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SETTING OPERATING POLICIES FOR SUPPLY HUBS

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Abstract

This paper deals with the joint optimisation of operations at the supply hub for the hub vendor and the upstream supplier. Different operating conditions are considered, namely, backordering, minimum and maximum specified inventory levels. Some analytical insights on better managing suppliers operating under a vendor managed inventory program are presented. Essentially, we show that the penalty cost imposed on over-and under stocking, and the min-max policy for hub inventory reside in the power of the hub operator while the order cost of dispatching and production reside with the supplier. A numerical example and an algorithm are included to highlight this result.

Keywords: Supply Chain Management, Supply hub, Logistics.

1. Introduction

There have been many new developments in the area of supply chain management, particularly those pertaining to cost reduction and customer demand responsiveness. In this regard, a key question that begs addressing in improving supply chain efficiency is on managing the suppliers. Erengüç et al. (1999) have highlighted that to support JIT production, suppliers are required to deliver frequently in small quantities. In the automotive industry, some suppliers of Toyota already deliver as frequently as four times a day. Many of these suppliers are single-source suppliers and are located close to points of delivery (Aderohnumu et al., 1995). This makes it relatively easy for them to synchronize their operations with those of Toyota. However, for the suppliers located outside Japan, these are usually located far from the points of delivery. The long haul, plus problems involving multiple sourcing, makes time based logistics management a complex and challenging problem for the suppliers (Celly et al., 1986).

Under such a setting, irrespective of the competitive positions of the companies in the supply chain, cooperative decision-making is advantageous. Today, when one wades through the literature on industry practices in relationships among the supply chain partners especially in the area of inbound logistics, one will find a spectrum of relationships ranging from Manufacturer Owned Inventory (MOI) through to Vendor Managed Inventory (VMI), and Supplier Owned

Inventory (SOI). The supply chain is constantly evolving and the supply chain a la mode as far as the time sensitive industrial sectors are concerned is that of using supply hubs (or material hubs or vendor hubs) to provide both leanness and agility for players in the supply chain. Typically, the use of supply hubs also comes with an increase in VMI arrangements. Already, many companies in the high tech and electronics industry have set up hubs or VMI facilities to house many of the components, parts, and raw materials, necessary for the assembling or manufacture of a product. Notable examples of this practice include those of Dell, Compaq and HP.

Before proceeding further, it is instructive to provide a working definition of a supply hub. We define a supply hub as a location sited very near a manufacturer's facility where all or some of its supplies are warehoused with the agreement that the materials will be paid for only when consumed (Zuckerman, 2000). Thus, a supply hub can be thought of as another model of the supplier-buyer relationship where VMI is actively pursued. Under the arrangement of VMI, the supplier manages the customer's inventory at the customer's distribution centre or at a manufacturing node. The onus is no longer on the customer but the supplier to decide how much and when to replenish the customer's inventory.

The use of supply hubs has merit for industries which suffer from varying, uncertain market demand, shortening product life cycles and uncertain supply lead times such as consumer electronics products or even in the food processing industry. For instance, for electronics contract manufacturers (CM), product life cycles have been shrinking drastically over the years. This increases the risk of obsolescence. Faster cycle time has become the order of the day. To reduce this risk, manufacturers are forced to keep no inventory or low inventory. A missed supply can become very critical to the manufacturer. In this regard, the electronics industry has discovered VMI to be a solution to both obsolescence and stock outs.

Thus, a strong motivation for a firm to consider moving to a the supply hub is typically to get rid of the high margin of error in forecasting and the uncertainty imposed on the suppliers to keep excessive safety stock throughout the supply chain. Today, supply hubs have become a streamlined approach to managing inventory by suppliers for customers. The purpose of these facilities basically is to have a ready supply of parts available to support an assembly or manufacturing operations undertaken either by the client or a contract manufacturer. The twist here is that the end manufacturer (who can be the customer) only takes ownership when the parts are used or received. In so doing, the manufacturers have access to a ready supply of inventory at little or no inventory carrying cost to them.

From the customer's perspective, slower moving inventory can be reduced significantly due to better observation of inventory levels by suppliers, more accurate tracking of inventory turns due to better planning and coordination at the hub level, leading to order cycle time reduction. From the supplier's perspective, this facilitates better inventory management in view of the inventory visibility offered by a VMI arrangement. The supplier monitors the inventory to coordinate the production and delivery schedules. An added benefit for a supplier in the VMI program is that it can observe the actual consumption rate of its parts by the CM. Over a period of time, the supplier can use this information to make an inference of the accuracy of the demand projection provided by the CM.

