

**The Stability of Bond  
Financed Deficits:  
A Critique of The Literature**

**By**

**Vivek Moorthy**

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**Please address all correspondence to:**

**Vivek Moorthy  
Professor  
Indian Institute of Management  
Bangalore-560 076.**

**Phone : 6632450  
Fax : 080 - 644050  
email : vivekm@iimb.ernet.in**

**Indian Institute of Management Bangalore**

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Comments Welcome

**Abstract**

A large body of literature has concluded that bond finance of the deficit is unstable because of rising interest payments on the debt. This paper criticizes this literature in detail, focussing on the Blinder-Solow and Sargent-Wallace papers. It argues that conclusions regarding a debt trap under bond finance stem from unrealistic assumptions about the interest rate, and not from the assumption of budget balance in the Blinder-Solow model, as commonly believed. An adaptive expectations Fisher-equation specification for how interest rates respond to the mode of debt financing and to inflation, implies that, in a growth framework, stability under bond finance can be easily achieved.

JEL Classification: E42, E43, E63

\*Economics and Social Sciences Area, Indian Institute of Management, Bangalore  
Bannerghatta Road, Bangalore 560076 India Phone: 080-6632450 Fax 080-6644050  
e-mail: [vivekm@iimb.ernet.in](mailto:vivekm@iimb.ernet.in)

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## The Stability of Bond Financed Deficits: A Critique of the Literature

There is an extensive debate on the relative stability and the expansionary and associated inflationary impact of bond-financed versus money financed deficits, with a major contradiction between much of the academic literature and what can perhaps be characterised as the empirical or monetarist view. The 'mainstream consensus' in the academic literature, insofar as there is one, primarily implies that bond finance should be avoided since, most likely, it leads to a debt trap. This conclusion is reached by deriving a long run equilibrium (LRE) condition for budget balance in a model incorporating interest payments on the debt, usually starting from an IS/LM framework. The seminal paper in this literature is "Does Fiscal Policy Matter?" by Blinder and Solow (1973). Others who have argued along the same lines are Tobin and Buiter (1976) and Turnovsky (1977). In a growth context, it has been emphasized by Sargent and Wallace (1981) in their influential article "Some Unpleasant Monetary Arithmetic" that under rational expectations concerning future interest payments on debt, the condition for the debt-to-GDP ratio to stabilize (real growth should exceed the real interest rate) is not likely to be satisfied under bond finance, and therefore the deficit should be monetized.

The opposing 'monetarist'<sup>1</sup> or classical or empirical view stresses that bond finance is less inflationary than money finance and that the debt can, and does, stabilize under bond finance. This viewpoint stresses that, as an **empirical** matter, stability holds. A good exposition of this view point is Darby's (1984) rejoinder to Miller (1983). However, an

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<sup>1</sup> Monetarism has different tenets, such as the Fisher equation, stable money demand and the ensuing belief that the central bank should follow a money growth rule. This author finds it useful to distinguish between monetarism versus the Quantity Theory, two views that are often treated as identical. A subset of these tenets can hold while others may not. Thus the Fisher equation (and underlying natural rate hypothesis) can be characterized as monetarism, while the Quantity Theory entails the **additional** belief that money demand is stable, and therefore the central bank should follow a money growth rule. Empirical evidence suggests that the Fisher equation is strong while money demand is unstable and the Quantity theory is weak. Alternatively, the monetarist view as described here can just be called 'classical.' Regarding bond and money finance of the deficit, the monetarist view would be that even if money demand is unstable, nevertheless money finance will invariably be more expansionary (in its impact on nominal GNP and thus inflation) than bond finance. This broad conclusion can follow even if we start from an IS/LM framework.

economic rationale for this monetarist viewpoint has not been clearly spelled out. Nor have the monetarists clearly identified the critical assumption(s) and weaknesses in the prevailing literature which concludes that bond finance leads to a debt trap.

This paper provides a critique of the mainstream consensus and seeks to provide theoretical justification for the monetarist or empirical view. It emphasizes the role of inflation expectations (based on a Fisher equation with adaptive expectations or similar response) in the determination of interest rates. Since the academic literature on debt stabilization has not correctly analyzed how inflation expectations and hence interest rates respond under bond financing, it has erroneously concluded that bond financing is unstable. Incorporating how inflation expectations and interest rates are linked to the mode of debt financing suggests that stability can be achieved far more easily under bond financing than implied by the literature. Broadly speaking, the divergent short and long run consequences of bond versus money finance of the deficit should be treated as an extension of the opposite short and long run consequences of changing money growth, first outlined in Milton Friedman's (1968) analysis, with the added twist of debt variables.

The paper is organised as follows. Part II begins by outlining the literature in models that deal with levels of income, focussing on the Blinder-Solow (1973) model. The prevailing analysis has concluded that this model leads to peculiar long-run results because of the assumption of long-run budget balance. However, I argue that their peculiar results instead stem from leaving out the Fisherian impact of inflation on interest rates. In addition, I show that, using a difference equation-discrete time version of the Blinder-Slow model, money finance is better, contrary to their conclusion. Part III outlines the stability condition in a growth framework and provides a critique of the influential Sargent-Wallace view that bond-financing leads to a 'debt trap'. Part IV outlines the debt-adjustment process from a monetarist perspective and also adduces some evidence suggesting how bond finance may actually facilitate debt adjustment in the long run by lowering the risk premium component of interest rates.

## II

### A. The Blinder-Solow model

The macroeconomic analysis of the interaction between monetary and fiscal policy using the government budget constraint comprises of two sets of models: those dealing with levels of macro variables and those with growth rates. The first set of models are sometimes called static and the latter dynamic. Classifying them as static and dynamic respectively is misleading since models with levels can have intrinsic dynamics: when the budget is not in balance, the stock of bonds or money changes every period. However, the long-run equilibrium for models with levels is a stationary state in which the level of variables does not change.

The Blinder-Solow 1973 paper arose out of the literature dealing with the asset effects of the government budget constraint on spending in an IS/LM framework. It was written to refute Milton Friedman's (1972) contention that the long-run impact of a bond financed (BF) increase in government spending is small or negative. Blinder and Solow proved that either:

- a) BF is more expansionary than Money Finance (MF) (i.e. raises nominal GNP), OR
- b) if BF is less expansionary than MF, as Friedman claimed, then it leads to a debt trap.

Whichever case (a) or (b) prevails, Blinder and Solow concluded that Friedman's position - regarding the outcome and desirability of BF - is logically untenable. In case (b) BF should be clearly eschewed because of the unstable debt trap. In case (a) if the economy is at full employment, BF is more inflationary than MF and thus is undesirable, as will be seen later. Since most often the economy is at full employment, the Blinder-Solow model and subsequent analysis along their lines implicitly implies that BF should generally be eschewed and the debt should be financed by printing money. This is a startling conclusion; it goes against all common sense, monetarist/classical and central bankers views on the subject.

