$ is the same as for the case without reporting (cf. (3)). The reason is that conditional on dealing with a good type manager, the backward looking report does not provide any additional information to the continuation decision. With a good type manager, $c_{t-1} = 1$ already reveals that $\lambda_{t-1} = \overline{\lambda}$. Consequently, the posterior $P_t(H_t)$ is also the same as in the benchmark setting (cf. (2)). The backward looking report does not reveal any additional information about manager type.

Corollary 2 *Conditional on dealing with a good type manager, the backward looking financial report z_t does not provide any information incremental to the liquidation/continuation decision c_{t-1}, i.e., $\phi_t^b(n_1|H_t,G) = \phi_t(n_1|c_{t-1},G)$. Consequently, the backward looking financial report also does not provide any information on manager type, i.e., $P_t(H_t) = P_t(c_{t-1})$.*

Observe that the posterior $\phi_t^b(n_1|H_t,B)$ does differ from the benchmark setting (cf. (4)). Conditional on dealing with a bad type manager, the backward looking financial report is informative. In particular, for $z_t = \overline{\lambda}$ the investor learns that the firm has been profitable over the past periods so that $\phi_t^b(n_1|H_t,B) = 0$ for all $n_1 < t-1$. In other words, the investor learns that $n_1 \geq t-1$. This in turn also affects the investor's beliefs about manager type compared to the no information case. In particular, the backward looking financial report also allows the investor to update his beliefs about n_1 for the case that he is dealing with a bad type manager. When $z_t = \lambda_{t-1} = \overline{\lambda}$, the investor now learns that $n_1 \geq t-1$ if he is dealing with a bad type manager.

The following proposition formalizes the optimal investment strategy.

Proposition 2 *With backward looking financial reports, the investor stops investing in period $\overline{n}_b$ or when an unprofitable financial report $z_t = \underline{\lambda}$ has been disclosed, whatever happens first. More specifically, for each period t, the optimal investment decision equals*

$$m_t = \begin{cases} e & \text{if } t \leq \overline{n}_b \text{ and } z_t = \overline{\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where

$$\overline{n}_b = max\left\{ t \,\middle|\, P_t(H_t)e \sum_{n_1=t}^{\infty} \phi_t^b(n_1|H_t, G)\overline{\lambda}^{n_1-t+1} \right.$$
$$\left. +(1-P_t(H_t))\beta_B e \sum_{n_1=t-1}^{\infty} \phi_t^b(n_1|H_t, B)\overline{\lambda}^{n_1-t+1} > e \right\} \tag{15}$$

and $H_t = H_{t-1} \cup \{z_t = \overline{\lambda}\}$.

Similar to Proposition 1, the existence of investment threshold period $\overline{n}_b$ crucially depends on assumption (A2). The effect of backward financial reporting follows from comparing the critical periods $\overline{n}_1$ and $\overline{n}_b$ when the investor stops investing. Observe that without financial reporting, the support for the posterior distribution $\phi_t(n_1|H_t, B)$ ranges from $\{1, 2, \ldots, \infty\}$. With backward looking financial reporting, this support shrinks to $\{t-1, t, \ldots, \infty\}$. This implies that with backward looking reporting, the investor assigns a higher posterior likelihood to n_1 than without backward looking reporting, i.e., $\phi_t^b(n_1|H_t, B) > \phi_0(n_1)$ for all $n_1 \geq t-1$. Consequently, the condition in (15) is weaker than the condition in (5):

Corollary 3 *Compared to the benchmark setting of no financial reporting, the investor continues investing in the firm longer with backward looking financial reporting in the sense that $\overline{n}_b \geq \overline{n}_1$.*

Backward looking financial reporting affects investment efficiency and social loss in the following way. It reduces underinvestment because the investor continues investing for $\overline{n}_b - \overline{n}_1 \geq 0$ additional periods. It also reduces the overinvestment problem to at most one period because the investor will stop investing in period t when he observes $z_t = \underline{\lambda}$.

5 Perfect forward looking financial reporting

We start with analyzing the hypothetical case of perfect forward looking information. This implies that the financial report perfectly reveals the profitability of the firm in the upcoming period, i.e. $z_t = \lambda_t$ for all periods t. Upon receiving the report z_t, the investor updates his beliefs about n_1 and manager type. When $z_t = \underline{\lambda}$, the investor obviously stops investing. Because the firm continued her operations into period t, he also learns that he has been dealing with a bad type manager (for a good type manager would have liquidated the firm at the end of period $t-1$). When $z_t = \overline{\lambda}$, the investor continues investing and he knows that $n_1 \geq t$ irrespective of manager type. Consequently, there is no updating of beliefs on manager type so that posterior beliefs regarding manager type equal prior beliefs, i.e., $P_t\left(H_{t-1}, \overline{\lambda}\right) = \theta$.

Lemma 3 *For each period t and each history H_t with $c_{t-1} = 1$ and $z_t = \overline{\lambda}$ it holds that*

$$\phi_t^f\left(n_1 \left| H_t\right.\right) = \frac{1_{n_1 \geq t} \phi_{t-1}^f\left(n_1 | H_{t-1}\right)}{1 - \phi_{t-1}^f\left(t-1 | H_{t-1}\right)} \tag{16}$$

$$P_t(H_t) = \theta. \tag{17}$$

Proposition 3 *With perfect forward looking financial reports, the investor stops investing in period $\overline{n}_f$ or when an unprofitable financial report $z_t = \underline{\lambda}$ has been disclosed, whatever happens first. More specifically, for each period t, the optimal investment* *decision equals*

$$m_t = \begin{cases} e & \text{if } t \leq \overline{n}_f \text{ and } z_t = \overline{\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

where

$$\overline{n}_f = max\left\{t \left| \theta e \sum_{n_1=t}^{\infty} \phi_t^f(n_1|H_t)\overline{\lambda}^{n_1-t+1} \right. \right. \\ \left. +(1-\theta)\beta_B e \sum_{n_1=t}^{\infty} \phi_t^f(n_1|H_t)\overline{\lambda}^{n_1-t+1} > e \right\} \tag{19}$$

and $H_t = H_{t-1} \cup \{z_t = \overline{\lambda}\}$.

The reason why the investor invests up to $\overline{n}_f$ rather than n_1 is because of the fact that the investor is still uncertain about manager type and he only receives a fraction β_B of the firm's cash flows when he is dealing with a bad type manager. Hence, even though the financial report reveals that the firm is profitable in the upcoming period (i.e., $z_t = \overline{\lambda}$), the investor's expected return may still be negative. For example, when the investor believes that the upcoming period is the final profitable period (i.e., $\phi_t^f(n_1|H_t) = 1$), the expected return of investment equals $e(\theta + (1-\theta)\beta_B)\overline{\lambda} - e$. Note that this expected return is negative when $(1-\theta)\beta_B$ is sufficiently small.

When comparing $\overline{n}_f$ to $\overline{n}_b$, observe that with backward looking financial information, the support for the posterior distribution $\phi_t^b(n_1|H_t)$ equals $\{t-1, t, \ldots, \infty\}$. With perfect forward looking financial information, the support shrinks to $\{t, t+1, \ldots, \infty\}$. Furthermore, because beliefs about manager type remain equal to the prior beliefs, the investor's beliefs of dealing with a good type manager are higher with perfect forward looking information than with backward looking information. By the same argument as before, we thus obtain that $\overline{n}_f \geq \overline{n_b}$.

Corollary 4 *With perfect forward looking financial reporting, the investor continues investment longer in the sense that* $\overline{n}_f \geq \overline{n}_b \geq \overline{n}_1$.

Perfect forward looking financial reporting further improves investment efficiency and reduces social loss. It reduces the underinvestment problem as the investor invests

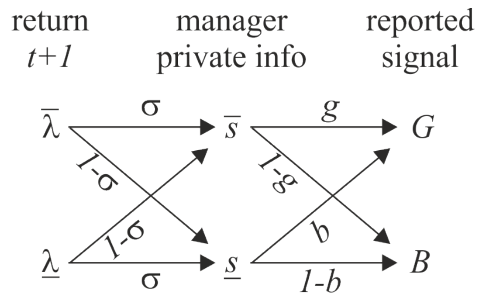

Figure 2: Information structure with imperfect forward looking financial reporting.

for $\overline{n}_f - \overline{n}_b \geq 0$ periods. Note that the overinvestment problem is completely resolved because the investor will stop investing in period t when he receives an unprofitable report.

