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A Quantity Flexibility Contract in a Supply Chain with Price Dependent Demand

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Abstract:

We consider a quantity flexibility contract in a supply chain, under price dependent demand. The demand is random and follows a multiplicative model. We show that the retail price that maximizes expected retailer profits, is higher than the price that would maximize profits if there was no demand uncertainty. We then show that if the wholesale price lies in a certain range, there is a positive buyback fraction that the manufacturer would prefer. This preference is in comparison to not entering into a quantity flexibility contract at all. Finally, through numerical work we find that the contract results in a win-win situation for both the manufacturer and the retailer in the case of price dependent demand.

Keywords: Supply Chain Management, Quantity Flexibility Contract, Price Dependent Demand

1. Introduction and Literature Review

Double marginalization often occurs in a supply chain with independent decision makers. Because of double marginalization, the retailer stocks fewer amount of product than is optimal for the supply chain. This reduces supply chain profits. Buyback contracts, Revenue Sharing contracts and Quantity Flexibility contracts are some of the contracts that have been proposed to overcome the problem of double marginalization. These contracts overcome the problem by offering incentives to the retailer to increase the order quantity. In a buyback contract, the manufacturer specifies a wholesale price and a buyback price at which to purchase unsold units at the end of the season. This increases the salvage value per unit for the retailer, who then orders more. In a revenue sharing contract, the manufacturer shares a fraction of the revenue of the retailer and offers the retailer a low wholesale price. The low wholesale price reduces the overstocking risk for the retailer, who then orders more. In the version of the quantity flexibility contract that we consider, the manufacturer offers to buy back unsold units from the retailer at the wholesale price. But there is an upper limit to the amount of unsold product that can be returned by the retailer. See Cachon (2001) for a description of this version of the quantity flexibility contract.

Quantity Flexibility contracts have found applications in industry. According to Lovejoy (1999), quantity flexibility contracts have been used by Toyota and by Nippon Otis, a manufacturer of elevator equipment. Connors et. al (1995) mention that quantity flexibility contracts have been used by IBM. According to Farlow (1995) such contracts have been used in the purchase of workstation components by Sun Microsystems. Masten and Crocker (1985) discuss a `Take or Pay' contract for natural resources which is similar to the quantity flexibility contract. Chopra and Meindl (2004) mention that quantity flexibility contracts are common for components in the electronic and computer industry. According to Chopra and Meindl (2004), Benetton a fashion clothing manufacturer has used these contracts with its retailers to increase supply chain profits.

It is well documented (Cachon (2001), Chopra and Meindl (2004)), that buyback contracts, revenue sharing contracts and quantity flexibility contracts increase profits for both parties resulting in a win-win situation, if properly implemented. But do these same benefits carry over if demand is price dependent. Emmons and Gilbert (1998), have shown in a model of price dependent demand, that buyback policies would benefit both the retailer and the manufacturer. The authors show that given that the wholesale price is in a certain range, the manufacturer chooses a positive buyback price over not offering a buyback price. Mahajan (2008) has used the same price dependent demand model of Emmons and Gilbert (1998) and considered a revenue sharing contract. It is shown that in this setting, there is a positive revenue sharing fraction that the manufacturer would prefer.

In the present paper, we use the same model of price dependent demand as Emmons and Gilbert (1998). We consider the quantity flexibility contract in this setting. We show that the retail price that maximizes the expected retailer profits, is higher than the price that would maximize profits if there was no demand uncertainty. We then show that if the wholesale price lies in a certain range, there is a positive buyback fraction that the manufacturer would prefer. This preference is in comparison to not entering into a quantity flexibility contract at all. Finally, through numerical work we find that the contract results in a win-win situation for both the manufacturer and the retailer, in the case of price dependent demand.

Eppen and Iyer (1997) consider a backup agreement, which is similar to a quantity flexibility contract in that the retailer can return only a portion of the unsold units. Tsay (1999) shows that under certain conditions the quantity flexibility contract can coordinate the supply chain. The author also studies how the quantity flexibility contract affects the behavior and profits of the manufacturer and the retailer in the supply chain. Tsay and Lovejoy (1999) model demand forecast updates and lead times along with the quantity flexibility contract. Expositions of the quantity flexibility contract can be found in Cachon (2001) and Chopra and Meindl (2004). Bassok and Anupindi (1995) study a quantity flexibility contract where the retailer produces a forecast of monthly demand and has the flexibility to revise his order within a certain specified percentage. Bassok and Anupindi (1997) analyze a similar problem where the contract turns out to be a minimum purchase agreement for cumulative orders over a longer horizon. Kostamis and Duenyas (2009) consider a contract where a firm buys a minimum quantity from the supplier. Once demand is realized, there is the possibility of ordering more from the supplier. The authors aim to find contract parameters under which the supplier would overproduce beyond the minimum quantity contracted for with the firm.