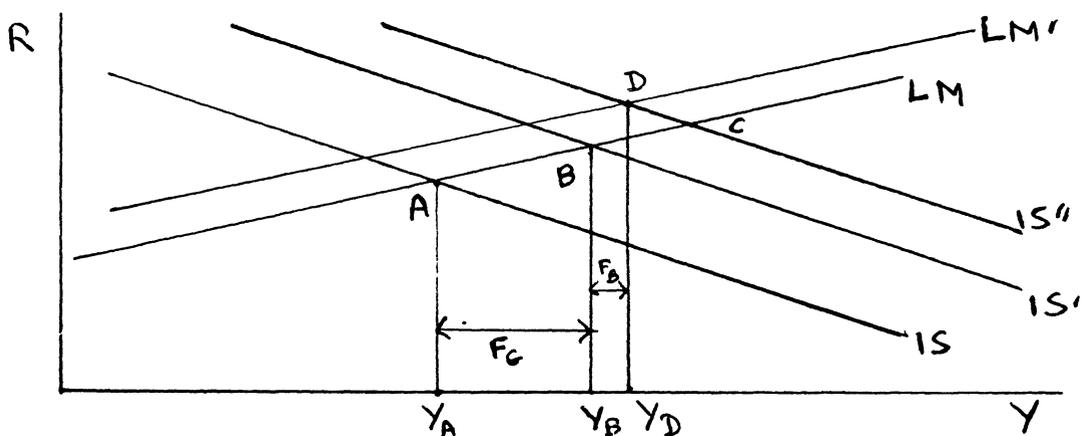
To critique the Blinder-Solow view, which this paper will do, requires first outlining the background literature and analysing the desirability of alternative policies implied by the model. Relevant variables are defined as they are introduced. Appendix A lists all variables and their definitions, and abbreviations. Alternative policies are defined below.

**Monetary Policy:** A change in  $M$  (exogenous or high-powered money), keeping Government expenditure ( $G$ ) constant. It is an open-market operation.

**BF (Bond Finance):** The financing of Government debt and expenditure by issuing bonds, keeping  $M$  constant. While the term BF can be and is used interchangeably with the term fiscal policy, the latter term refers to government spending rather than its mode of financing per se.

**MF (Money Finance):** The financing of  $G$  by printing money, or Monetization of deficit.

The graph below explains the long-run impact of financing  $G$  by issuing bonds, or BF. For convenience, in the diagram below, government spending rises by Rs. 1 and is financed by issuing a (normalized) bond worth Rs. 1. (In the full model presented later and in Appendix B and C, the bonds are priced differently.)



$F_G$  is the IS/LM Keynesian multiplier, incorporating crowding out (cf. Appendix A).  $F_G$  is the 'first round' multiplier effect from the flow of government spending (equals  $Y_A - Y_B$ ), irrespective of how this is financed.  $F_B$  represents the net bond multiplier. It incorporates the combined impact of shifts in the IS & LM curves. If government bonds are perceived to be net wealth, which is generally believed to be the case, the IS curve shifts out leading to a new equilibrium at point C. But the rise in wealth also increases money demand,

which is a function of wealth, thus shifting up the LM curve. Depending on which effect is larger, the final point D can be to the right or left of C. Every period, income changes by  $F_B$  in response to issue of bonds. These are continuing effects; even if  $F_B$  is small, the cumulative effect is larger than  $F_G$ .<sup>2</sup>

Milton Friedman's statement that "The empirical question is how important the first-round effects are compared to the ultimate effects. Theory cannot answer that question (1972)" implies that  $F_B < 0$ . The lacunae with Friedman's argument is that while it stresses long run effects, it does not look at the Long Run Equilibrium (LRE) solution. Much earlier, Ott and Ott (1965) and later Christ (1968) had worked out a long-run equilibrium corresponding to budget balance, showing that  $Y^*$  (\* denotes LRE Value) would be the same irrespective of the mode of financing government expenditure. The logic is extremely simple: under budget balance  $G = t \cdot Y$  (t is the flat tax rate on all income). So

$$\Delta Y^* = 1/t \Delta G \quad \text{II(1)}$$

The LRE multiplier is always  $1/t$ , under both BF and MF, according to Ott and Ott.

Blinder and Solow carried this analysis one step further. They showed that putting interest payments into the debt into the budget constraint - omitted by Ott and Ott and also by Christ - changes the result. With interest payments, LRE  $Y^*$  has to be higher under BF than under MF. The logic is that under BF, total G (composed of the same

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<sup>2</sup> Under Ricardian equivalence, in which private agents anticipate higher taxes to offset the current deficit and so save correspondingly more, government bonds are not perceived to be net wealth. So  $F_B = 0$ , and there are no wealth effects on consumption or money demand and so no shifts in the IS or LM curves after the one-time effect of higher G. There are various reasons why Ricardian equivalence is not likely to hold, such as liquidity constraints on consumption. Barro (1989) summarizes and discusses these reasons. This paper ignores the case of Ricardian equivalence. The conclusions here are compatible with  $F_B > 0$ .

primary G plus interest payments on the debt) is higher. So for taxes to match total G, in LRE Y has to rise **more** under BF. They derived the following result:

$$dY^*/dG \text{ (under BF)} = [F_B - (1-t)F_G]/[t.F_B - (1-t)] \quad \text{II(2)}$$

which exceeds  $dY^*/dG = 1/t$  which is the LRE multiplier under MF of the deficit..

The derivation of the above multiplier and related results are shown in Appendix B.

For this LRE to be reached, the following stability condition needs to be met:

$$t . F_B > 1 - t$$

Taxes from income generated from bond > Debt service or net interest paid on bond.

While Blinder and Solow felt that the stability condition was likely to be met,<sup>3</sup> their goal was to lay out the logical possibilities implied by their model and expose the contradictions in Friedman's position. As they summarize their argument in the concluding section,

“ if such an economy is stable at all under bond finance, fiscal policy is normally effective. If the monetarists are right, the system must be unstable. And then fiscal policy is worse than impotent: bond-financed spending drives down income without limit. (1973, p. 336)”

The subsequent literature has concluded, contrary to the conjecture of Blinder and Solow, that this stability condition is **not** likely to be met, and that a debt trap will result under BF. In other words, although BF is likely to be less expansionary than MF, as Friedman asserted, it is dangerous because of falling income and rising interest payments on debt.

## B. Prevailing Analysis of the Blinder-Solow model

A critical evaluation of the Blinder-Solow paper and subsequent literature entails identifying which are the crucial assumptions that drive their results, which of these assumptions can be relaxed without altering their conclusions and which of these can be

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<sup>3</sup> “A number between 1 and 2 seems plausible for  $F_B$  at least for the United States. The appropriate interpretation of  $t$  is as the marginal propensity to tax and reduce income-conditioned transfers payments as GNP rises...it would appear that  $t > 0.5$ . And this would imply that any  $F_B$  greater than unity would mean that the system is stable.” (Blinder & Solow, p.335)

rejected as being empirically invalid. Three assumptions will be discussed (a) fixed prices (b) fixed capital stock and (c) the budget balance condition. The general consensus of the academic literature is that their results stem from the last assumption of budget balance.

(a) Fixed Prices. The initial analysis of Blinder and Solow is with fixed prices and no aggregate supply constraints. Can putting in flexible prices, certainly a realistic assumption in the IS/LM model, reverse their stability conclusion? As it turns out, the answer is no. As Turnovsky (1977) and others conclude, instability is more likely under flexible prices. The main reason for this conclusion is as follows:

If  $F_B$  is defined in nominal terms, then with flexible prices, stability requires:

$$t F_B > (1-t) + \Delta P^* g \text{ (where } g \text{ is real primary Government expenditure)}$$

Keeping real primary  $g$  fixed, nominal  $G$  rises with the price level, captured by the second term, making stability harder to achieve than under fixed prices. Alternatively, one could consider the bond multiplier in real terms, or  $F_b$ . With flexible prices and supply constraints,  $F_b$  will be lower due to real crowding out. When the economy cannot exceed potential or full-employment output,  $F_b$  is zero and so there is zero tax revenue.<sup>4</sup> Scarth, in his incisive summary of the literature states, "All studies extending the B-S analysis lead to this strong conclusion, i.e., the prospect of instability is even greater under flexible prices" (1989, p. 85).