6 Imperfect forward looking financial reporting

Perfect forward looking financial reporting is not feasible in practice. Nevertheless, accounting standards do make attempts in making financial reporting more forward looking. Accrual accounting and, in particular, fair value accounting aim to report expected future cash flows in a timely way. To analyze the effect of imperfect forward looking financial reporting, we impose the following structure on the financial report z_t (see Figure 2).

First, we consider a mixed attribute financial reporting system that issues a perfectly measured backward looking report in conjunction with an imperfectly measured forward looking report, i.e., $z_t = (z_t^b, z_t^f)$ where the backward looking report satisfies $z_t^b = \lambda_{t-1}$. Second, for the forward looking report we assume that at the start of period t the manager of the firm receives a noisy signal $s_t \in \{\overline{s}, \underline{s}\}$ about the profitability λ_t of the upcoming period. The precision of this signal is $\sigma \in [\frac{1}{2}, 1]$ with the interpretation that $Pr\left(s_t = \overline{s} \,\middle|\, \lambda_t = \overline{\lambda}\right) = Pr\left(s_t = \underline{s} \,|\, \lambda_t = \underline{\lambda}\right) = \sigma$. For $\sigma = \frac{1}{2}$, the signal is completely uninformative; for $\sigma = 1$, the signal is perfectly informative. Third, reporting standards govern how the private signal s_t is transformed into the financial report $z_t^f \in \{\overline{z}, \underline{z}\}$. Reporting standards are characterized by a pair (g, b) with $g, b \in [0,1]$ such that $Pr\left(z_t^f = \overline{z}\,|s_t = \overline{s}\right) = g$ and $Pr\left(z_t^f = \overline{z}\,|s_t = \underline{s}\right) = b$ with $g \geq b$ so that report $\overline{z}$ indeed conveys better news than $\underline{z}$. One can interpret the manager's private signal s_t as the verifiable evidence that can be used to justify the financial report z_t^f. For example, s_t can be verifiable information about expected future cash flows to justify a fair value estimate. Alternatively, s_t can be bad news that requires the recognition of an impairment expense.

Observe that the parameters (g, b) determine the informativeness of the forward looking financial report z_t^f. When $g = b$, the report z_t^f is completely uninformative about the manager's private signal s_t; when $g = 1$ and $b = 0$, the report z_t^f perfectly reveals the manager's private signal s_t. We classify the forward looking report z_t^f as conservative or aggressive in the same way as Bagnoli and Watts (2005) and Venugopalan (2001). A conservative report satisfies $g \leq 1$ and $b = 0$ and implies that there is no over-reporting of the manager's private signal $s_t = \underline{s}$. The degree of conservatism is measured by the probability of under-reporting the good news private signal, i.e., $1 - g$ (cf. Chen, Hemmer and Zhang 2007, Gigler and Hemmer 2001). An aggressive report satisfies $g = 1$ and $b \geq 0$ and implies that there is no under-reporting of the manager's private signal $s_t = \overline{s}$. The degree of aggressiveness is measured by the probability of over-reporting the bad news private signal, i.e., b.

Lemma 4 *For each period t and each history H_t with $c_{t-1} = 1$ and $(z_t^b, z_t^f) = (\overline{\lambda}, z)$ it holds that*

$$\phi_t^z\left(n_1\,|H_t\right) = \frac{\phi_{t-1}^z(n_1|H_{t-1},G)P_{t-1}(H_{t-1}) + \phi_{t-1}^z(n_1|H_{t-1},B)(1-P_{t-1}(H_{t-1}))}{1-\phi_{t-1}^z(t-1|H_{t-1},G)P_{t-1}(H_{t-1})} \tag{20}$$

$$P_t(H_t) = \frac{P_{t-1}\left(H_{t-1}\right) - \phi_{t-1}^z\left(t-1|H_{t-1},G\right)P_{t-1}\left(H_{t-1}\right)}{1-\phi_{t-1}^z\left(t-1|H_{t-1},G\right)P_{t-1}\left(H_{t-1}\right)} \tag{21}$$

where

$$\phi_t^z\left(n_1|H_t,G\right) = 1_{n_1\geq t}\frac{\phi_{t-1}^z\left(n_1|H_{t-1},G\right)}{1-\phi_{t-1}^z\left(t-1|H_{t-1},G\right)}, \tag{22}$$

$$\phi_t^z\left(n_1|H_t,B\right) = $$

$$1_{n_1\geq t-1}\frac{Pr(z_t^f=z|n_1)\phi_{t-1}^z(n_1|H_{t-1},B)}{Pr(z_t^f=z|\lambda_t=\underline{\lambda})\phi_{t-1}^z(t-1|H_{t-1},B)+Pr(z_t^f=z|\lambda_t=\overline{\lambda})\sum_{n=t}^{\infty}\phi_{t-1}^z(n|H_{t-1},B)}. \tag{23}$$

The investor updates his beliefs as follows. When $z_t^b = \underline{\lambda}$, the investor learns that the preceding period was unprofitable so that he stops investing. When $z_t^b = \overline{\lambda}$, he learns that the preceding period was profitable. When the investor presumes that he is dealing with a bad type manager, the forward looking report does reveal some information about n_1. Because the investor knows that $n_1 \geq t-1$, he knows that $\lambda_t = \underline{\lambda}$ for $n_1 = t-1$ and that $\lambda_t = \overline{\lambda}$ for $n_1 > t-1$. Hence, a bad news forward looking report $z_t^f = \underline{z}$ is more likely to occur when $n_1 = t-1$ whereas a good news forward looking report $z_t^f = \overline{z}$ is more likely to occur when $n_1 > t-1$. In other words, a good news forward looking report makes the investor more optimistic about n_1 than a bad news forward looking report. In contrast, when the investor presumes that he is dealing with a good type manager, the forward looking report z_t^f does not reveal any information about n_1. Because the firm has continued its operations, the investor knows that $\lambda_t = \overline{\lambda}$ so that the likelihood of receiving the report z_t^f is independent of n_1. Observe that this also implies that the investor does not use the forward looking report to update his beliefs about manager type (cf. (21)) as the beliefs $\phi_t^z(n_1|H_t, G)$ only depend on the backward looking report.

Observe that one can also represent the updating of beliefs as a two step process. In the first step, the investor updates his beliefs on n_1 using the backward looking report z_t^b; this results in the beliefs $\phi_t^z(n_1|H_{t-1}, z_t^b, \cdot)$ where

$$\phi_t^z\left(n_1|H_{t-1}, z_t^b, G\right) = 1_{n_1 \geq t} \frac{\phi_{t-1}^z\left(n_1|H_{t-1}, G\right)}{1 - \phi_{t-1}^z\left(t-1|H_{t-1}, G\right)}, \tag{24}$$

$$\phi_t^z\left(n_1|H_{t-1}, z_t^b, B\right) = 1_{n_1 \geq t-1} \frac{\phi_{t-1}^z\left(n_1|H_{t-1}, B\right)}{1 - \phi_{t-1}^z\left(t-1|H_{t-1}, B\right)}. \tag{25}$$

In the second step, the investor updates $\phi_t^z(n_1|H_{t-1}, z_t^b, \cdot)$ to $\phi_t^z(n_1|H_t, \cdot)$ using the forward looking report z_t^f. This yields

$$\phi_t^z\left(n_1|H_t, G\right) = \phi_{t-1}^z\left(n_1|H_{t-1}, z_t^b, G\right), \tag{26}$$

$$\phi_t^z\left(n_1|H_t, B\right) =$$
$$1_{n_1 \geq t-1} \frac{Pr(z_t^f = z|n_1)\phi_{t-1}^z\left(n_1|H_{t-1}, z_t^b, B\right)}{Pr(z_t^f = z|\lambda_t = \underline{\lambda})\phi_{t-1}^z\left(t-1|H_{t-1}, z_t^b, B\right) + Pr(z_t^f = z|\lambda_t = \overline{\lambda})\sum_{n=t}^{\infty}\phi_{t-1}^z\left(n|H_{t-1}, z_t^b, B\right)}. \tag{27}$$

Observe that substituting (24) into (26) yields expression (22). Similarly, for expressions (25) and (27); they are equivalent to (23).