There are also models of price dependent demand in which a pricing decision and an inventory decision is jointly considered. Lau and Lau (1988), consider the newsboy problem with price dependent demand distributions. Gerchak and Parlar (1987), consider a model in which a joint decision is made on inventory levels and marketing effort. Weng (1997) considers a manufacturer retailer supply chain with price dependent random demand. The three decision variables for the supply chain are the retail price, the manufacturer's wholesale price and the retailer's order quantity. There is also the possibility of a second more costly production run during the season. The author finds that a contract similar to a two-part tariff coordinates the supply chain. Pelin et al (2008) study a firm where a pricing decision is made by the marketing department and a leadtime decision is made by the manufacturing department. The firm's customer cares about both these decisions. They find the present setup is more inefficient and lowers profits as compared to a situation when both decisions are centralized. The authors show that coordination can be achieved using a transfer price contract with bonus payments. Finally, Petruzzi and Dada (1999), have a review article on pricing and the newsvendor problem.

This paper is organized as follows. In Section 2, we formulate the model and consider the retailer's decision. In Section 3, we consider the manufacturer's decision. In Section 4 we carry out numerical work. Finally, Conclusions are presented in Section 5.

2. Model Formulation: The Retailer's Decision

The manufacturer charges a wholesale price c for the product sold at the start of the selling season. Left over units at the end of the season, are bought back at the full price c. However, there is an upper limit to the number of units that the manufacturer can buy back. If the manufacturer sells x units at the start of the selling season, then he buys back at most δx units at the end of the season with $0 < \delta < 1$. A production cost, ν, is incurred by the manufacturer per unit. Given the wholesale price, c, and the buyback fraction, δ, the retailer chooses the order quantity, x, and the price, p, at which to sell the product to the consumer.

The retailer faces a price dependent demand in accordance with a multiplicative demand model. In particular, the demand,

$$X(p,Y) = D(p)*Y$$

Here Y is a positive random variable, with $E[Y] = 1$ and with cumulative distribution F(.) and density f(.). The function D(p) is linear and decreasing in the price, p. We assume $D(p) = b(p-k)$, with $b<0$ and $k>0$ are constants. Then, we have that,

$$E[X(p,Y)] = D(p)*E[Y] = D(p)$$

To analyze the problem, we first assume that the wholesale price, c, and the buyback fraction, δ, of the manufacturer are fixed. Given these two parameters, we determine the retailer's decisions. That is, we determine the order quantity, x, and the price, p, that the retailer would offer to the consumer. Once we determine the retailer's decision for a fixed c and δ, we then analyze the manufacturer's decision.

Let $\pi_R(p,x;c,\delta)$ denote the retailer's expected profit, given the manufacturer's decisions, c and δ. Then,

$$\pi_R(p,x;c,\delta) = pE[\min\{x,X\}] - cx + c(\delta x)F((1-\delta)x) + cE[(x-X)1_{\{(1-\delta)x < X < x\}}] \quad (1)$$

The third term in the above is the amount the retailer gets back from the manufacturer at the end of the season, if the retailer's demand is less than $(1-\delta)x$. Since the retailer's demand is less than $(1-\delta)x$, he would have more than δx units left over. He claims the price c from the manufacturer for δx units, as the manufacturer does not accept back more. In the last term above, the retailer's demand is more than $(1-\delta)x$, and he is able to get the buyback price, c, from the manufacturer for all unsold units.