(b) Fixed Capital Stock: If bringing in flexible prices does not weaken but strengthens the case against BF, can any other change in assumption drastically alter their results? Blinder and Solow show that allowing investment to be a function of not just the interest rate as in their basic IS-LM model, but also of the capital stock ( $K$ ), which varies like the stock of bonds and money, changes the stability condition. In the last part of their paper

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<sup>4</sup> In a footnote, Blinder and Solow discuss flexible prices and point out three effects that reduce the value of the fiscal multiplier but "none of them have any bearing on its sign, which is what is at issue here." (p. 324)

("Crowding out when the capital stock may vary") they show that "it is logically possible for the economy to be stable and fiscal policy ineffective" but go on to state "we regard this as a curiosum rather than a vindication of monetarism" (p 336)

Tobin and Buitier (1976) further extend the investment function by adding a 'propensity to invest' term i.e  $I = I(r, K, Y)$ . They show that although the stability condition is much more stringent,  $dY^*/dG$  can be  $< 0$  in a stable long-run equilibrium under BF. Thus, varying the capital stock can salvage the monetarist view about debt stability under BF in the Blinder-Solow model.

This paper completely avoids the complications of various outcomes under BF that arise from different investment functions. In many cases, the original Blinder-Solow results prevail under different investment functions. The opposing monetarist or empirical view, that BF is stable and less expansionary than MF, is hardly tenable if such an outcome is predicated on some specific investment function and/or parameter values. In short, the simplest IS-LM investment function  $I = I_r$  with  $I'_r < 0$  is reasonable and should not be considered as being the unrealistic assumption that drives their results. Letting both prices and/or capital stock does not fundamentally alter their results.

(c) Budget Balance: In a footnote, Blinder and Solow state an intriguing result,

"An interesting corollary of this is that an open-market purchase i.e. a swap of B for M by the government with G unchanged will be contractionary! This is because, with less debt service, the existing levels of G and Y will imply a budgetary surplus which, in turn, must lead to a reduction in the supplies of money and/or bonds" (1973, Footnote #9, p. 325)

Put differently, when the stability condition is met, a one-time increase in money supply leads to an ongoing reduction in the supply of bonds that reduces  $Y(GDP)$  in the long run. This corollary is proved in Appendix B. Tobin and Buitier(1976) in their extension of the Blinder-Solow model come to a similar conclusion and acknowledge it is peculiar.<sup>5</sup>

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<sup>5</sup> "It may seem paradoxical that monetary expansion is, in the long run, contractionary. We do not think that the result should be taken seriously given that it depends on the assumption that monetary expansion entails a fiscal contraction via reduction of debt interest transfers." (Tobin & Buitier, 1976, p. 293) In effect, they are attributing this peculiar result of their model to the long-run budget balance assumption.

Pursuing the logic of the Blinder-Solow model further, this paper shows that an increase in private investment is also contractionary in the long run. This is proved in Appendix B. An increase in private investment raises  $Y$  in the short run, which leads to a budgetary surplus and hence a reduction in bond supplies that lowers  $Y$  in the long run. Neither Blinder and Solow nor Tobin and Buiter point this result out. The result itself is trivial, given the corollary discussed above, but the economic implication is not. A surge in animal spirits induces a recession in the long run! A Keynesian model that leads to such an extremely peculiar conclusion should not be used as the analytical basis for making inferences about the stability of BF versus MF.

Tobin and Buiter and others reflecting on these peculiar results attribute them to the assumption of long run budget balance. For instance, commenting on the Tobin-Buiter paper which was presented at the Conference on Monetarism, Stanley Fischer states,

“Thus, the surprises in the Tobin-Buiter paper are due almost entirely to the power of the long-run government budget constraint.” (1976, p. 326)

Sargent, much earlier had argued along the same lines,

“Christ has criticized the static Keynesian multiplier formulae because they do not agree with the long-run multipliers he obtains by imposing some (in our opinion, strange) stationarity conditions - in particular, the requirement that in the long run the government deficit be zero.” (1979, p. 111)

### C. The Missing Fisher effect, Not Budget Balance

In the opinion of this author, contrary to the general views expressed above, the peculiar results of the Blinder-Solow and related models do not stem from the long-run government budget constraint. While in an economy constantly subject to change, the budget may never actually balance, a budget balance condition (or a stable debt/GDP ratio in a growing economy) rules out a debt trap, constitutes an equilibrium and is therefore justified as a long-run condition. Those who have emphasised that budget

balance is crucial to generate the Blinder-Solow result have never specified appropriate alternative assumptions about the budget that would lead to realistic results.

It is worth noting that a long-run equilibrium condition in which the balance of payments is zero (**analogous** to budget balance) has been an integral part of open economy macroeconomics since Mundell's (1963) pioneering analysis introduced assets into the prevailing Keynesian flow analysis. Yet this LRE zero balance of payments condition has not led to results that are considered counter-intuitive or unrealistic.

The real flaw in the above models, I would argue, is they are based on the wrong assumption about interest rates: these are determined in an IS/LM framework, leaving out the Fisher effect in which nominal rates rise with expected inflation. The Fisher effect is central to the classical/monetarist view that BF can be **both** less expansionary and stable than MF. Even when flexible prices are brought into the analysis, since in the stationary state (LRE) the price level does not change, there is no difference between nominal and real interest rates. Hence total interest payments on the debt are **higher** under BF than under MF in the Blinder-Solow model, with associated results regarding debt stability.

But if there were a Fisher effect present, their fundamental result need not follow. Under BF with a Fisher effect, inflation and hence interest rates, interest payments on the debt and taxes **can** be lower, although the stock of bonds is higher. Hence  $Y^*(BF)$  can be  $< Y^*(MF)$  with budget balance. The structure of models dealing with levels of variables precludes inflation in the steady state. Thus by their very construction these models leave out the fundamental economic phenomenon (the response of interest rates to expected inflation) that can ensure non-inflationary and stable BF. In effect, Blinder and Solow derived a stability condition that is not likely to be satisfied, but is most often irrelevant.

In the flexible price case of the Tobin-Buiter paper, during the transition to the new LRE, there is inflation, with the distinction between nominal and real rates of return. They state,

“If  $x$  is the expected instantaneous proportional rate of change of the price level, and  $R$  the nominal rate of return on bonds, the real rate of return on bonds is  $R - x$ ” (T-B p. 291)

In the equations that summarise their dynamic flexprice model (38 through 43, p. 297) although investment (IS curve) depends on the real rate ( $R - x$ ) the nominal rate does not respond to inflation expectations. But introducing inflation is not the same as keeping the real interest rate fixed and letting the nominal rate vary with expected inflation, as in a monetarist model with a Fisher effect. In his comments on the Tobin-Butter paper, even Milton Friedman does not identify the absence of the Fisher effect as the fundamental assumption underlying their results. Nor does he invoke the Fisher effect to justify his statement that the long-run effect of fiscal policy is likely to be small or negative<sup>6</sup>.