Lemma 5 *The backward looking report z_t^b is sufficient for the forward looking reports $z_1^f, z_2^f, \ldots, z_{t-1}^f$ received in the past periods, i.e., $\phi_t^z(n_1|H_{t-1}, z_t^b, G) = \phi_t^z(n_1|c_{t-1}, z_t^b, z_t^f, G)$ and $\phi_t^z(n_1|H_{t-1}, z_t^b, B) = \phi_t^z(n_1|c_{t-1}, z_t^b, z_t^f, B)$.*

Lemma 5 is intuitive as the backward looking report resolves all uncertainty regarding the firm's profitability of periods in the past. Hence, there is no added value to knowing the history of forward looking financial reports. Lemma 5 is essential for the derivation of the optimal investment strategy; it implies that the investment thresholds do not depend on the complete history of forward looking reports.

Proposition 4 *With imperfect forward looking financial reports characterized by (g, b), the optimal investment decision equals*

$$m_t = \begin{cases} e & \text{if } t \leq \overline{n}_{\underline{z}}(g,b) \text{ and } z_t^b = \overline{\lambda} \\ e & \text{if } \overline{n}_{\underline{z}}(g,b) < t \leq \overline{n}_{\overline{z}}(g,b) \text{ and } (z_t^b, z_t^f) = (\overline{\lambda}, \overline{z}) \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

where

$$\overline{n}_z(g,b) = max \left\{ t \,\middle|\, P_t(H_t) e \sum_{n_1=t}^{\infty} \phi_t^z(n_1|H_t, G)\overline{\lambda}^{n_1-t+1} + (1 - P_t(H_t))\beta_B e \sum_{n_1=t-1}^{\infty} \phi_t^z(n_1|H_t, B)\overline{\lambda}^{n_1-t+1} > e \right\} \tag{29}$$

with $z = \overline{z}, \underline{z}$ and $H_t = H_{t-1} \cup \{z_t^b = \overline{\lambda}, z_t^f = z\}$.

Similar to Proposition 1, observe that the existence of investment threshold periods $\overline{n}_{\overline{z}}$ and $\overline{n}_{\underline{z}}$ crucially depends on assumption (A2). Proposition 4 states that up to period $\overline{n}_{\underline{z}}(g, b)$, the investment decision is independent of the forward looking report. The investor invests as long as the backward looking report reveals that the firm was profitable, i.e., $\lambda_{t-1} = \overline{\lambda}$. It implies that a bad news forward looking report has insufficient impact on the investor's beliefs to make him stop investing. During the time interval $(\overline{n}_{\underline{z}}(g,b), \overline{n}_{\overline{z}}(g,b)]$, the forward looking report is relevant. The investor only invests in period t when the backward looking report reveals that the firm was still profitable in the preceding period *and* the forward looking report contains good news, i.e., $z_t^f = \overline{z}$. Hence, during the time interval $(\overline{n}_{\underline{z}}(g,b), \overline{n}_{\overline{z}}(g,b)]$, the investor may sometimes invest and sometimes not. After period $\overline{n}_{\overline{z}}(g,b)$, the investor never invests. At that point in time, his beliefs about n_1 and manager type are too pessimistic to continue investment; even when he would receive a good news forward looking report.

To illustrate the incremental value of forward looking disclosures to backward looking disclosures, denote by $E^b(t)$ the expected payoff of the investor at the start of period t, i.e.,

$$E^b(t) = P_t(H_t) e \sum_{n_1=t}^{\infty} \phi_t^b(n_1|H_t, G)\overline{\lambda}^{n_1-t+1} + (1 - P_t(H_t))\beta_B e \sum_{n_1=t-1}^{\infty} \phi_t^b(n_1|H_t, B)\overline{\lambda}^{n_1-t+1}. \tag{30}$$

Denote by $E^z(t)$ the expected payoff of the investor at the start of period t when the forward looking report $z_t^f = z$ has been received, i.e.,

$$E^z(t) = P_t(H_t) e \sum_{n_1=t}^{\infty} \phi_t^z(n_1|H_t, G)\overline{\lambda}^{n_1-t+1} + (1 - P_t(H_t))\beta_B e \sum_{n_1=t-1}^{\infty} \phi_t^z(n_1|H_t, B)\overline{\lambda}^{n_1-t+1}. \tag{31}$$

Lemma 6 *For each $t \geq 1$ it holds that*

$$E^z(t) = E^b(t) + (1 - P_t(H_t))\beta_B e a_z(t) \left(\sum_{n=t}^{\infty} \phi_t^b(n|H_t, B)(\overline{\lambda}^{n-t+1} - 1) \right) \tag{32}$$

where

$$a_{\overline{z}}(t) = \frac{(2\sigma - 1)(g - b)\phi_t^b(t-1|H_t, B)}{\sigma g + (1-\sigma)b - (2\sigma - 1)(g - b)\phi_t^b(t-1|H_t, B)} \tag{33}$$

$$a_{\underline{z}}(t) = \frac{(2\sigma - 1)(b - g)\phi_t^b(t-1|H_t, B)}{\sigma(1-g) + (1-\sigma)(1-b) - (2\sigma - 1)(b - g)\phi_t^b(t-1|H_t, B)} \tag{34}$$

Lemma 6 expresses the incremental value of the forward looking financial report. Observe that the effect of the forward looking financial report only arises through the multiplier $a_z(t)$.

Corollary 5 *For any imperfect forward looking disclosure characterized by* (g, b) *with* $g \geq b$ *it holds that* $\overline{n}_{\underline{z}}(g, b) \leq \overline{n}_b \leq \overline{n}_{\overline{z}}(g, b)$.

Compared to the setting with only backward looking financial reporting, it follows intuitively that $\overline{n}_{\underline{z}}(g, b) \leq \overline{n}_b \leq \overline{n}_{\overline{z}}(g, b)$. Forward looking good news makes the investor more optimistic about n_1 so that he will stop investing (weakly) later than in the case with only backward looking information. In particular, to maximize the expected payoff $E^{\overline{z}}(t)$ conditional on having received a good news forward looking report at the start of period t, one needs to maximize the multiplier in (33) with respect to the reporting parameters g and b. It is straightforward to show that the expected payoff is maximized for full disclosure, i.e., $(g, b) = (1, 0)$. A full disclosure reporting system perfectly reveals the manager's private signal s_t so that it provides the most information on the profitability λ_t of the firm. A similar argument applies to forward looking bad news. A bad news forward looking financial report makes the investor more pessimistic about n_1 so that he will stop investing (weakly) earlier. Furthermore, full disclosure minimizes the expected payoff $E^{\underline{z}}(t)$.

6.1 Underinvestment and aggressive reporting

This subsection analyzes how forward looking financial reporting can address underinvestment. Our starting point is that there is undervestment with a backward looking financial report, i.e., $\overline{n}_b < n_1$, i.e., the investors stops investing prematurely. By definition of $\overline{n}_b$, it holds that $E^b(\overline{n}_b) \geq e > E^b(\overline{n}_b + 1)$. Introducing forward looking financial reporting has two effects. First, investment may continue for more periods as $\overline{n}_{\overline{z}}(g, b) \geq \overline{n}_b$. Second, during the time interval $(\overline{n}_{\underline{z}}(g, b), \overline{n}_{\overline{z}}(g, b)]$ investment only arises when a good news report is received. Hence, the expected number of periods that investment occurs with forward looking reporting equals

$$\overline{n}_{\underline{z}}(g, b) + Pr(z_t^f = \overline{z} | \lambda_t = \overline{\lambda}) \left(\overline{n}_{\overline{z}}(g, b) - \overline{n}_{\underline{z}}(g, b) \right).$$

In contrast, with backward looking financial reporting, investment occurs for $\overline{n}_b$ periods. Consequently, forward looking financial reporting reduces underinvestment when there exists a reporting system (g, b) such that

$$Pr(z_t^f = \overline{z}|\lambda_t = \overline{\lambda})\left(\overline{n}_{\overline{z}}(g,b) - \overline{n}_b\right) \geq \Big(1 - Pr(z_t^f = \overline{z}|\lambda_t = \overline{\lambda})\Big)\Big(\overline{n}_b - \overline{n}_{\underline{z}}(g,b)\Big).\quad(35)$$

The left hand side of the inequality expresses the benefit of forward looking reporting; it equals the expected number of periods that investment occurs beyond period $\overline{n}_b$. This benefit is increasing in both the likelihood of receiving a good news forward looking report and the investment threshold $\overline{n}_{\overline{z}}(g,b)$. The right hand side expresses the cost of forward looking reporting: it equals the expected number of periods that investment does not arise before period $\overline{n}_b$. The cost is increasing in the likelihood of receiving a bad news forward looking report and decreasing in the investment threshold $\overline{n}_{\underline{z}}(g,b)$.