To find the optimal order quantity, x^*, given p,c and δ, we differentiate (1). We have

$$d/dx[\pi_R(p,x;c,\delta)] = pP(X>x) - c + c\delta F((1-\delta)x) + c\delta(1-\delta)xf((1-\delta)x)$$

$$+ cd/dx[E[(x-X)1_{\{(1-\delta)x < X < x\}}]] \qquad (2)$$

We have that,

$$d/dx[E[(x-X)1_{\{(1-\delta)x < X < x\}}]] = [F(x) - F((1-\delta)x)] - \delta(1-\delta)xf((1-\delta)x) \qquad (3)$$

Substituting (3) in (2), we have

$$d/dx[\pi_R(p,x;c,\delta)] = (p-c)[1-F(x)] - c(1-\delta)F((1-\delta)x) \qquad (4)$$

If $\delta=0$, in the above, then there is no contract and we have the regular newsboy problem. Then equation (4), represents the standard newsboy fractile. If we differentiate equation (4) again, we see that the second order condition is satisfied and so setting equation (4) to zero provides the optimal x^*, given p,c and δ.

To make the expression tractable, we now assume that Y has a uniform distribution between (0,2). The same simplifying assumption has also been made in Emmons and Gilbert [3].

With this assumption, using equation (4), we then have that the optimal order quantity for the retailer, x^*, which maximizes $\pi_R(p,x;c,\delta)$, is given by,

$$x^* = 2(p-c)D(p)/[(p-c) + c(1-\delta)^2] \qquad (5)$$

With the assumption that Y has a uniform distribution between (0,2), equation (1) becomes,

$$\pi_R(p,x;c,\delta) = (p-c)x - px^2/(4D(p)) + cx^2\delta(1-\delta)/(2D(p)) + cx^2\delta^2/(4D(p)) \qquad (6)$$

Substituting the optimal value of x^*, from (5) in (6), we have,

$$\pi_R(p,x^*;c,\delta) = (p-c)^2D(p)/[(p-c) + c(1-\delta)^2] \qquad (7)$$

If $\delta = 0$, $\pi_R(p,x^*;c,\delta) = (p-c)^2D(p)/p$. This is the same as the expression for $\pi_R(p,x^*;c,f)$, when f=0 in the revenue sharing contract in Mahajan [7].

We then have the following result, similar to Emmons and Gilbert [3].

Proposition 1: The retail price p^* that maximizes the expected retailer profits, $\pi_R(p,x;c,\delta)$, is higher than the price that would maximize profits if there was no demand uncertainty. In the case of no demand uncertainty, X(p) = D(p).

Proof: Let g(p) be the retailer profit if there was no demand uncertainty. Then g(p) = (p-c)D(p) = (p-c)b(p-k). We have,

$g'(p) = b(2p-c-k)$. Let p_d be the optimal price if there is no demand uncertainty. Then, $p_d = (c+k)/2$.

We denote $\pi_R(p,x^*;c,\delta)$ as determined in (7) by h(p). Then

$$h(p) = g(p)[1- c(1-\delta)^2/[(p-c) + c(1-\delta)^2]] \qquad (8)$$

$$\text{Let } f(p) = 1- c(1-\delta)^2/[(p-c) + c(1-\delta)^2] \qquad (9)$$

Then h(p) = g(p)f(p) with $f'(p) > 0$ and $g'(p) > 0$ for $p <= p_d$.

$$\text{So, for } p <= p_d,\ h'(p) = g'(p) f(p) + f'(p) g(p) > 0 \qquad (10)$$

We differentiate $h'(p)$ again to get,

$$h''(p) = g''(p) f(p) + 2g'(p) f'(p) + g(p)f''(p) \qquad (11)$$

We have, $g''(p) <0$, $f''(p) <0$ and $f'(p) >0$. Also, $g'(p) <0$ for $p > p_d$. Using these relations and (11) , we have, that for $p > p_d$.

$$h''(p) < 0 \tag{12}$$

Summarizing (10) and (12), we have that $h'(p) > 0$ for $p <= p_d$ and $h''(p) < 0$ for $p > p_d$. This means that $h'(p)$ is a decreasing function for $p > p_d$.

If we can show that $h'(p) < 0$ for some $p_1 > p_d$, it would mean that there is a price $p_d < p^* < p_1$, at which $h'(p^*) = 0$. This is because $h'(p)$ is continuous.