In all likelihood, it is not possible to include the Fisher effect in the transition to the new LRE in the flexprice case of the Tobin-Butter model. If it could be modelled, it would be a fruitful exercise to show that the debt can stabilize under BF without it being more expansionary<sup>7</sup>. If not, one can just conclude that the structure of these models make them irrelevant to analyze the implications of debt stability under BF.

#### D. Policy Implications of Time Periods

There is another vital dimension to the choice of BF versus MF: the welfare implications of how soon the economy responds to demand stimulus. Since the Blinder-Solow framework uses differential equations it cannot shed light on this issue: all it says is that

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<sup>6</sup> Friedman (1976) lists four differences between monetarists and Keynesians in approach (Money versus credit, Build up or down, stocks and flows, substitution versus wealth effects). He also states that “I have tried to present monetarist analysis in IS-LM terms, even though recognizing that this was a cumbersome theoretical structure for the purpose..” (pp 315-317). However, he does not explicitly reject the Tobin-Butter analysis for lacking a Fisher effect, a more fundamental problem than cumbersome.

<sup>7</sup> The simulation using difference equations and discrete time periods in Appendix C presents the path of nominal GNP for the B-S model. It can be seen that for the (‘reasonable’) numerical values chosen, although  $Y^*(MF) < Y^*(BF)$ ,  $Y$  increases more slowly under BF than MF. During this transition, under flexible prices, the price level and possibly inflation will be lower under BF than under MF. With a Fisher effect, this could feedback into lower interest rates and interest payments so total  $Y$  may need to grow less for budget balance under BF than under MF. So the expected inflation during the transition period could determine the actual outcome.

$Y^*(BF) > Y^*(MF)$  when the former is stable. However, the difference equation discrete-time period framework used in the simulation in Appendix C can be used to discuss this issue. The Blinder-Solow model implies that except when the economy is below full employment **and** the stability condition is satisfied, BF is better. The matrix below outlines these cases.

Implications of Blinder-Solow model

	BF Stable ( $t.FB > 1 - t$ )	BF Unstable
Full Employment	Avoid BF. More inflationary	Avoid BF
Below Full Employment	BF is Desirable	Avoid BF

However, even assuming BF is stable and the economy is below full employment, this conclusion of the model that BF is desirable does **not** follow <sup>8</sup>. The reason is that it takes longer (for reasonable parameter values) under BF to get close to  $Y_{Full}$  (the level of nominal income -and real income, since prices are fixed- corresponding to full employment). With a typical social welfare or loss function used in stabilization policy analysis, in which welfare loss is proportional to the variance of the deviation of output from full-employment output, MF can yield more welfare than BF since it brings  $Y$  closer to  $Y_{Full}$  soon. If  $Y^*(MF) < Y_F$ , another MF increase in  $G$  can be used to close the gap.

As the chart in Appendix C shows, starting with same increase in  $G$  and with an identical first-round effect, upto 12 periods  $Y(MF)$  exceeds  $Y(BF)$ , even though  $Y^*(BF)$  is higher. Thus, when response time is taken into account, the only state of the world (under unemployment with fixed prices) in which the Blinder-Solow model implies that MF should be eschewed is, ironically, the only one in which MF is preferable. A central tenet

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<sup>8</sup> It is unlikely that the economy will ever be in the top left hand quadrant, since, as discussed earlier, at full employment the stability condition is not likely to hold, even allowing for a short-run situation in which the economy is above potential output and  $F_b > 0$ .

of the monetarists, i.e., for getting out of a recession, MF of the deficit is better than BF because the demand response is quicker, thus seems vindicated.<sup>9</sup>

### III

#### A. The Stability Condition with Growth

To arrive at a 'monetarist' conclusion regarding the stability of BF versus MF requires a growth framework. With steady inflation that gets incorporated into interest rates and interest payments on the debt, it is easy to show that in LRE, BF can be less expansionary than MF and the debt can stabilize. While the stability condition is well known, it may be helpful to outline the relevant formulae in order to discuss how inflationary expectations and dynamics of adjustment affect the outcome and also to evaluate the Sargent and Wallace argument that MF of the deficit is preferable. As will be seen, an adaptive expectations Fisher effect on interest rates reverses the short-run debt instability that results under BF.

The government budget constraint can be expressed as in Darby (1984), scaled by income

$$G/Y - T/Y = g_M (M/Y) + [g_d - r(1-t)]D/Y \quad \text{III(1)}$$

$D$  is the stock of government debt outstanding

$r(1-t)$  is the after-tax real interest rate. It equals  $R(1-t) - \Pi$

$g_M.M$  is the amount of money printed to finance the deficit in that period

$[g_d - r(1-t)]D$  is borrowing that period in excess of real after-tax interest paid on debt<sup>10</sup>.

Rearranging III(1) yields

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<sup>9</sup> In fact, Friedman (1948) originally advocated MF of the deficit for short-run stabilization policy but later switched to advocating a money growth rule in 1959. Perhaps this switch was partly based on a judgement that MF deficits would vitiate an independent monetary policy, with loss of long-run inflation control.

<sup>10</sup> Most variables above are expressed in nominal terms. Being ratios they could be expressed in real terms and the price level would cancel. But  $g_d$  and  $r(1-t)$  are in real terms. Under perfect foresight  $r = r^e$ , and with a tax-adjusted Fisher equation that keeps the post-tax real rate constant,  $R = r + \Pi/(1-t)$ . Therefore  $R(1-t)$  equals  $r(1-t) + \Pi$  while  $g_D = g_d + \Pi$ . Hence  $g_D$  minus  $R(1-t)$ , by subtracting  $\Pi$  from both terms, is the same as  $g_d$  minus  $r(1-t)$ .

$$(D/Y) = [G/Y - T/Y - g_M(M/Y)]/[g_d - r(1 - t)] \quad \text{III(2)}$$

In the steady state, the debt/GDP ratio stabilizes, i.e.  $g_{D/Y} = g_D - g_Y = g_d - g_y = 0$ .

Substituting the steady state condition  $g_d = g_y$  in (2) above yields

$$(D/Y)^* = [G/Y - T/Y - g_M(M/Y)]/[g_y - r(1-t)] \quad \text{III(3)}$$

Thus as long as  $g_y$  exceeds  $r(1-t)$  any rise in  $G/Y$  can be financed by issuing bonds indefinitely and the debt will stabilize.