Proposition 5 *If the full disclosure reporting system reduces underinvestment, then there exists an aggressive reporting system that reduces underinvestment even better.*

Aggressive reporting is better at addressing underinvestment because it increases the likelihood of disclosing a good news forward looking financial report compared to full disclosure. In particular, Proposition 5 exploits the fact that one can introduce a small degree of overreporting without affecting the investment thresholds, i.e., there generally exists $b > 0$ such that $\overline{n}_{\overline{z}}(1,b) = \overline{n}_{\overline{z}}(1,0)$ and $\overline{n}_{\underline{z}}(1,b) = \overline{n}_{\underline{z}}(1,0)$.

Observe that Proposition 5 does not imply that an aggressive reporting system is preferred over a conservative reporting system. An aggressive reporting system affects the cost and benefits of forward looking reporting in a different way than a conservative reporting system. First, an aggressive reporting system increases the likelihood of disclosing a good news report which in turn increases the benefit and decreases the cost. A conservative reporting system decreases the likelihood of disclosing a good news report. Second, an aggressive reporting system reduces the investment threshold $\overline{n}_{\overline{z}}(g,b)$ which in turn decreases the benefit of forward looking reporting. In contrast, a conservative reporting system increases the investment threshold $\overline{n}_{\underline{z}}(g,b)$ which in turn decreases the cost of forward looking reporting. Summarizing, the trade-off between aggressive and conservative reporting depends on how the effect on the likelihood of disclosing a good news report trades off against the effects on the investment thresholds. This trade-off likely depends on the presumed prior distribution ϕ_0 of n_1.

6.2 Overinvestment and conservative reporting

This subsection analyzes how forward looking financial reporting can address overinvestment. For this purpose, presume that overinvestment arises in the setting with only backward looking financial report, i.e., $\overline{n}_b > n_1$, i.e., the investors continues investing for too many periods. Introducing forward looking financial reporting has two effects. First, investment may continue for more periods as $\overline{n}_{\overline{z}}(g,b) \geq \overline{n}_b$. Second, during the time interval $(\overline{n}_{\underline{z}}(g,b), \overline{n}_{\overline{z}}(g,b)]$ investment does not arise when a bad news report is received. By the same argument as in Subsection 6.2, one can derive that forward looking financial reporting reduces overinvestment when there exists a reporting system (g,b) such that

$$Pr(z_t^f = \overline{z}|\lambda_t = \overline{\lambda})\left(\overline{n}_{\overline{z}}(g,b) - \overline{n}_b\right) \leq \left(1 - Pr(z_t^f = \overline{z}|\lambda_t = \overline{\lambda})\right)\left(\overline{n}_b - \overline{n}_{\underline{z}}(g,b)\right). \quad (36)$$

The left hand side of the inequality now expresses the cost of forward looking reporting: it equals the expected number of periods that investment occurs beyond period $\overline{n}_b$. This cost is increasing in both the likelihood of receiving a good news forward looking report and the investment threshold $\overline{n}_{\overline{z}}(g,b)$. The right hand side expresses the benefit of forward looking reporting: it equals the expected number of periods that investment does not arise before period $\overline{n}_b$. The benefit is increasing in the likelihood of receiving a bad news forward looking report and decreasing in the investment threshold $\overline{n}_{\underline{z}}(g,b)$.

Proposition 6 *If the full disclosure reporting system reduces overinvestment, then there exists a conservative reporting system that reduces overinvestment even better.*

Conservative reporting is better at addressing overinvestment because it decreases the likelihood of disclosing a good news forward looking financial report compared to full disclosure. In particular, Proposition 6 exploits the fact that one can introduce a small degree of underreporting without affecting the investment thresholds, i.e., there generally exists $g < 1$ such that $\overline{n}_{\overline{z}}(g, 0) = \overline{n}_{\overline{z}}(1, 0)$ and $\overline{n}_{\underline{z}}(g, 0) = \overline{n}_{\underline{z}}(1, 0)$.

7 Concluding remarks

We introduce the possibility of reinvesting project returns in a repeated sender-receiver game. We also allow for a production technology that features positive returns to a certain point in time after which it switches to negative returns. The receiver / manager is aware of the switching point but the sender/investor is not. Such information asymmetry leads to underinvestment or overinvestment in the absence of financial reports. Both backward looking and forward looking financial information that are perfect in nature ameliorate the overinvestment and underinvestment problems from a social standpoint. However, forward looking financial information does a better job of ameliorating these problems. We consider aggressive and conservative reporting in a mixed attribute financial reporting system that combines perfect backward looking financial information with noisy forward looking financial information.

We find that aggressive reporting better addresses the issue of underinvestment while conservative reporting better addresses the issue of overinvestment. This contrasts the conventional opinion of standard setters that neutral or conservative reporting is always the preferred option.

Our findings seem consistent with practice. Underinvestment is primarily a concern for start-up and growth firms where investors may be holding back investment because of higher uncertainty and/or risk. Such firms seem to engage in more aggressive reporting by focusing on alternative financial performance measures. During the 1990's, technology companies claimed that profit figures were understated because accounting standards did not properly reflect their investments in intangibles. Standard setters responded to this by making the accounting standards more aggressive in the sense that it would allow for the capitalization of internally developed software. More recently, internet-based companies like Google and Facebook put more emphasis on non-financial information (e.g., web-traffic, number of daily users) rather than financial information when searching for external financing.

In contrast, overinvestment is more of a concern for mature firms with little growth opportunities. Standard setters have increased the level of conservative reporting to reduce opportunistic reporting behavior. For example, lease accounting reduced the opportunities for off-balance sheet financing and following Enron, consolidation rules for special purpose entities became more stringent. Currently, standards setters are also debating the use of non-GAAP reporting practices.

Finally, observe that our findings suggest that reporting standards should be counter-cyclical in order to address over and underinvestment issues. In bull markets, when investor sentiment or optimism is high, overinvestment is more of a concern than underinvestment so that conservative reporting may be more desirable. Conversely for bear markets, when investor sentiment is low, underinvestment is more of a concern than overinvestment so that aggressive reporting may be more desirable. This highlights an important drawback of fair value based accounting standards, which tend to be pro-cyclical. Standard setters should be aware that fair value based standards may increase over and underinvestment problems in capital markets.

Appendix

Proof of Lemma 1. Let $t \geq 1$. It holds that

$$\phi_t \left(n_1 \left| H_t \right. \right) = \frac{Pr \left(n_1, H_t \right)}{Pr \left(H_t \right)}.$$

Using that $H_t = H_{t-1} \cup \{c_{t-1}\}$ we can write

$$\begin{aligned} Pr \left(n_1, H_t \right) &= Pr \left(n_1, H_{t-1}, c_{t-1} \right) = Pr \left(n_1, H_{t-1}, c_{t-1}, G \right) + Pr \left(n_1, H_{t-1}, c_{t-1}, B \right) \\ &= Pr \left(c_{t-1} | n_1, H_{t-1}, G \right) Pr \left(n_1 | H_{t-1}, G \right) Pr \left(G | H_{t-1} \right) \\ &\quad + Pr \left(c_{t-1} | n_1, H_{t-1}, B \right) Pr \left(n_1 | H_{t-1}, B \right) Pr \left(B | H_{t-1} \right) \\ &= Pr \left(c_{t-1} | n_1, H_{t-1}, G \right) \phi_{t-1} \left(n_1 | H_{t-1}, G \right) P_{t-1} \left(H_{t-1} \right) \\ &\quad + Pr \left(c_{t-1} | n_1, H_{t-1}, B \right) \phi_{t-1} \left(n_1 | H_{t-1}, B \right) \left(1 - P_{t-1} \left(H_{t-1} \right) \right). \end{aligned}$$

Substituting $Pr\left(c_{t-1}|n_1, H_{t-1}, G\right) = 1_{n_1 \geq t}$ and $Pr\left(c_{t-1}|n_1, H_{t-1}, B\right) = 1$, we obtain

$$Pr\left(n_1, H_t\right) = 1_{n_1 \geq t}\phi_{t-1}\left(n_1|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) + \phi_{t-1}\left(n_1|H_{t-1}, B\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right).$$