We next show $h'(p) < 0$ for some $p_1 > p_d$. Let $L = c(1-\delta)^2$. Then,

$$h'(p) = b[2p-c-k][1- L/(p-c+L)] + b(p-c)(p-k)L/(p-c+L)^2 \tag{13}$$

Since $b<0$, this is equivalent to showing,

$$h'(p)\,(p-c+L)^2/[b(p-c)] > 0 \quad \text{for some } p_1 > p_d \tag{14}$$

The above is equivalent to,

$$(p-c)[(p-c)+ L]/(p-c+2L) > k-p \tag{15}$$

Choosing $p_1 = k - \varepsilon$ for $\varepsilon > 0$ and suitably small, showing (15) is equivalent to showing,

$$(p_1-c)[(p_1-c)+ L]/(p_1-c+2L) > (p_1-c)^2/(p_1+3c) > \varepsilon \tag{16}$$

In the above for the first inequality, we have used that $L = c(1-\delta)^2$. We now need to show the second inequality in (16). We have that,

$$(k - \varepsilon -c)^2/(k - \varepsilon +3c) \cong (k -c)^2/(k +3c) > \varepsilon \tag{17}$$

where we have neglected second order terms since ε is small.

We have thus shown (15). We then have, $h'(p_1) < 0$ for some $p_1 > p_d$. So there is a price p^* with $p_d < p^* < p_1$, at which $h'(p^*) = 0$.

Thus in the presence of demand uncertainty the retailer prices the product higher and sells lesser units as compared to the case when demand is certain.

The retailer makes two decisions, the optimal order quantity and the optimal price for a given c and δ. The optimal order quantity decision has been defined in (5). The retailer's profit at the optimal order quantity is given by (7) and is shown below. We have,

$$\pi_R(p,x^*;c,\delta) = (p-c)^2D(p)/[(p-c) + c(1-\delta)^2] \qquad (18)$$

To find the optimal price p^*, we differentiate (18). This results in the optimal price p^* satisfying the following quadratic equation.

$$2p^2 + [3c(1-\delta)^2 - 3c - k]p + (c^2 + ck - 2ck(1-\delta)^2 - c^2(1-\delta)^2) = 0 \qquad (19)$$

If $\delta = 0$, the above equation becomes $2p^2 - pk - ck = 0$. This is the same equation for the revenue sharing contract, when f=0 in Mahajan [7].

Let $h(\delta) = \delta(\delta-2)$ and

Discriminant = $h(\delta)[9c^2 h(\delta) + 10kc + 8c^2] + k^2 + 8kc$. Then, solving (19),

$$p^* = ([k - 3ch(\delta)] \pm \text{sqrt}(\text{Discriminant}))/4 \qquad (20)$$

Let $B = 3ch(\delta) - k$ and $\delta^* = 1 - \text{sqrt}((k+c)/(2k+c))$.

We have that for $0 < \delta < \delta^*$, Discriminant $> B^2$. So, if we take the negative root in (20), we would get a negative p^*, which is not possible. So we discard the negative root in (20) and we have that,

$$p^* = ([k - 3ch(\delta)] + \text{sqrt}(\text{Discriminant}))/4 \qquad (21)$$

If $\delta = 0$ in (21), we have ,

$$p^* = k/4 + \mathrm{sqrt}(k^2/16 + ck/2\ ) \qquad (22)$$

which corresponds to the case for the revenue sharing contract when f=0, in Mahajan [7].

In the next section, we discuss the manufacturer's decision.

3. The Manufacturer's Decision

The manufacturer charges a wholesale price c for the product sold at the start of the selling season. The manufacturer also buys back unsold units at the full price c. But he does this for at most a fraction δ of the units sold at the start of the season. A production cost, ν, is incurred by the manufacturer per unit. Given the wholesale price, c, and the buyback fraction, δ, the retailer chooses the order quantity, x^*, and the price, p^*, at which to sell the product to the consumer. The optimal order quantity and the optimal price of the retailer have been defined above using equations (5) and (21) . This has been done with the assumption of a uniform distribution for the random variable Y.