The numerical example in Appendix D works out the impact of an increase in  $G/Y$  paid for by BF and MF respectively. As can be seen, relative to MF, BF raises the debt/GDP ratio but inflation is lower. Starting at  $Y_{t_0}$ , in the long run equilibrium steady state,

$$g_{Y^*}(\text{BF}) < g_{Y^*}(\text{MF}) \quad \text{since } \Pi(\text{BF}) < \Pi(\text{MF}) \text{ and } g_y \text{ is same in both cases}$$

Hence  $Y^*_{t+i}(\text{BF}) < Y^*_{t+i}(\text{MF})$  for all periods in the steady-state. Although the debt/GDP ratio is higher, nominal interest rates and interest payments are lower under BF. Debt is stable under BF, with lower nominal interest payments, contrary to Blinder-Solow.<sup>11</sup>

### B. The Sargent-Wallace scenario

Bond finance is viable as long as the stability condition is satisfied. However, many vital issues concerning stability have to do with the behaviour of  $g_y$  and  $r$  during the transition period, and the probable consequences of the outcomes during the transition period for

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<sup>11</sup> In this example, there is a permanent deficit which is financed by money growth. Strict budget balance does not hold. However, it is easy to construct an example without any money growth to finance the deficit and the overall budget in balance every period. In such a set-up, inflation can result only if there are sources of money growth other than the government deficit. For brevity and convenience, this more general set-up has not been modelled.

the long-run stability of BF.<sup>12</sup> The Sargent-Wallace paper carries the following implications, derived as they emphasize, from a monetarist model,

- 1) The interaction between fiscal and monetary policy through the government budget constraint implies that, contrary to standard monetarist views, tight money now leads to higher inflation in the future<sup>13</sup>
- 2) In a modified monetarist model, explained later, “tighter money today leads to a higher inflation rate and price level not only eventually but starting from today” (p. 161)
- 3) This modified monetarist model implies that “the easier monetary policy is uniformly better than the tighter monetary policy. In terms of the model of Appendix I in Sargent-Wallace, “the equilibrium for the easier monetary policy is Pareto superior to that for tighter monetary policy.” (p. 167) In effect, Sargent and Wallace were advocating MF of the deficit<sup>14</sup> McCallum (1978) also points to debt instability under a money-growth rule.

### 3. Critique of the Sargent-Wallace dynamics

At a very basic level, the Sargent-Wallace analysis can be rejected on empirical grounds, as Darby (1984) did in his rejoinder to Sargent and Miller. If  $g_y$  exceeds  $r(1-t)$ , which has been the **long-run** historical record for the US, then their entire argument, made in the context of the US economy, falls apart. In the context of other economies, the long-run relationship between  $g_y$  and  $r(1-t)$  has to be examined before coming to any conclusions

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<sup>12</sup> For convenience,  $g_y$  is often compared to  $r$  in general discussion, ignoring the tax term.

<sup>13</sup> This main conclusion is in their introduction, “More generally, given the time path of fiscal policy and given that government interest-bearing debt can be sold only at a real interest rate exceeding the (real) growth rate, the tighter is current monetary policy, the higher must inflation eventually be.” (p. 160)

<sup>14</sup> Although presented as a purely logical implication of the budget constraint, their 1981 paper was motivated by the Reagan deficits. These deficits, partly due to tax cuts, arose at a time, when a year earlier, US monetary policy had just turned extremely tight to reduce double-digit inflation, with the ensuing situation of  $r$  vastly exceeding  $g_y$ , which turned out to be transitional. Based on the observed outcomes during the 1980s, the S-W interpretation of the Reagan deficits was off on many counts: in their judgement about the ability of the Federal Reserve to persevere with tight monetary policy, as a prediction about how bond markets would respond to declining inflation coupled with rising debt, and above all as a policy prescription of how a central bank should respond to lax fiscal policy.

about a debt-trap<sup>15</sup>. The long-run historical record implies that fiscal and monetary policy can be independent, with the debt adjusting passively. As Darby puts it, “Dynamic inconsistencies do not result from treating government expenditures, taxes and money growth as simultaneously exogenous. A current deficit is therefore not per se inflationary in the sense of requiring future increased money growth(Darby, 1984, p. 271)”

The above analysis has not looked at the demand functions for bonds and money. For analytical convenience, it is assumed that the demand for money does not depend on the interest rate; the numerical examples in Appendix D are worked out on this assumption.<sup>16</sup> The amount of seignorage a government can extract from bonds or money depends on the underlying demand functions by the public for these assets. As Sargent and Miller (1984) point out in their reply to Darby, under certain conditions, even if  $g_y$  exceeds  $r$ , monetization of the deficit is necessary. But the reverse is also true: if the demand for money balances is elastic, the inflation tax cannot be used to pay for the deficit. Depending on the specific demand functions for money and bonds, different degrees of monetary accommodation represent the optimal outcomes, with multiple equilibria as well.<sup>17</sup>

It is useful to analyze other assumptions and the economic responses underlying the Sargent-Wallace conclusions to understand the various possible dynamics of adjustment for debt stability. In their model the Fisher effect is present, but is embedded in some arbitrary assumptions in a rational expectations model. The implicit sequence of responses to a rise in  $G/Y$  financed by issuing bonds is:

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<sup>15</sup> In the Indian context, the S-W recommendations have been recently made, “For any desired fiscal deficit, there would exist an optimal level of monetary accommodation and vice versa.” (Rao & Rao, 1998) However, this analysis is based on an additional solvency constraint, ie. some binding level, legal perhaps, on the debt/GDP ratio that is not part of the S-W model. It is undoubtedly true that in developing countries such as India, without well developed debt markets, MF of the deficit is often required. But it is also true that avoiding MF in such cases keeps **primary spending in check** and the debt stable.

<sup>16</sup> In Darby’s (1984) numerical examples of stable BF, money demand varies with the interest rate.

<sup>17</sup> Bruno and Fischer (1990) show that if there are discrete jumps in the inflation process, then the same fiscal deficit could result in two alternative inflation rates. Rao and Rao (1998) extend this model and claim that the economy can find itself in a high interest trap if there is excessive borrowing. Implicitly, a higher degree of MF is better, as in the Sargent-Wallace paper.

- 1) agents (bond market participants) see the real interest rate, interest payments and the debt ratio rising as the central bank refuses to monetize and/or lowers money growth.
- 2) agents assume that the path of government expenditures (G/Y) is unchanged
- 3) they conclude that future money growth has to rise to satisfy the budget constraint
- 4) they bid up nominal interest rates, a rational expectations Fisher effect based on 1,2 & 3
- 5) the rise in nominal interest rates leads to a fall in current money demand, which leads to higher inflation, without even current money growth rising<sup>18</sup>.

However, what Sargent and Wallace claim is the outcome of forward-looking expectations, in the sequence of events described above, is also predicated on the assumption (2) that agents assume that future G is unchanged. But when bond market participants see a rising debt ratio due to a tight central bank, they could very well look ahead and conclude that that in the future primary G will have to be lower, not that future money growth will have to be higher, with G unchanged. Hence real and nominal rates can fall, and the debt can stabilize without a rise in future money growth.<sup>19</sup> Alternatively, they could assume that the paths of G and M are unchanged, but that the stability condition holds, and a stable rational expectations jump equilibria could result. Their conclusion involves assuming more than just that forward-looking behaviour.

While the expectation that future G will fall (when current money growth is tight) can lead to falling interest rates, an adaptive response to past inflation is enough to generate stability under bond financing, and is probably the most likely outcome. With an adaptive

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<sup>18</sup> The last response, corresponding to their “more sophisticated and dynamic description of the demand for base money (p. 161)” is the Cagan-Bresciani-Turroni effect, often observed during hyper-inflation. As inflation expectations rise, so do interest rates, in turn reducing money demand relative to supply and leading to further inflation without a change in current money supply.