Nonetheless, there are some misgivings expressed by some of the players, especially the suppliers. For example, large manufacturers like Solectron are driven to the use of the supply hubs by its customers like Compaq and would therefore tend to push their own suppliers to subscribe to the same practice. However, the tier two suppliers being smaller in firm size and competitive attributes might not be inclined to follow suit. As a result, larger players like Solectron would have to entice such suppliers with free warehouse space. Another reason put forth against the supply hubs is the creation of another intermediation point in the supply chain. Associated with this is the burden of managing the cost of inventory at two different locations.

2 Current typical mode of operations for the supply hub

As a supply hub is a place where all the participating suppliers place their materials, the management of the hub requires expertise. The operation and management of a supply hub are usually handed over to a third party logistics provider (3PL). The responsibility for the ownership of the inventory and inventory management in a hub resides with the supplier at present. The suppliers continue to feed their clients' manufacturing forecasts. Usually, a change of ownership is recognised only at the point where the goods physically enter the production line or out of the hub. There is an interesting variation in the ownership of materials. Some vendors have apprehensions that they may ultimately own the inventory for too long and so are reluctant to join the supply hub. To lure such vendors, some customers have a "freshness clause" in their agreements whereby the customer assumes ownership of materials that have been in the warehouse beyond a specified period.

The client provides bounds for the inventory levels which to be maintained at the supply hubs, which are usually agreed upon at the point of signing the management contracts. Though the prevailing policy is a min-max policy, customers are usually more insistent on maintaining the minimum level, which is typically a prescribed two weeks' of stock. The supply hub operator has the responsibility to oversee this arrangement, with an information system to trigger messages to the vendors whenever the inventory stocked at the hub falls below a certain minimum level.

Outbound transportation from the 3PL's warehouse to the customer's production site is managed by the 3PL itself. This cost is usually insignificant due to the proximity of the two locations. But inbound transportation into the 3PL's warehouse becomes either the responsibility of the suppliers or in some cases the 3PL manages this also.

From the dynamics described above, it is clear that the underlying principle in the supply hubs concept is "postponement of procurement" (Zuckerman, 2000) which comes after the now famous "postponement strategies" in the final configuration of products.

3. Literature Review

The recent literature on supply chain management has been stressing the need for better customer satisfaction at lower cost (see Fearne, 1996). One key attribute as mentioned by Towill (1996) is to time compress the supply chain through outsourcing key management activities of inventory to the suppliers. In this regard, Silver (1991) points out that most models in SCM consider only inventory and backordering costs, but ignore the cost of transportation and handling, in particular the interaction effects between these costs. Supply chain researchers have thus undertaken to study the role of vendors in managing their customers' inventory, and across different industries too. For instance, Achabal et al. (2000) have presented a decision support system to help a major apparel manufacturer improve channel coordination between its 30 retail partners. In particular, they found that VMI helped to underscore the supplier's goal of better product availability and the customer's goal of more effective inventory management. This conclusion however is not new. Cottrill (1997) had already alluded to this when he highlighted the example of P&G and the VMI suppliers. In the case of P&G, the literature has clearly reported that vendors had borne most of the development and implementation costs of the supply hub at the customer's locations. The benefits as reported by Cottrill (1997) include more efficient production and distribution, and less excessive inventory.

One key issue identified by Holmstrom (1998) in a VMI partnership project between a supplier of packaged goods and a grocery wholesaler is to find an effective way for the vendor to take responsibility of the wholesaler's inventory. In this regard, Holmstrom reports that the information to help focus the responsibility include the reorder point, minimum replenishment batch, and the amount of free stock. Recently, Waller et al. (1999) also reiterated that as buyers relinquish control of the key re-supply decisions under VMI, the vendor is obliged to meet a specific customer service goal and this is usually expressed as an in-stock target.

Apart from this, various other works have also been undertaken to understand the effects of VMI on supply chain inventories (not always at the hub). For instance, Cachon and Fisher (1998) examined forecasting and inventory management using simulation, and showed that inventory at the supplier (manufacturer) and buyer (retailer) could be reduced while improving downstream service (no stock-outs). However, they did not consider issues related to allocating inventory across the buyers. In another study, Narayanan and Raman (1997), using an analytic inventory model, found that transferring the stock decisions to the supplier (manufacturer) could lead to increased channel profits. Waller et al. (1999) further note that, in a simulation involving the computer/ consumer electronics industry, most of the inventory reductions in the chain is achieved through more frequent inventory reviews (by the supplier), more order intervals, and more frequent deliveries made by the supplier.