We now consider the manufacturer's decision. The manufacturer's profit $\pi_M(c, \delta)$ is given by,

$$\pi_M(c, \delta) = (c-\nu)^* x^*(c,\delta) - c(\delta x^*(c,\delta))F((1-\delta) x^*(c,\delta)) - cE[(x^*(c,\delta) -X)1_{\{(1-\delta)x < X < x\}}] \qquad (23)$$

In the above, the distribution F(.) is Uniform (0, 2D(p^*(c, δ))). The second and third terms in (23), correspond to the last two terms in (1). This is because the additional revenue to the retailer from returning unsold units is the loss to the manufacturer from paying for the unsold units. Using the fact that F(.) is Uniform (0,2D(p^*)), and writing x^* for $x^*(c, \delta)$ and p^* for $p^*(c, \delta)$, we have that (23) becomes,

$$\pi_M(c, \delta) = (c-\nu)^* x^* - c\delta(1-\delta)(x^*)^2/(2D(p^*)) - c\delta^2(x^*)^2/(4D(p^*)) \qquad (24)$$

We can rewrite (24) as,

$$\pi_M(c, \delta) = (c-\nu)* x^* - c\delta(2-\delta)(x^*)^2 /(4D(p^*)) \quad (25)$$

We then have the following Proposition.

Proposition 2: If the wholesale price c, lies in the range $(p^*+2\nu)/3 < c < k/4$, there exists a buyback fraction δ at which the manufacturer receives a higher profit than when $\delta = 0$.

Proof: We need to show that $d/d\delta\ (\pi_M(c, \delta))$ evaluated at $\delta = 0$ is positive.

The derivative of the second term of (25) evaluated at $\delta = 0$ is $c(x^*)^2 /(2D(p^*))$.

Substituting for x^* and $D(p^*)$, the derivative of the second term of (2) evaluated at $\delta = 0$ is $(p-c)cx^*/p$. Using this and (25), we have,

$$d/d\delta\ (\pi_M(c, \delta)) = (c-\nu)* d/d\delta\ (x^*) - (p-c)cx^*/p \quad (26)$$

The second term of the above has been evaluated at $\delta = 0$.

Using (5), we have that $d/d\delta\ (x^*)$ evaluated at $\delta = 0$ is given by,

$$d/d\delta\ (x^*) = 2b/(p^*)^2[\ 2c(p^*-c)(p^*-k) + (p^*)^2\ d/d\delta\ (p^*) - ck\ d/d\delta\ (p^*)\ ] \quad (27)$$

We have that $d/d\delta\ (p^*)$ evaluated at $\delta = 0$ is,

$$d/d\delta\ (p^*) = 6c - 2[10ck + 8c^2]/(8p^* - 2k) \quad (28)$$

Using (26) and (27), to show $d/d\delta\ (\pi_M(c, \delta))$ evaluated at $\delta = 0$ is positive, we need to show that,

$$4bc(c - \nu)(p^*-c)(p^*-k) + 2b(c - \nu)\ d/d\delta\ (p^*) >$$

$$2bck(c - \nu)/(p^*)^2\ d/d\delta\ (p^*) + (p^*-c)cx^*/ p^* \quad (29)$$

Multiplying both sides of (27) by $(p^*)^2/2b$ (we have $b<0$) and noting that

$x^* = 2b(p^*-c)(p^*-k)/ p^*$ evaluated at $\delta = 0$, we have that to show (28), we need to show that,

$$c(p^* - c)(k - p^*)[2(c-\nu) - (p^* - c)] > [(p^*)^2 - ck](c - \nu)\, d/d\delta\, (p^*) \qquad (30)$$

We show (30) by finding conditions under which the LHS of (30) is positive and the RHS is negative.

We have that $c < p^* < k$ and we introduce the condition

$$c > (p^*+2\nu)/3 \qquad (31)$$

This results in the LHS of (30) being positive.

We next show that $(p^*)^2 > ck$ and introduce a condition for $d/d\delta\, (p^*) < 0$. This makes the RHS of (30) negative. With this, (30) would be satisfied and the result would be shown.

We first show that $(p^*)^2 > ck$. Here p^* is evaluated at $\delta = 0$. It is written as,

$$p^* = k/4 + \text{sqrt}(k^2/16 + ck/2\,) \qquad (32)$$

We have that $(p^*)^2 > ck$ implies,

$$k^2/4 + k\, \text{sqrt}(k^2/16 + ck/2\,) > ck \qquad (33)$$

We have that $k^2/16 + ck/2 > c^2/16 + c^2/2 = (3c/4)^2$

Using the above the LHS of (33) is,

$$k^2/4 + k\, \text{sqrt}(k^2/16 + ck/2\,) > k^2/4 + 3ck/4 > ck/4 + 3ck/4 = ck$$

We have then shown that $(p^*)^2 > ck$.