<sup>19</sup> In their introduction, Sargent-Wallace (1981) discuss two coordination schemes: in the first, monetary authority is powerful and fiscal policy is set residually; in the second, fiscal policy is first decided and monetary policy adjusts to that. They analyse the second scheme based on their ‘empirical judgement’ about Western economies. In a latter article, Sargent (1985) discusses the outcome of the first scheme in which fiscal policy is set residually and so a lower rate of money growth sooner or later requires lower fiscal deficits. However, even when fiscal policy is set residually, a lower rate of money growth does not necessarily require lower fiscal deficits, as analysis in this section has shown.

expectations Fisher effect, agents need not make assumptions about whether the fiscal or monetary authority will back down in the future or conjecture what the future path of  $G$  will be. Keeping the future path of  $G$  fixed, nominal interest rates can fall as nominal GNP growth and inflation can be lower under bond financing.

#### 4. Theoretical Consistency of Debt Stability under Bond Finance

There is another theoretical aspect of debt stability that needs to be discussed. It has been argued that  $g_y > r$  is theoretically inconsistent since it implies that the present value of a project is infinite and precludes cost-benefit analysis.<sup>20</sup> However this viewpoint ignores the distinction between the risky rate that should be used in cost-benefit analysis and the risk-free rate on government bonds. In finance theory the well-known Gordon growth model is used for computing the present value (price) of a stock. Consider a stock that pays a dividend  $d$  that grows by  $g$  every period. If this is the representative stock for the economy then  $g$  is the growth rate of the economy. The stock price is merely the discounted stream of future dividend payments. If the risky rate for discounting the stream of dividends is  $r(\text{risky})$  then the price of the stock, obtained as the sum of the infinite series is merely  $d/[r(\text{risky}) - g]$ . Stability (for the stock to have a finite price) requires that  $r(\text{risky})$  exceed  $g$ . There is no contradiction in having an economy in which  $R(\text{risky}) > g > r(\text{risk-free})$ , with both stability conditions being met.

## IV

The 'monetarist' approach to debt adjustment can be best summarized by the schematic below that traces out the adjustment path of different variables following a decrease in money growth used to finance the deficit, keeping  $G/Y$  constant. In the monetarist view, during the transition, the real rate will rise due to tight money and simultaneously real growth will decline. Indeed, from this perspective, an abnormal gap between  $r$  and  $g_y$  has more to do with lower money growth than with rising or large deficits.

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<sup>20</sup>This point has been made by Scarth (1989) along the lines of earlier literature which concludes that optimizing behaviour precludes  $g_y$  exceeding  $r$ .

Impact of a decrease in money growth previously used to finance government expenditure

	Nominal Rate	Inflation	Real Rate	Real GDP Growth	D/Y Ratio
Short Run	Rises	Falls/Same	Rises	Falls	Rises, unstable $g_y < r(1-t)$
Transition/ Intermediate Run	Falls	Falls	Falls	Rises	Rises stably, $g_y > r(1-t)$
L.R.E. (perhaps)	Lower	Lower (lower)	Same (higher)	Same	Higher

While stability can be achieved under BF, are there any long-term benefits to be derived from lower inflation resulting from lower money growth, keeping fiscal policy unchanged, or from preventing rising inflation by not monetizing a rise in  $G/Y$ ? Inflation implies real transactions costs, as Okun (1980) has persuasively argued, some of which cannot be captured by GDP. Hence a permanent reduction in inflation leads to welfare gains.<sup>21</sup>

In the context of debt stability, the long-run benefits or costs of inflation would depend on any potential impact of lower inflation on  $g_y$  and  $r$ . The nominal rate can be expressed as:  
 $R = \text{Real risk-free rate} + \text{Expected Inflation} + (\text{Inflation}) \text{ Risk Premium}.$

If low inflation and/or the higher credibility of the central bank reduces the **risk premium** in bond yields (a long run effect), then BF deficits facilitate debt adjustment in the long run.<sup>22</sup> Similarly, if real GDP growth is negatively related to inflation, then BF of the

<sup>21</sup> Further, inflation leads to financial and business practices and activities that enter GDP but, from a utility standpoint, are a deadweight loss, similar to rent-seeking activity. Hence tests of the impact of inflation on real GDP growth, such as that of Barro (1995) are likely to underestimate the adverse impact of inflation on welfare.

<sup>22</sup> The tendency for the real rate to fall as inflation falls due to a lower risk-premium can be called a Keynes effect, who was probably the first to stress this phenomenon. "One of the heaviest and arguably the most avoidable burden of production is that of risk. This element of risk is greatly aggravated by instability in the standard of value. Currency reforms, which led to the adoption by this country and the

deficit lowers the burden of the debt in the long run by raising  $g_y$ .<sup>23</sup> If both these two effects are present, relative to MF, BF raises  $r$  and lowers  $g_y$  in the short run, but does the opposite in the long run, thus facilitating debt stability.

These two long-run effects have not been given much attention in the literature on the debt burden, even by those advocating BF. Insofar as long run effects are considered, the 'realistic' presumption is that often debt financing will raise real rates, reduce capital stock and thus reduce growth.<sup>24</sup> While this may be the case, it should be reemphasized that it is even more realistic to stress that BF can ensure that primary government spending is kept in check, which is far more crucial for debt stabilization than anything else.

The issue of the long-run effects of BF on real growth and the real interest rate needs empirical investigation. There is a wide array of broad evidence suggesting a risk premium effect on interest rates from historical trends, as well as from recent developments. It is worth noting that under the gold standard, yields on very long-term bonds were under 3% in UK for lone periods of time. In Switzerland, the only country in the world whose inflation record closely approaches price-stability, ten-year bond yields are currently just below 3%. During the 1950s in the USA, despite a debt-GDP ratio in excess of 100% for some years, ten-year bond yields averaged 2.75%.<sup>25</sup>

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world at large of sound monetary principles, would diminish the wastes of *risk*, which consume at present too much of our estate." (Preface, 1923, p. xiv)

<sup>23</sup> In a 100 country study that controls for other variables, Barro (1995) found that inflation reduced growth, although by a small amount, and that evidence was clear only for countries with inflation over 10%.

<sup>24</sup> In empirical simulations of the debt burden in India under BF and MF, the authors state, "...it is assumed throughout the study that the authorities maintain a constant real GDP growth: also, while developing the debt-financing (i.e. BF) scenario, inflation rate is assumed to be unchanged. In reality, growing debt-GDP ratio and concomitant debt-service burden may stifle the economic growth and aggravate inflation. These feedback effects are ignored in the debt-financing scenario...Actual outcomes are therefore likely to be worse. On the other hand, in the MF scenario, feedback effects on inflation have been duly incorporated" (Rangarajan, Basu and Jadhav, 1989)

<sup>25</sup> Many of the OECD countries have, like the USA, successfully lowered inflation over the last two decades and now have low real rates, despite a temporary debt-trap during disinflation and the ensuing rise in the debt-GDP ratio. Ten-year bond yields in most European countries are currently in a 4.5 to 5% range, with inflation in a 1.5-2% range. After-tax real rates are now below, or close to, long-term growth rates for most of these countries. Even Belgium, despite a debt-GDP ratio over 100%, has an adequately low real rate for stable debt, perhaps because its monetary policy is closely tied to the Bundesbank, which

Although it may be difficult to find strong links between the real interest rate on regular bonds and the inflation rate from statistical time-series or cross-section studies, it is worthwhile to test for a risk premium effect on interest rates. Data on inflation-indexed bonds, which have been issued in UK, USA and Canada in recent years, could provide useful information about this 'inflation risk-premium' hypothesis. In short, not only can debt be stable under bond finance, it can also lower the real rate of interest and be welfare enhancing if there is a significant risk-premium component of the real interest rate.