In a similar way, we can derive that

$$\begin{aligned} Pr\left(H_t\right) &= \sum_n Pr\left(n, H_t\right) \\ &= \sum_n 1_{n \geq t}\phi_{t-1}\left(n|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) \\ &\quad + \sum_n \phi_{t-1}\left(n|H_{t-1}, B\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right) \\ &= \sum_{n=t}^{\infty} \phi_{t-1}\left(n|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) + \left(1 - P_{t-1}\left(H_{t-1}\right)\right) \\ &= \left(1 - \phi_{t-1}\left(t-1|H_{t-1}, G\right)\right) P_{t-1}\left(H_{t-1}\right) + \left(1 - P_{t-1}\left(H_{t-1}\right)\right) \\ &= 1 - \phi_{t-1}\left(t-1|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right). \end{aligned}$$

Hence, the posterior is equal to

$$\phi_t\left(n_1\,|H_t\right) = \frac{1_{n_1 \geq t}\phi_{t-1}\left(n_1|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) + \phi_{t-1}\left(n_1|H_{t-1}, B\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right)}{1 - \phi_{t-1}\left(t-1|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right)}.$$

For the posterior beliefs of manager type, it holds that:

$$P_t(H_t) = Pr\left(G|H_{t-1}c_{t-1}\right) = \frac{Pr\left(G, H_{t-1}, c_{t-1}\right)}{Pr\left(H_t\right)}.$$

Rewriting gives

$$\begin{aligned} Pr\left(G, H_{t-1}, c_{t-1}\right) &= Pr\left(c_{t-1}|G, H_{t-1}\right) Pr\left(G|H_{t-1}\right) Pr\left(H_{t-1}\right) \\ &= Pr\left(c_{t-1}|G, H_{t-1}\right) P_{t-1}\left(H_{t-1}\right) Pr\left(H_{t-1}\right) \end{aligned}$$

and

$$\begin{aligned} Pr\left(H_t\right) &= Pr\left(G, H_t\right) + Pr\left(B, H_t\right) = Pr\left(G, H_{t-1}, c_{t-1}\right) + Pr\left(B, H_{t-1}, c_{t-1}\right) \\ &= Pr\left(c_{t-1}|G, H_{t-1}\right) P_{t-1}\left(H_{t-1}\right) Pr\left(H_{t-1}\right) \\ &\quad + Pr\left(c_{t-1}|B, H_{t-1}\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right) Pr\left(H_{t-1}\right) \end{aligned}$$

so that

$$P_t(H_t) = \frac{Pr\left(c_{t-1}|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right)}{Pr\left(c_{t-1}|H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) + Pr\left(c_{t-1}|H_{t-1}, B\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right)}.$$

Substituting

$$\begin{aligned} Pr\left(c_{t-1}|H_{t-1},G\right) &= \frac{Pr\left(c_{t-1},H_{t-1},G\right)}{Pr\left(H_{t-1},G\right)} = \frac{\sum_{n=1}^{\infty} Pr\left(c_{t-1},H_{t-1},G,n\right)}{Pr\left(H_{t-1},G\right)} \\ &= \sum_{n=1}^{\infty} Pr\left(c_{t-1}|H_{t-1},G,n\right) Pr\left(n|H_{t-1},G\right) \\ &= \sum_{n=1}^{\infty} Pr\left(c_{t-1}|H_{t-1},G,n\right) \phi_{t-1}\left(n|H_{t-1},G\right) \\ &= \sum_{n=t}^{\infty} \phi_{t-1}\left(n|H_{t-1},G\right) = 1-\phi_{t-1}\left(t-1|H_{t-1},G\right) \end{aligned}$$

and

$$\begin{aligned} Pr\left(c_t|H_{t-1},B\right) &= \sum_{n=1}^{\infty} Pr\left(c_{t-1}|H_{t-1},B,n\right) \phi_{t-1}\left(n|H_{t-1},B\right) \\ &= \sum_{n=1}^{\infty} \phi_{t-1}\left(n|H_{t-1},B\right) = 1 \end{aligned}$$

yields

$$P_t(H_t) = \frac{\left(1-\phi_{t-1}\left(t-1|H_{t-1},G\right)\right) P_{t-1}\left(H_{t-1}\right)}{1-\phi_{t-1}\left(t-1|H_{t-1},G\right) P_{t-1}\left(H_{t-1}\right)}.$$

The posterior beliefs $\phi_t\left(n_1|H_t,G\right)$ and $\phi_t\left(n_1|H_t,B\right)$ are updated as follows:

$$\phi_t\left(n_1|H_t,G\right) = \frac{Pr\left(n_1,H_{t-1},c_{t-1},G\right)}{Pr\left(H_{t-1},c_{t-1},G\right)}.$$

Substituting

$$\begin{aligned} Pr\left(n_1,H_{t-1},c_{t-1},G\right) &= Pr\left(c_{t-1}|n_1,H_{t-1},G\right) Pr\left(n_1|H_{t-1},G\right) Pr\left(H_{t-1},G\right) \\ &= 1_{n_1\geq t}\phi_{t-1}\left(n_1|H_{t-1},G\right) Pr\left(H_{t-1},G\right) \end{aligned}$$

and

$$\begin{aligned} Pr\left(H_{t-1},c_{t-1},G\right) &= \sum_{n=1}^{\infty} Pr\left(n,H_{t-1},c_{t-1},G\right) \\ &= \sum_{n=1}^{\infty} Pr\left(c_{t-1}|n,H_{t-1},G\right) Pr\left(n|H_{t-1},G\right) Pr\left(H_{t-1},G\right) \\ &= \sum_{n=t}^{\infty} \phi_{t-1}\left(n|H_{t-1},G\right) Pr\left(H_{t-1},G\right). \end{aligned}$$

yields

$$\phi_t\left(n_1|H_t,g\right) = 1_{n_1\geq t}\frac{\phi_{t-1}\left(n_1|H_{t-1},G\right)}{\sum_{n=t}^{\infty}\phi_{t-1}\left(n|H_{t-1},G\right)}.$$

In a similar way one derives that

$$\phi_t\left(n_1|H_t,B\right)=\phi_{t-1}\left(n_1|H_{t-1},B\right).$$

Proof of Proposition 1. At the start of period t and given $c_{t-1}=1$, the expected payoff of investing m_t equals

$$\begin{aligned}
E_{\beta,n_1}\left(\beta\overline{\lambda}^{\max(n_1-t+1,0)}\middle| H_t\right) &= P_t(H_t)E_{n_1}\left(\overline{\lambda}^{\max(n_1-t+1,0)}\middle| H_t,G\right)\\
&\quad+(1-P_t(H_t))\beta_B E_{n_1}\left(\overline{\lambda}^{\max(n_1-t+1,0)}\middle| H_t,B\right)\\
&= P_t(H_t)\sum_{n_1=t}^{\infty}\phi_t(n_1|H_t,G)\overline{\lambda}^{\max(n_1-t+1,0)}\\
&\quad+(1-P_t(H_t))\beta_B\sum_{n_1=1}^{\infty}\phi_t(n_1|H_t,B)\overline{\lambda}^{\max(n_1-t+1,0)}\\
&= P_t(H_t)\sum_{n_1=t}^{\infty}\phi_t(n_1|H_t,G)\overline{\lambda}^{\max(n_1-t+1,0)}\\
&\quad+(1-P_t(H_t))\beta_B\sum_{n_1=1}^{\infty}\phi_0(n_1)\overline{\lambda}^{\max(n_1-t+1,0)}
\end{aligned}$$

where the second equality follows from (3) and the third equality follows from (4). Assumption (A1) implies that this expected payoff exceeds one for $t=1$. Assumption (A2) implies that this expected payoff is decreasing in t with an asymptotic value below one. Hence, investment $m_t=e$ is optimal for all periods $t\leq\overline{n}_1$ with $\overline{n}_1$ as defined in (5).