We lastly introduce a condition for $d/d\delta\, (p^*) < 0$. From (28), we have that,

$$d/d\delta\, (p^*) = (48cp^* - 32ck - 16c^2) / (8p^* - 2k) \qquad (34)$$

Using (32) we have that $8p^* - 2k > 0$. We have that,

$$48cp^* = 12ck + 3c*\text{sqrt}(16k^2 + 128ck)$$

$$< 12ck + 3c*(4k + 16c)$$

$$= 24ck + 48c^2 \qquad (35)$$

The numerator of the RHS of (34) is,

$$48cp^* - 32ck - 16c^2 < -8c(k-4c) \qquad (36)$$

We have that the LHS of (36) is negative if $c< k/4$.

So if $c< k/4$, $d/d\delta\ (p^*) < 0$.

We then have that if, $(p^*+2\nu)/3 < c < k/4$, the LHS of (30) is positive and the RHS is negative. We have then shown the result.

We have thus shown that under price dependent demand, the manufacturer benefits by entering into a quantity flexibility contract. This is not clear apriori. As the manufacturer increases the buyback fraction, δ, it provides an incentive for the retailer to order more units from the manufacturer. This is because the retailer can return a higher number of unsold units at the full price. If the retailer orders more units, the manufacturer makes a margin on a larger number of units and this increases his profits. But as the manufacturer increases the buyback fraction, δ, he has to accept back a larger number of unsold units at the full price and this decreases his profits. So there is a tradeoff.

In the next section we discuss numerical work.

4. Numerical Work

It is not possible to analytically solve the manufacturer's problem in (25). We carry out some numerical work to see if the quantity flexibility contract results in a win-win situation for both the manufacturer and the retailer.

For numerical work, we have chosen ν=1, b=-2 and k=6. For comparison purposes, we first look at the case where $\delta = 0$, which corresponds to no contract. We determine the manufacturer profit, the retailer profit and the total supply chain profit as we vary c from 1.5 to 5.5 (as $\nu < c < k$). Results are shown in Figure 1 below.

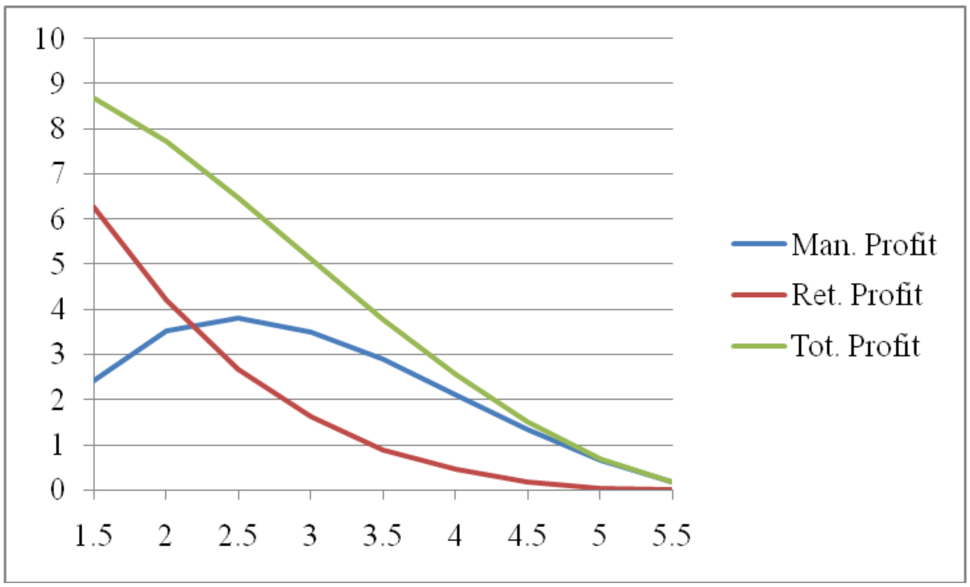

Figure 1: The manufacturer profit, retailer profit and supply chain profit as a function of the wholesale price c, when $\delta = 0$

We see that supply chain profits are maximized at c=1.5 (it would actually be c=1, but we have not calculated that). This illustrates the phenomenon of double marginalization. Supply chain profits are maximized when $c = \nu$. The retailer margin then corresponds to the supply chain margin, and the retailer makes the ordering decision that is optimal for the integrated supply chain.