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has strong inflation fighting credentials. When the Bank of England was formally granted independence in April 1997, long-term yields fell significantly.

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## Appendix A

### List and explanation of variables used in this paper

Capital letters usually denote nominal variables and the small letters the corresponding real variable. Superscript 'e' denotes expected value,  $\Delta$  denotes change in that period.

\* denotes LRE (Long Run Equilibrium) values corresponding to budget balance or stable debt-GDP ratio. (This may differ from full-employment-values denoted Full).

P denotes the price level

Y denotes nominal GNP (y denotes real GNP)  $Y = P \cdot y$

R is the nominal interest rate and r is the real rate of interest (R - Inflation)

G is government expenditure

T is taxes. T equals  $t \cdot Y$ , where t is the flat income-tax rate

M is (exogenous) money supply.  $\Delta M$  is a change in M via open-market operations.

BF denotes bond finance of Government debt and expenditure, keeping M constant.

MF denotes the financing of G by printing money, or monetization of the deficit or debt.

The bonds issued are all perpetuities with a coupon of 1. So  $\text{Price}(\text{Bond}) = 1/R$  and

B is the number of bonds outstanding **and** also the interest payments on the debt.

$F_G$  is the IS/LM spending (fiscal) multiplier including the nominal crowding out effect.

$F_B$  denotes the **net** impact on Y from the issue of one bond due to the wealth impact of this bond on IS & LM curves.

$F_M$  is the impact on Y of a unit change in M, i.e. the 'monetary' multiplier

$\Pi$  is the inflation rate  $= (P_{t+1} - P_t)/P_t$ .

D denotes the outstanding value of nominal debt and d denotes real debt.

$g_Y$ ,  $g_y$ ,  $g_D$  and  $g_M$  denote the growth rates of nominal GNP, real GNP, stock of debt, and stock of money respectively.

## Appendix B

### Derivation of Blinder-Solow result and related results

$\Delta B/R$  is the value of new bonds issued to finance the deficit in a period (cf. Note below)

$\Delta M$  is the money printed to finance the deficit in a period.

$(G + B) = t(Y + B)$  is the LRE budget balance.  $Y$  is net income and  $Y + B$  gross income.

Equilibrium income  $Y^*$  is obtained as the solution of the two basic equations:

$$Y = F(t, G, B, M) \tag{1}$$

$$G + B = T(Y + B) + \Delta B/R + \Delta M \tag{2}$$

In equilibrium  $\Delta B = \Delta M = 0$

For Money Finance, the simple inverse of the tax-rate formula applies:

$$dY^*/dG \text{ (MF)} = 1/t$$

Also, by differentiating (1),  $dY = F_G dG + F_M dM$ . So  $dM^* = (dY^* - F_G dG)/F_M$ .

For Bond Finance,  $Y^*$  is obtained by totally differentiating (1) and (2)

$$dY = F_G dG + F_B dB \tag{1}' \quad \text{From this substitute } dB = (dY^* - F_G dG)/F_B \text{ in (2)'}$$

$$dG + dB = t dY + t dB \tag{2}' \quad \text{since } \Delta M = \Delta B = 0 \text{ in LRE. Solving (2)'} \text{ yields}$$

$$dY^*/dG \text{ (BF)} = [F_B - (1-t)F_B]/[tF_B - (1-t)].$$

This holds when the stability condition  $[tF_B - (1-t)] > 0$  is satisfied.

Also  $dB^* = (dY^* - F_G dG)/F_B$ . It can be shown that  $dY^*/dG \text{ (BF)} > dY^*/dG \text{ (MF)}$ .

The main result above and related ones are proved in Scarth (1988). This Appendix uses his notation and his exposition for the basic results above.

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Note:  $\Delta(B/R)$  is the change in the value of existing bonds. This differs from  $\Delta B/R$  if interest rates change with associated capital gains and losses. For convenience,  $\Delta R$  is assumed to be zero, which is the LRE condition. Although this is an IS/LM model in which changes in income and mode of financing do affect  $R$  in the short run, these short run changes do not affect the LRE budget balance level of  $Y^*$ .

Blinder-Solow Corollary: If a BF deficit is stable, then an increase in money supply via an open market operation ( $\Delta M = - \Delta B/R$ ), with subsequent changes in the budget financed by bonds reduces LRE income, i.e.  $dY^*/dM < 0$ .

Proof: By totally differentiating (1) and (2) yields for this case:

$$dY = F_B dB + F_M dM \quad (1)' \quad \text{and} \quad dB = t dY + t dB \quad (2)' \quad \text{since} \quad dG = 0$$

Substituting  $dB^* = [t/(1-t)]dY$  in from (2)' in (1)' and solving yields

$$dY^*/dM = (1-t)F_M / [(1-t) - tF_B] < 0$$

since denominator  $< 0$  by the stability condition and also  $F_M > 0$ ,  $(1-t) > 0$ .

$$\text{Also } dB^* = t/(1-t)dY^* < 0 \text{ since } dY^* < 0$$

$$\text{and } dB^*/dM = (dB^*/dY^*)(dY^*/dM) = t.F_M / [(1-t) - t.F_B] < 0$$

Investment Corollary: If a BF deficit is stable, then an autonomous rise in private investment (I) reduces income in the long run.

$Y = F(I, G, t, B, M)$  (1) yields by differentiating

$$dY = F_G dI + F_B dB \quad (1)' \quad (\text{Since the } F_G \text{ multiplier also applies to Investment or I}).$$

$$dB = t.dY + t.dB \quad (2)' \quad \text{Substituting } dB \text{ from (2)' into (1)', as above, yields}$$

$$dY^* = F_G dI + F_B [t/(1-t)]dY^* \text{ or}$$

$$dY^* [1 - tF_B/(1-t)] = F_G dI \quad \text{or}$$

$$dY^*/dI = (1-t) F_G / [(1-t) - t.F_B] < 0 \text{ since by the stability condition, denominator } < 0.$$

$$\text{Also } dB^*/dI = (dB^*/dY^*)(dY^*/dI) = t.F_G / [(1-t) - t.F_B] < 0$$

## Appendix C

### Discrete-time Simulations with the Blinder-Solow model

This Appendix presents numerical simulations from the Blinder-Solow model to show their basic results and the time path of income during the transition. Start with an original equilibrium (in which there are no bonds) with the following values:

$$G = 250, t = 0.25, Y^* = 1,000, M = 500, R = 10\%, F_G = 2, F_B = 4, F_M = 2.$$

A. Consider an increase in  $G$  by 50 financed by  $M$

$$dY^*/dG (MF) = 1/t = 4. \text{ So } dY^* = 200 \text{ and } dM^* = dG(1/t - F_G)/F_M = 50.$$

B. Consider the same increase in  $G$  financed by issuing bonds.

$$dY^*/dG (BF) = [F_B - F_G(1-t)]/[tF_B - (1-t)] = 10.$$

$$\text{So } dY^* = 500 \text{ and } dB^* = (dY^* - F_G dG)/F_B = 100.$$

The budget balance  $(G + B) = t(Y + B)$  is satisfied  $(300 + 100) = .25(1500 + 100)$

The following graph on the next page shows the time path of  $Y$  under both modes of finance. The simulation is set up such that the deficit equals this period's primary expenditure ( $G$ ) plus last period's interest payments minus taxes based on last period's values. This is an artificial set-up: for expositional convenience, the deficit affects the stock of bonds and money only from period 3 onwards. The first round of  $G$  in period 2 does not need to be financed and has the same impact under BF and MF. As can be seen,  $Y$  grows more rapidly under MF than under BF. The implications are discussed in page 15. Insofar as financing would need to take place in period 2, income would rise even quicker under MF than BF.