Recently, Cetinkaya and Lee (2000) presented an analytical model for coordinating inventory and transportation decisions in VMI systems. They treated a time-based consolidation policy situation whereby the vendor experiences a sequence of random demands from a group of retailers, and determined the optimal replenishment quantity and dispatch frequency.

While some interesting work have been done to address the supply hub and VMI arrangement and optimising for cost and so on, interesting research issues still reside in this problem of supply hubs. First, though the industry trend is towards the establishment of the supply hubs, we find examples of lukewarm response (for e.g. Solectron) from the suppliers. Further, if a supplier is a monopolist (for example Intel), it cannot be easily lured into joining the supply hub. So, there is a need to design appropriate strategies to sell the concept of the supply hub to suppliers. As mentioned earlier, Solectron's strategy of free warehouse space is one way to entice suppliers. Obviously, a proper pricing strategy needs to be worked out for the benefit of all parties concerned.

Second, as mentioned, the current industry model is to have a 3PL to manage the supply hub. The customer specifies a min-max policy according to which the supplier has to maintain an inventory between the minimum and the maximum levels specified in the contract. Although a maximum is specified, the customer is usually not concerned about the maximum. It is the minimum level against which the supplier's stock and performance are monitored. Two weeks' stock requirement is the minimum level practised across the industry. There is no documented evidence of how this magic number of two weeks has been arrived at. Hence, there appears to be a need for a systematic quantitative analysis of finding the minimum requirement expected of the suppliers. Other questions are also of interest to this study. These are (i) should it be 2 weeks for all parts? (ii) is this the optimal policy? It is the second issue that this paper proposes to examine using a simple one supplier, one hub, and one product model.

4. Problem Formulation

The problem of a supplier, one supply hub, and one product can be characterised as a two-stage problem with centralized control. We further assume that the supplier has real time information about inventory at the hub. The objective then is for the supplier to optimise his performance given the min-max policy specified at the hub. The supplier pays a penalty for violating the minimum and maximum inventory norms specified at the supply hub. Given that there are fixed costs of production and transport, the supplier has to optimally decide when to produce, how much to produce, when to dispatch and how much to dispatch.

We now make following assumptions:

- Demand is deterministic and constant over time
- There are no capacity constraint at the supplier in terms of production or storage
- Lead-time for replenishment at both production and dispatch is zero. This is not a strong assumption because if lead-time is finite, all we need to do is to offset the reorder quantities (r_p and r_d respectively) to take care of demand during lead-time. Under a constant lead-time situation, this assumption is made only to simplify the exposition.

- Cost structure is linear in both production and dispatch. In a deterministic case, under a linear cost structure, one can ignore variable costs as they are not going to affect the supplier policy decisions. As such, we need to include only the fixed cost of production and dispatch.

The following notations will be used:

Q_d = Dispatch batch quantity (replenishment at the supply hub)
Q_p = Production batch quantity (to be supplied by the supplier)
r_d = Reorder point for dispatch
r_p = Reorder point for production
A_d = Fixed cost of dispatch
A_p = Fixed cost of production
h = Inventory carrying costs at the supply hub per unit item per time.
h_d = Inventory carrying cost per unit at the supplier's warehouse per unit per time.
p_s = Penalty for under stocking (below a customer specified minimum) per unit per time.
p_e = Penalty for over stocking (above a hub specified maximum) per unit per time
I = Minimum inventory which the supplier is supposed to keep, below which p_s is incurred
I^+ = Maximum inventory which the supplier is supposed to keep, above which p_e is incurred
D = Demand rate
s = Under-inventory (stock which is below minimum level) at the hub just before replenishment
T= Cycle-time for replenishment (time between two replenishment) at the hub
TC = Total average annual relevant costs.
= Production costs + Dispatch cost + Carrying cost at hub + Carrying cost at supplier warehouse +Penalty cost for under-inventory + Penalty cost for over-inventory.

The two-stage problem faced by a supplier can thus be characterized by 4 parameters (Q_p, r_p, Q_d, r_d). When the echelon stock level declines to r (r_p and r_d respectively) a batch of size Q (Q_p and Q_d respectively) is ordered. All stockouts are backordered. Put simply, whenever the inventory at the supply hub reaches a level r_d, the supplier would dispatch a quantity Q_d to the hub. Similarly, the supplier would monitor echelon inventory (inventory at the supply hub plus inventory at the supplier warehouse). Whenever it reaches level r_p, the supplier would produce quantity Q_p. In this paper, we seek to characterize this policy and suggest an algorithm to arrive at an optimal policy. From here after unless specified, all costs used are the average annual costs.