The manufacturer profit is maximized at c = 2.5 and has the value π_M = 3.8. The retailer profit is the highest at c=1.5 (it would actually be c=1, at which the retailer profit equals the supply chain profit). The retailer profit at the wholesale price of c=2.5, chosen by the manufacturer is π_R = 2.68.

We next consider the supply chain with the quantity flexibility contract. For each value of c, we choose δ which maximizes the manufacturer profit. Results are shown in Figure 2 below.

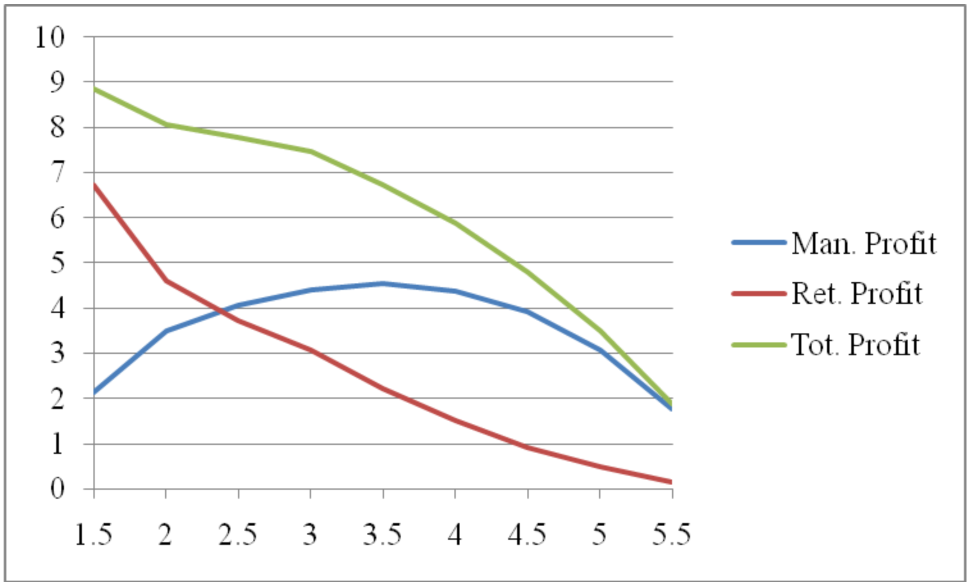

Figure 2: The manufacturer profit, retailer profit and supply chain profit as a function of the wholesale price c, with the optimal δ

We first compare the retailer profits in Figure 1 and Figure 2. We see that the retailer profits are higher for all values of c, in Figure 2 with the optimal δ, as compared to Figure 1. Thus the retailer benefits from entering into a quantity flexibility contract. Proposition 2 shows that the manufacturer would also like to enter into the contract. Thus we see that the contract provides a win-win situation for the retailer and the manufacturer, even in the case of price dependent demand. If we compare the manufacturer profits between Figures 1 and 2, we see that the manufacturer profits are higher for all values of c in Figure 2, except for c=1.5 and c=2.0. This agrees with Proposition 2, as the values of c have to be in a certain range before the manufacturer agrees to enter into the contract.

In Figure 2, the manufacturer profit is maximized at c=3.5 and is $\pi_M = 4.53$. The retailer profit at the wholesale price of c=3.5, chosen by the manufacturer is $\pi_R = 2.2$. If we compare with the retailer profit of $\pi_R = 2.68$, when δ=0, we see that the retailer is worse off at the actual contract terms offered by the manufacturer. But this can be rectified quite easily. We see that the manufacturer profit in Figure 2 is quite flat over a large range. If the manufacturer agrees to take a slight hit in profits, he can improve the retailer profits by a substantial amount. So, if the manufacturer chooses the wholesale price as c=3.0, we have $\pi_M = 4.4$. At the value of c=3.0, $\pi_R = 3.07$. By changing his wholesale price decision to c=3.0, from the optimal decision of c=3.5, the manufacturer profit has reduced by only about 3%. But this has increased the retailer profits by about 40%. Also this contract makes the retailer better off (with π_R = 3.07 as compared to $\pi_R = 2.68$ when δ=0), as compared to having no contract. Thus, we have a win-win situation for both parties.

In the next section, we present Conclusions.