## Appendix C (contd.)

### Money Finance (from period 3 onwards)

1.  $\text{deficitmf} = 300 - .25\text{ynetmf}(-1)$
2.  $\text{delmoney} = \text{deficitmf}$
3.  $\text{dynetmf} = 2(\text{delmoney})$  [since  $F_M = 2$ ]
4.  $\text{ynetmf} = \text{dynetmf} + \text{ynetmf}(-1)$
5.  $\text{money} = \text{money}(-1) + \text{delmoney}$

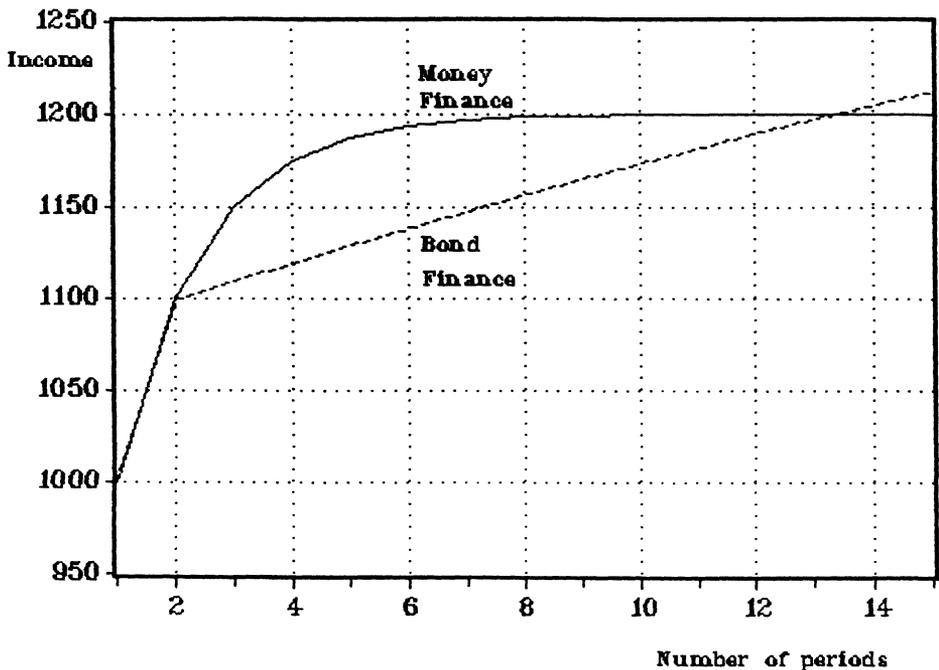
Period	deficitmf	delmoney	money	dynetmf	ynetmf
1	0	0	500.00	0	1000.00
2	50.00	0	500.00	100.00	1100.00
3	25.00	25.00	525.00	50.00	1150.00
4	12.50	12.50	537.50	25.00	1175.00
14	0.01	0.01	549.99	0.02	1199.98
15	0.01	0.01	549.99	0.01	1199.99

### Bond Finance (from period 3 onwards)

1.  $\text{deficit} = 300 + \text{nbond}(-1) - .25\text{ynet}(-1) - .25\text{nbond}(-1)$
2.  $\text{delnbond} = \text{deficit}/10$  [Since  $R = 10\%$ ]
3.  $\text{nbond} = \text{delnbond} + \text{nbond}(-1)$
4.  $\text{delynet} = 4(\text{delnbond})$  [since  $F_B = 4$ ]
5.  $\text{ynet} = \text{ynet}(-1) + \text{delynet}$
6.  $\text{ygross} = \text{ynet} + \text{nbond}(-1)$

Period	Deficit	delnbond	nbond	delynet	ynet	ygross
1	0	0	0	0	1000.00	1000.00
2	50.00	0	0	100.00	1100.00	1100.00
3	25.00	2.50	2.50	10.00	1110.00	1110.00
4	24.38	2.44	4.94	9.75	1119.75	1122.25
199	0.18	0.02	99.32	0.07	1497.28	1596.59
200	0.17	0.12	99.34	0.07	1497.35	1596.67

**Response of Income to Higher Spending under BF and MF**



## Appendix D

This Appendix works out numerical examples of long run equilibrium debt ratios and related variables for different fiscal and monetary policy combinations.

$$(D/Y)^* = [G/Y - T/Y - g_M(M/Y)]/[g_Y - r(1-t)] \quad \text{III.3}$$

Let  $G/Y = 0.20$ ,  $T/Y = 0.15$ ,  $g_M = .05$ ,  $(M/Y) = 0.4$ ,  $g_Y = 4\%$ ,  $r = 3\%$ ,  $t = 1/3$ .

In this example, for simplicity it is assumed that  $(M/Y)$  is fixed. It does not vary with the nominal interest rate and with real income growth, as in a normal money demand function. These extra factors can be introduced, as in Darby (1984) without altering basic results.

For the above values,  $(D/Y)^* = [.20 - .15 - .05(.4)]/[.04 - .02] = 1.5$  years or 150%.

Since  $M/Y$  is fixed,  $g_Y = g_M = 5\%$  and  $\Pi = g_Y - g_Y = 1\%$ . So  $R = r + \Pi / (1-t) = 4.5\%$

Real interest payments ratio to real income =  $r(D/Y) = .03(150\%) = 4.5\%$

Nominal interest payments ratio to income =  $R(D/Y) = 4.5 \times 1.5 = 6.75\%$ .

Let initial  $y_{t_0} = 1,000$ . Since there is 1% inflation,  $Y_{t_0}$  is 1,010.

Now suppose that  $G$  rises to 22%, financed fully either by bonds or money.

For BF, the debt/GDP ratio rises but inflation stays at 1%. For MF, money growth rises by 5% to meet the rise in  $G/Y$  [ $\Delta g_M(0.4) = \Delta G/Y = 0.2$ ] Hence  $g_M$  rises to 10% and  $\Pi$  rises to 6%. From the tax-adjusted Fisher equation,  $R$  is now =  $3\% + 6\%/(2/3) = 12\%$ .

The equilibrium values of some variables are listed in the Table below. Suppose convergence to the new debt-ratio is achieved in three periods. It can be seen that although in **real** terms, the debt-GDP ratio and the debt burden (ratio of interest paid to income) are higher under BF, in **nominal** terms both income and nominal interest payments are lower under BF. This contradicts the Blinder-Solow result that  $Y^*(BF) > Y^*(MF)$  for BF debt to stabilize. In the example below, it is assumed that interest is paid at the end of the period.

Period	(D/Y)* (%)	r(D/Y)	$\Pi$ %	R %	R(D/Y)	y	Y	Real Int. Paid	Nominal Int. Paid
Start $t_0$	150	.045	1.0	4.5	.0675	1000.0	1010.0	45.00	68.18
BF(t+3)	250	.075	1.0	4.5	.1125	1124.8	1157.6	84.36	130.20
MF(t+3)	150	.045	5.0	12.0	.1800	1124.8	1331.0	50.61	239.50