We now provide certain propositions, to help us in simplifying the problem further. These are the propositions derived from standard multistage serial systems (Zipkin, 2000) and we show that they are valid in the case under discussion too.

Proposition 1: $Q_p = kQ_d$ i.e. production batch sizes are always integer multiples of the dispatch batch quantity and every instance of production should lead to dispatch.

Consider a feasible, non-nested policy. Suppose at a particular time t, the supplier carries out production but does not dispatch the quantity to the hub. Let t_1 be the earliest time after t when the supplier carries out dispatch. Thus, the entire stock Q_p will remain at the supplier's end till time t_1 when stock will reduce to Q_p-Q_d. Now consider an alternative policy where the supplier postpones production till time t_1. Clearly, the inventory at the hub is same in both policies but inventory at the supplier warehouse would be higher by Q_p-Q_d for the time period from t to t_1 in the first policy. So, the supplier can decrease costs by postponing production till time of dispatch. This would mean that each cycle is repetitive where for every production, k dispatches are carried out. It can be shown that the optimal policy involves carrying out k equally spaced dispatches so as to minimize the overall inventory carrying costs in the system. So the optimal policy would involve Q_p= k Q_d where k is a positive integer.

Proposition 2: The supplier should follow a zero inventory policy for production timing decisions.

The supplier should produce only when the inventory at the supplier warehouse is zero. This is easy to prove. Suppose, production is carried out when inventory at the supplier's warehouse is I. Since $Q_p = kQ_d$, this inventory I would serve no useful purpose and the supplier can bring this down to zero without affecting any dispatches and in the process reduce carrying costs. So, any inventory policy that requires the supplier to maintain positive inventory at the supplier's warehouse at the time of production would be sub-optimal. Further, the supplier should monitor echelon inventory and produce Q_p whenever echelon inventory level reaches r_d.

From the above discussion, it is clear that optimal inventory policy obeys the following characteristics: r_p= r_d, Q_p= kQ_d.

As argued in the inventory literature (Zipkin, 2000), it is optimal for a supplier to have a nonzero under-inventory at the time of dispatch. Hence, the reorder point for the supply hub is set at I_- - s.

Given that $k^* = Q_p^*/Q_d^*$ and $r_d^* = L - s^*$, the optimal supply policy can be characterized by three parameters Q_d^*,k^* and s^*.

As discussed, we have five components of cost i.e. production cost, dispatch cost, carrying costs at the supply hub, inventory carrying costs at the supplier's warehouse, penalty cost for under-inventory, and penalty cost for over-inventory. We now need to work out expressions for each one of them in terms of the decision variables and parameters outlined earlier. First,

Production cost = $A_p D/Q_p = A_p D/kQ_d$

Dispatch cost = $A_d D/Q_d$

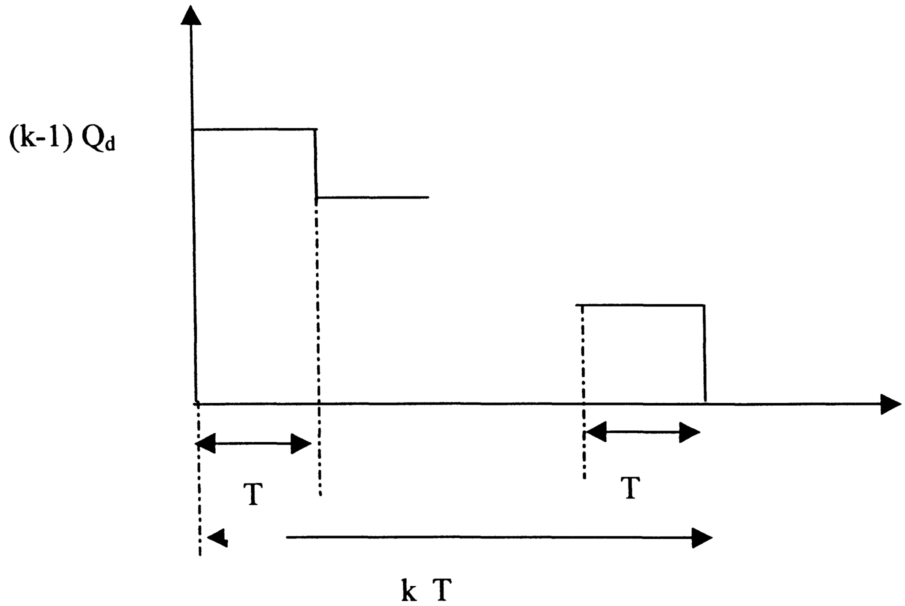

Figure 1: Inventory at supplier's warehouse

As shown in Figure 1, the inventory at the supplier's warehouse obeys a step function. During any production cycle, immediately after production, it would hold $(k-1)Q_d$ inventory for first time period Q_d/D, $(k-2)Q_d$ for next time period Q_d/D, and so on. There would be D/kQ_d cycles in a year. So, the inventory costs at the supplier's warehouse is