5. Conclusions

We considered a quantity flexibility contract, under price dependent demand. The demand is random and follows a multiplicative model. We showed that the retail price that maximizes expected retailer profits, is higher than the price that would maximize profits if there was no demand uncertainty. We then showed that if the wholesale price lies in a certain range, there is a positive buyback fraction that the manufacturer would prefer. This preference is in comparison to not entering into a quantity flexibility contract at all. Finally, through numerical work, we found that the contract results in a win-win situation for both the manufacturer and the retailer in the case of price dependent demand. It is well documented (Cachon (2001), Chopra and Meindl (2004)), that buyback contracts, revenue sharing contracts and quantity flexibility contracts increase profits for both parties resulting in a win-win situation, if properly implemented. But do these same benefits carry over if demand is price dependent? In this paper, we have shown that it is the case for the quantity flexibility contract.

Similar results have been obtained for three of the contracts under price dependent demand, the buyback, the revenue sharing and the quantity flexibility contract. It remains to be seen if the results also hold for other contracts such as the sales rebate contract.

References

1. Bassok Y and Anupindi, R. (1995) Analysis of Supply Contracts with Forecasts and Flexibility. Working Paper, Northwestern University. Evanston, Illinois.

2. Bassok Y. and Anupindi, R. (1997) Analysis of Supply Contracts with Total Minimum Commitment. *IIE Transactions*, **29**, 373-381.

3. Cachon G.P. (2001) Supply Chain Coordination with Contracts in *Handbooks in Operations Research and Management Science: Supply Chain Management*, Graves, S. and de Kok Ton (eds) North Holland 2001

4. Chopra, S. and Meindl, P. (2004) *Supply Chain Management*, Second Edition, Pearson Education.

5. Connors, D., An, C., Buckley, S., Feigen, G., Levas, A., Nayak, N., Petrakian, R. and Srinivasan, R.(1995) *Dynamic Modeling for Reengineering Supply Chains*. IBM Research Report, T.J. Watson Research Center, Yorktown Heights, New York.

6. Emmons, H. and Gilbert, S.M. (1998) Note. The Role of Returns Policies in Pricing and Inventory Decisions for Catalogue Goods. *Management Science*, **44**, 276-283 .

7. Eppen, G. and Iyer, A. (1997) Backup Agreements in Fashion Buying- the Value of Upstream Flexibility. *Management Science*, **43**, 1469-84.

8. Farlow, D., Schmidt, G., and Tsay, A.A. (1995) Supplier Management at Sun Microsystems. Case Study, Graduate School of Business, Stanford University, Stanford, California.

9. Gerchak, Y. and Parlar, M. (1987) A Single Period Inventory Problem with Partially Controllable Demand. *Computers and Operations Research*, **14**, 1-9.

10. Kostamis, D. and Duenyas, I. (2009) Quantity Commitment, Production and Subcontracting with Bargaining. *IIE Transactions*, **41**, 677-686.

11. Lau, A.H.L and Lau, H.S. (1988) The Newsboy Problem with Price Dependent Demand Distributions. *IIE Transactions*, **20** , 168-175.

12. Lovejoy, W.S. (1999) *Integrated Operations*, Southwestern College Publishing, Cincinnati, Ohio.

13. Mahajan S. (2008) A Revenue Sharing Contract with Price Dependent Demand, Working Paper, Indian Institute of Management, Bangalore.

14. Masten, S.E and Croker, K.J. (1985) Efficient Adaption in Long Term Contracts: Take-or-Pay Provisions for Natural Gas. *American Economic Review*, **75**,,1083-1093.

15. Pelin, P., Griffin, P.M. and Keskinocak, P. (2008) Coordination of Marketing and Production for Price and Leadtime Decisions. *IIE Transactions*, **40**, 12-30.

16. Petruzzi, N. and Dada, M. (1999) Pricing and the Newsvendor Problem: A Review with Extensions. *Operations Research*, 183-194.

17. Tsay ,A.A. and Lovejoy, W.S. (1999) Quantity Flexibility Contracts and Supply Chain Performance. *Manufacturing and Service Operations Management* ,**1**.

18. Tsay, A.A. (1999) The Quantity Flexibility Contract and Supplier-Customer Incentives. *Management Science*, **45**, 1339-1358.

19. Weng Z.K. (1997) Pricing and Ordering Strategies in Manufacturing and Distribution Alliances. *IIE Transactions*, **29**, 681-692.