Proof of Lemma 2. The proof is similar to the proof of Lemma 1. The only difference is in the updating of the posteriors $\phi_t^b(n_1|H_t,G)$ and $\phi_t^b(n_1|H_t,B)$. To start with the former, assume $c_{t-1}=1$ and $z_t=\overline{\lambda}$. It holds that

$$\phi_t^b\left(n_1|H_t,G\right) = \frac{Pr\left(n_1,H_{t-1},c_{t-1},z_t,G\right)}{Pr\left(H_{t-1},c_{t-1},G\right)}.$$

Observe that

$$Pr\left(n_1,H_{t-1},c_{t-1},z_t,G\right) = Pr\left(c_{t-1},z_t|n_1,H_{t-1},G\right)Pr\left(n_1|H_{t-1},G\right)Pr\left(H_{t-1},G\right)$$

and that $c_{t-1}=1$ implies $\lambda_t=\overline{\lambda}$ and thus $n_1\geq t$. Consequently, $Pr\left(c_{t-1},z_t|n_1,H_{t-1},G\right)=Pr\left(c_{t-1}|n_1,H_{t-1},G\right)$ so that the resulting posterior is equivalent to the one in Lemma 1.

For the posterior $\phi_t^b(n_1|H_t, B)$, it holds that

$$\phi_t^b(n_1|H_t, B) = \frac{Pr(n_1, H_{t-1}, c_{t-1}, z_t, B)}{Pr(H_{t-1}, c_{t-1}, B)}.$$

Observe that

$$Pr(n_1, H_{t-1}, c_{t-1}, z_t, B) = Pr(c_{t-1}, z_t|n_1, H_{t-1}, B)\, Pr(n_1|H_{t-1}, B)\, Pr(H_{t-1}, B)$$

and that $c_{t-1} = 1$ does not provide any information on λ_t. However, the backward looking report $z_t = \lambda_{t-1} = \overline{\lambda}$ implies that $n_1 \geq t-1$. Substituting $Pr(c_{t-1}, z_t|n_1, H_{t-1}, B) = 1$ for $n_1 \geq t-1$ and $Pr(c_{t-1}, z_t|n_1, H_{t-1}, B) = 0$ for $n_1 \leq t-2$ and continuing the calculus as in Lemma 1 completes the proof.

Proof of Proposition 2. This proof is similar to Proposition 1.

Proof of Lemma 3. The proof is similar to the proof of Lemma 2. The only difference is in the updating of the posterior $\phi_t^f(n_1|H_t, B)$ and $P_t(H_t)$. To start with the former, assume $c_{t-1} = 1$ and $z_t = \lambda_t = \overline{\lambda}$. It holds that

$$\phi_t^f(n_1|H_t, B) = \frac{Pr(n_1, H_{t-1}, c_{t-1}, z_t, B)}{Pr(H_{t-1}, c_{t-1}, B)}.$$

Observe that

$$Pr(n_1, H_{t-1}, c_{t-1}, z_t, B) = Pr(c_{t-1}, z_t|n_1, H_{t-1}, B)\, Pr(n_1|H_{t-1}, B)\, Pr(H_{t-1}, B)$$

and that $z_t = \lambda_t = \overline{\lambda}$ implies $n_1 \geq t$. Consequently, $Pr(c_{t-1}, z_t|n_1, H_{t-1}, B) = 1_{n_1 \geq t}$ so that the resulting posterior is equivalent to $\phi_t(n_1|H_t, G)$.

For the beliefs of manager type, one can show in a similar way as in the proof of Lemma 1 that

$$P_t(H_t) = \frac{Pr(c_{t-1}, z_t|H_{t-1}, G)\, P_{t-1}(H_{t-1})}{Pr(c_{t-1}, z_t|H_{t-1}, G)\, P_{t-1}(H_{t-1}) + Pr(c_{t-1}, z_t|H_{t-1}, B)\,(1 - P_{t-1}(H_{t-1}))}.$$

Using that $z_t = \lambda_t = \overline{\lambda}$ implies $n_1 \geq t$ for both manager types G and B, one derives that

$$Pr(c_{t-1}, z_t|H_{t-1}, G) = \frac{Pr(c_{t-1}, z_t, H_{t-1}, G)}{Pr(H_{t-1}, G)} = \frac{\sum_{n=1}^{\infty} Pr(c_{t-1}, z_t, H_{t-1}, G, n)}{Pr(H_{t-1}, G)}$$

$$\begin{aligned} &= \sum_{n=1}^{\infty} Pr\left(c_{t-1}, z_t | H_{t-1}, G, n\right) Pr\left(n | H_{t-1}, G\right) \\ &= \sum_{n=1}^{\infty} Pr\left(c_{t-1}, z_t | H_{t-1}, G, n\right) \phi_{t-1}^{f}\left(n | H_{t-1}, G\right) \\ &= \sum_{n=t}^{\infty} \phi_{t-1}^{f}\left(n | H_{t-1}, G\right) = 1 - \phi_{t-1}^{f}\left(t-1 | H_{t-1}, G\right) \end{aligned}$$

and

$$Pr\left(c_{t-1}, z_t | H_{t-1}, B\right) = \sum_{n=t}^{\infty} \phi_{t-1}^{f}\left(n | H_{t-1}, B\right) = 1 - \phi_{t-1}^{f}\left(t-1 | H_{t-1}, B\right).$$

Because $\phi_1^f(0|H_0, G) = \phi_1^f(0|H_0, B) = \phi_0(0)$, it follows by induction that$\phi_{t-1}^{f}\left(t-1 | H_{t-1}, G\right) = \phi_{t-1}^{f}\left(t-1 | H_{t-1}, B\right)$ for all $t \geq 2$. Consequently, $Pr\left(c_{t-1}, z_t | H_{t-1}, G\right) = Pr\left(c_{t-1}, z_t | H_{t-1}, B\right)$ for all $t \geq 1$ so that

$$\begin{aligned} P_t(H_t) &= \frac{Pr\left(c_{t-1}, z_t | H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right)}{Pr\left(c_{t-1}, z_t | H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) + Pr\left(c_{t-1}, z_t | H_{t-1}, B\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right)} \\ &= \frac{Pr\left(c_{t-1}, z_t | H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right)}{Pr\left(c_{t-1}, z_t | H_{t-1}, G\right) P_{t-1}\left(H_{t-1}\right) + Pr\left(c_{t-1}, z_t | H_{t-1}, G\right)\left(1 - P_{t-1}\left(H_{t-1}\right)\right)} \\ &= P_{t-1}(H_{t-1}) \end{aligned}$$

for all $t \geq 1$. Consequently, $P_t(H_t) = P_0 = \theta$.

Proof of Proposition 3. This proof is similar to Proposition 1 and 2.

Proof of Lemma 4. It suffices to prove expressions (22) and (23). The proof of the former is similar to the proof in Lemma 2. For expression (23), observe that

$$Pr\left(n_1, H_{t-1}, c_{t-1}, z_t, B\right) = Pr\left(c_{t-1}, z_t | n_1, H_{t-1}, B\right) Pr\left(n_1 | H_{t-1}, B\right) Pr\left(H_{t-1}, B\right)$$

so that

$$\begin{aligned} \phi_t^z\left(n_1 | H_{t-1}, z_t, B\right) &= \frac{Pr\left(n_1, H_{t-1}, c_{t-1}, z_t, B\right)}{Pr\left(H_{t-1}, c_{t-1}, z_t, B\right)} \\ &= \frac{Pr\left(c_{t-1}, z_t | n_1, H_{t-1}, B\right) Pr\left(n_1 | H_{t-1}, B\right) Pr\left(H_{t-1}, B\right)}{\sum_{n_1=t-1}^{\infty} Pr\left(c_{t-1}, z_t | n_1, H_{t-1}, B\right) Pr\left(n_1 | H_{t-1}, B\right) Pr\left(H_{t-1}, B\right)} \\ &= \frac{Pr\left(c_{t-1}, z_t | n_1, H_{t-1}, B\right) \phi_{t-1}^z\left(n_1 | H_{t-1}, B\right)}{\sum_{n_1=t-1}^{\infty} Pr\left(c_{t-1}, z_t | n_1, H_{t-1}, B\right) \phi_{t-1}^z\left(n_1 | H_{t-1}, B\right)} \\ &= 1_{n_1 \geq t-1} \frac{Pr\left(z_t^f = z | n_1, H_{t-1}, B\right) \phi_{t-1}^z\left(n_1 | H_{t-1}, B\right)}{\sum_{n_1=t-1}^{\infty} Pr\left(z_t^f = z | n_1, H_{t-1}, B\right) \phi_{t-1}^z\left(n_1 | H_{t-1}, B\right)}, \end{aligned}$$

where the last equality follows from the fact that $c_{t-1} = 1$ and $z_t^b = \overline{\lambda}$ imply $n_1 \geq t-1$. Using that $\lambda_t = \underline{\lambda}$ for $n_1 = t-1$ and $\lambda_t = \overline{\lambda}$ for $n_1 \geq t$, it follows that

$$Pr\left(z_t^f = z|n_1, H_{t-1}, B\right) = \begin{cases} Pr(z_t^f = z|\lambda_t = \underline{\lambda}) & \text{if } n_1 = t-1 \\ Pr(z_t^f = z|\lambda_t = \overline{\lambda}) & \text{if } n_1 \geq t \end{cases} \tag{37}$$

Substituting this in the expression above for $\phi_t^z\left(n_1|H_{t-1}, z_t, B\right)$ completes the proof.

Proof of Lemma 5. The proof of the first part uses the same argumentation as Corollary 2. The proof for $\phi_t^z(n_1|H_t, B)$ goes by induction. Assume that $\phi_\tau^z(n_1|H_\tau, B) = \phi_\tau^z(n_1|c_{\tau-1}, z_\tau^b, B)$ for all $\tau \leq t-1$. Recall from the proof of Lemma 4 that

$$\phi_t^z\left(n_1|H_{t-1}, z_t, B\right) = \\ 1_{n_1 \geq t-1} \frac{Pr\left(z_t^f = z|n_1, H_{t-1}, B\right) \phi_{t-1}^z\left(n_1|H_{t-1}, B\right)}{\sum_{n=t-1}^{\infty} Pr\left(z_t^f = z|n, H_{t-1}, B\right) \phi_{t-1}^z\left(n|H_{t-1}, B\right)}. \tag{38}$$

Similarly, we have for all $n_1 \geq t-1$ that

$$\begin{aligned} \phi_{t-1}^z\left(n_1|H_{t-2}, z_{t-1}, B\right) &= \\ &= 1_{n_1 \geq t-2} \frac{Pr\left(z_{t-1}^f = z|n_1, H_{t-2}, B\right) \phi_{t-2}^z\left(n_1|H_{t-2}, B\right)}{\sum_{n=t-2}^{\infty} Pr\left(z_{t-1}^f = z|n, H_{t-2}, B\right) \phi_{t-2}^z\left(n|H_{t-2}, B\right)} \\ &= 1_{n_1 \geq t-2} \frac{Pr(z_{t-1}^f = z|\lambda_t = \overline{\lambda}) \phi_{t-2}^z\left(n_1|H_{t-2}, B\right)}{\sum_{n=t-2}^{\infty} Pr\left(z_t^f = z|n, H_{t-2}, B\right) \phi_{t-2}^z\left(n|H_{t-2}, B\right)} \\ &= 1_{n_1 \geq t-2} \alpha \phi_{t-2}^z\left(n_1|H_{t-2}, B\right) = 1_{n_1 \geq t-2} \alpha \phi_{t-2}^z\left(n_1|c_{t-3}, z_{t-2}^b, B\right) \end{aligned}$$

with

$$\alpha = \frac{Pr(z_{t-1}^f = z|\lambda_t = \overline{\lambda})}{\sum_{n=t-2}^{\infty} Pr\left(z_t^f = z|n, H_{t-2}, B\right) \phi_{t-2}^z\left(n|H_{t-2}, B\right)}.$$

Substituting this expression into (38) yields

$$\phi_t^z\left(n_1|H_{t-1}, z_t, B\right) = \\ 1_{n_1 \geq t-1} \frac{Pr\left(z_t^f = z|n_1, H_{t-1}, B\right) \phi_{t-2}^z\left(n_1|c_{t-3}, z_{t-2}^b, B\right)}{\sum_{n=t-1}^{\infty} Pr\left(z_t^f = z|n, H_{t-1}, B\right) \phi_{t-2}^z\left(n|c_{t-3}, z_{t-2}^b, B\right)}.$$

From expression (37) it then follows that $\phi_t^z\left(n_1|H_{t-1}, z_t, B\right)$ does not depend on $z_1^f, z_2^f, \ldots, z_{t-1}^f$.

To complete the proof, observe that by the same argument as above and using that $\phi_0(n_1)$ does not depend on any financial reports, it follows that $\phi_2^z(n_1|H_1, z_2, B)$ does not depend on z_1^f.

Proof of Corollary 5. Denote by $\hat{\phi}_t^z(n_1|H_{t-1}, G)$ and $\hat{\phi}_t^z(n_1|H_{t-1}, B)$ the posterior beliefs based on the backward looking financial report z_t^b only. Because of Lemma 5, it follows immediately that $\hat{\phi}_t^z(n_1|H_{t-1}, G) = \phi_t^z(n_1|H_{t-1}, G)$. For $\hat{\phi}_t^z(n_1|H_{t-1}, B)$ it holds that

$$\hat{\phi}_t^z(n_1|H_{t-1}, z_t^b, B) = 1_{n_1 \geq t-1} \frac{\phi_{t-1}^z(n_1|H_{t-1}, B)}{\sum_{n=t-1}^{\infty} \phi_{t-1}^z(n_1|H_{t-1}, B)}.$$

For $n_1 \geq t-1$ it holds that $\phi_{t-1}^z(n_1|H_{t-1}, B) = \alpha\hat{\phi}_t^z(n_1|H_{t-1}, B)$ with $\alpha = \sum_{n=t-1}^{\infty} \phi_{t-1}^z(n_1|H_{t-1}, B)$. Substituting this into (23) yields

$$\phi_t^z(n_1|H_t, B) =$$
$$1_{n_1 \geq t-1} \frac{Pr(z|n_1)\hat{\phi}_t^z\left(n_1|H_{t-1}, z_t^b, B\right)}{Pr(z|\lambda_t = \underline{\lambda})\hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right) + Pr(z|\lambda_t = \overline{\lambda}) \sum_{n=t}^{\infty} \hat{\phi}_t^z\left(n|H_{t-1}, z_t^b, B\right)}.$$

The expected payoff in expression (29) can now be written as

$$P_t(H_t)e \sum_{n_1=t}^{\infty} \hat{\phi}_t^z(n_1|H_{t-1}, z_t^b, G)\overline{\lambda}^{n_1-t+1}$$
$$+(1 - P_t(H_t))\beta_B e \sum_{n_1=t-1}^{\infty} \alpha(n_1)\hat{\phi}_t^z(n_1|H_{t-1}, z_t^b, B)\overline{\lambda}^{n_1-t+1} \tag{39}$$

where

$$\alpha(n_1) = \frac{Pr(z|n_1)}{Pr(z|\lambda_t = \underline{\lambda})\hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right) + Pr(z|\lambda_t = \overline{\lambda})\left(1 - \hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right)\right)}. \tag{40}$$

Recall that the expected payoff with only backward looking financial reporting equals (cf. 15))

$$P_t(H_t)e \sum_{n_1=t}^{\infty} \hat{\phi}_t^z(n_1|H_{t-1}, z_t^b, G)\overline{\lambda}^{n_1-t+1}$$
$$+(1 - P_t(H_t))\beta_B e \sum_{n_1=t-1}^{\infty} \hat{\phi}_t^z(n_1|H_{t-1}, z_t^b, B)\overline{\lambda}^{n_1-t+1} \tag{41}$$

Let $z = \overline{z}$. To show that $\overline{n}_{\overline{z}} \geq \bar{b}$, it suffices to show that the expected payoff in (39) exceeds the expected payoff in (41) for every $t \geq 1$. For this, it is again sufficient to show that $\alpha(t-1) < 1$ and $\alpha(n_1) \geq 1$ for $n_1 \geq t$. The proof of this proceeds as follows. Observe that $\alpha(n_1) \geq 1$ is equivalent to

$$(Pr(z|n_1) - Pr(z|\lambda_t = \underline{\lambda}))\,\hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right) \geq \\ \left(Pr(z|\lambda_t = \overline{\lambda}) - Pr(z|n_1)\right)\left(1 - \hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right)\right).$$

For $n_1 = t-1$, it holds that $\lambda_t = \underline{\lambda}$ so that the inequality reduces to

$$0 \geq (2\sigma - 1)(g - b)\left(1 - \hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right)\right).$$

Because this inequality does not hold for $g > b$, it follows that $\alpha(t-1) < 1$. For $n_1 \geq t$, it holds that $\lambda_t = \overline{\lambda}$ so that the inequality reduces to

$$(2\sigma - 1)(g - b)\hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right) \geq 0.$$

Hence, $\alpha(n_1) \geq 1$ for $n_1 \geq t$.

Let $z = \underline{z}$. To show that $\overline{n}_{\underline{z}} \leq \overline{b}$, it suffices to show that the expected payoff in (39) is less than the expected payoff in (41) for every $t \geq 1$. For this, it is again sufficient to show that $\alpha(t-1) > 1$ and $\alpha(n_1) \leq 1$ for $n_1 \geq t$. The proof of this proceeds as follows. Observe that $\alpha(n_1) \geq 1$ is equivalent to

$$(Pr(z|n_1) - Pr(z|\lambda_t = \underline{\lambda}))\,\hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right) \geq \\ \left(Pr(z|\lambda_t = \overline{\lambda}) - Pr(z|n_1)\right)\left(1 - \hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right)\right).$$

For $n_1 = t-1$, it holds that $\lambda_t = \underline{\lambda}$ so that the inequality reduces to

$$0 \geq (2\sigma - 1)(b - g)\left(1 - \hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right)\right).$$

Because $g > b$, it follows that $\alpha(t-1) < 1$. For $n_1 \geq t$, it holds that $\lambda_t = \overline{\lambda}$ so that the inequality reduces to

$$(2\sigma - 1)(b - g)\hat{\phi}_t^z\left(t-1|H_{t-1}, z_t^b, B\right) \geq 0.$$

Because $g > b$ this inequality does not hold. Hence, $\alpha(n_1) \geq 1$ for $n_1 \geq t$.

Proof of Lemma 6. Using that $\phi_t^z(n|H_t,B) = \alpha_z(n)\phi_t^b(n|H_t,B)$ with $\alpha_z(n)$ as defined in (40) it follows that

$$E^z(t) = E^b(t) + (1-P_t(H_t))\beta_B e* \\ \left(\sum_{n=t-1}^{\infty}(\alpha_z(n)-1)\phi_t^b(n|H_t,B)\overline{\lambda}^{n-t+1}\right).$$

From

$$\alpha_z(n)-1 = \begin{cases} \frac{\left(Pr(z|\lambda_t=\underline{\lambda})-Pr(z|\lambda_t=\overline{\lambda})\right)(1-\phi_t^b(t-1|H_t,B))}{Pr(z|\lambda_t=\underline{\lambda})\phi_t^b(t-1|H_t,B)+Pr(z|\lambda_t=\overline{\lambda})(1-\phi_t^b(t-1|H_t,B))} & \text{if } n=t-1 \\ \frac{\left(Pr(z|\lambda_t=\overline{\lambda})-Pr(z|\lambda_t=\underline{\lambda})\right)\phi_t^b(t-1|H_t,B)}{Pr(z|\lambda_t=\underline{\lambda})\phi_t^b(t-1|H_t,B)+Pr(z|\lambda_t=\overline{\lambda})(1-\phi_t^b(t-1|H_t,B))} & \text{if } n\geq t \end{cases} \tag{42}$$

it follows that $\frac{1-\alpha_z(t-1)}{(1-\phi_t^b(t-1|H_t,B))} = \frac{\alpha_z(n)-1}{\phi_t^b(t-1|H_t,B))}$ for all $n \geq t$. Defining $\hat{a}_z(t) = \frac{1-\alpha_z(t-1)}{(1-\phi_t^b(t-1|H_t,B))}$, we derive that

$$\begin{aligned} E^z(t) = E^b(t) + (1-P_t(H_t))\beta_B e * \Big(&-(1-\alpha_z(t-1))\phi_t^b(t-1|H_t,B) \\ &+\sum_{n=t}^{\infty}(\alpha_z(n)-1)\phi_t^b(n|H_t,B)\overline{\lambda}^{n-t+1}\Big) \\ = \quad E^b(t) + (1-P_t(H_t))\beta_B e * \Big(&-\hat{a}_z(t)(1-\phi_t^b(t-1|H_t,B))\phi_t^b(t-1|H_t,B)) \\ &+\sum_{n=t}^{\infty}\hat{a}a_z(t)\phi_t^b(t-1|H_t,B)\phi_t^b(n|H_t,B)\overline{\lambda}^{n-t+1}\Big) \\ = \quad E^b(t) + (1-P_t(H_t))\beta_B e\hat{a}_z(t)&\phi_t^b(t-1|H_t,B) * \\ \Big(-(1-\phi_t^b(t-1|H_t,B)) &+ \sum_{n=t}^{\infty}\phi_t^b(n|H_t,B)\overline{\lambda}^{n-t+1}\Big) \\ = \quad E^b(t) + (1-P_t(H_t))\beta_B e\hat{a}_z(t)&\phi_t^b(t-1|H_t,B) * \\ \Big(\sum_{n=t}^{\infty}\phi_t^b(n|H_t,B)&(\overline{\lambda}^{n-t+1}-1)\Big) \end{aligned}$$

To complete the proof, define $a_z(t) = \hat{a}_z(t)\phi_t^b(t-1|H_t,B = \alpha_z(n)-1$ for any $n \geq t$. Expression (33) then follows from substituting $Pr(\overline{z}|\lambda_t = \overline{\lambda}) = \sigma g + (1-\sigma)b$ and $Pr(\overline{z}|\lambda_t = \underline{\lambda}) = (1-\sigma)g + \sigma b$ into (42). Expression (34) follows from substituting $Pr(\underline{z}|\lambda_t = \overline{\lambda}) = \sigma(1-g)+(1-\sigma)(1-b)$ and $Pr(\underline{z}|\lambda_t = \underline{\lambda}) = (1-\sigma)(1-g)+\sigma(1-b)$ into (42).

Proof of Proposition 5. It suffices to show that there exists a reporting system $(1,b)$ such that

$$\overline{n}_{\underline{z}}(1,b) + Pr(z_t^f = \overline{z}|\lambda_t = \overline{\lambda})(\overline{n}_{\overline{z}}(1,b) - \overline{n}_{\underline{z}}(1,b))$$

$$> \overline{n}_{\underline{z}}(1,0) + Pr(s_t = \overline{s}|\lambda_t = \overline{\lambda})(\overline{n}_{\overline{z}}(1,0) - \overline{n}_{\underline{z}}(1,0)).$$

Because for reporting system $(1,b)$ with $b > 0$ it holds that $Pr(z_t^f = \overline{z}|n_1) = \sigma + (1-\sigma)b > \sigma = Pr(s_t = \overline{s}|\lambda_t = \overline{\lambda})$, the above inequality holds when there exists $b > 0$ such that $\overline{n}_{\underline{z}}(1,b) = \overline{n}_{\underline{z}}(1,0)$ and $\overline{n}_{\overline{z}}(1,b) = \overline{n}_{\overline{z}}(1,0)$. For the full disclosure reporting system it holds that

$$\begin{aligned} E^{\overline{z}}(t) &= E^b(t) + (1 - P_t(H_t))\beta_B e a_{\overline{z}}(t)\phi_t^b(t-1|H_t,B) * \\ &\left(\sum_{n=t}^{\infty} \phi_t^b(n|H_t,B)(\overline{\lambda}^{n-t+1} - 1)\right) \geq e \end{aligned}$$

for $t = \overline{n}_{\overline{z}}(1,0)$ and

$$\begin{aligned} E^{\underline{z}}(t) &= E^b(t) + (1 - P_t(H_t))\beta_B e a_{\underline{z}}(t)\phi_t^b(t-1|H_t,B) * \\ &\left(\sum_{n=t}^{\infty} \phi_t^b(n|H_t,B)(\overline{\lambda}^{n-t+1} - 1)\right) \geq e \end{aligned}$$

for $t = \overline{n}_{\underline{z}}(1,0)$ where

$$\begin{aligned} a_{\overline{z}}(t) &= \frac{(2\sigma-1)(1-b)\phi_t^b(t-1|H_t,B)}{\sigma + (1-\sigma)b - (2\sigma-1)(1-b)\phi_t^b(t-1|H_t,B)} \\ a_{\underline{z}}(t) &= \frac{(2\sigma-1)(b-1)\phi_t^b(t-1|H_t,B)}{(1-\sigma)(1-b) - (2\sigma-1)(b-1)\phi_t^b(t-1|H_t,B)}. \end{aligned}$$

If both inequalities are strict for $b = 0$, then both inequalities are still satisfied for some $b > 0$ sufficiently small as $a_{\overline{z}}(t) > 0$ and $a_{\underline{z}}(t) < 0$ are both decreasing in b.

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