$$= \{(k-1)Q_d^2/D) + (k-2)Q_d^2/D) + \ldots + Q_d^2/D\} (D/kQ_d)\, h_p$$
$$= \{k(k-1)/2\}\, Q_d\, h_p / k$$
$$= (k-1)\, Q_d\, h_p / 2.$$

Next, Figure 2 depicts the inventory at the hub. We define T1, T2 and T3 as follows:

T1 = time at which the hub would have under-inventory situation during a replenishment cycle.

T2 = time at which the hub would face a back order situation during a replenishment cycle.

T3 = time at which the hub would have an over-inventory situation during a replenishment cycle.

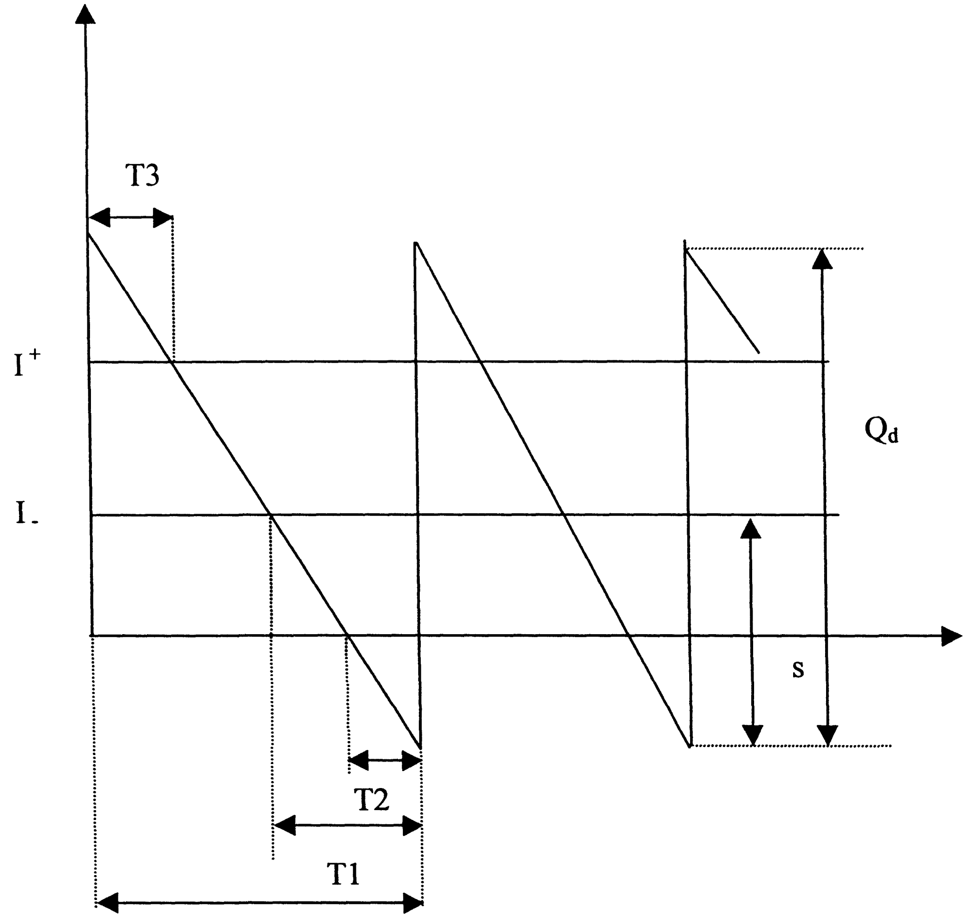

Figure 2: Inventory at Supply Hub

Obviously, the inventory carrying cost at the hub depends on the value of s. If $s \leq I_-$, we would not have any backordered demand and T2 = 0. We first derive an expression under the condition $s \leq I$-. Then, we derive an expression for inventory carrying cost where $s \geq I$-.

Inventory costs = h{Average inventory during relevant time in cycle* relevant time * Number of cycles in a year}

If $s \leq I_-$, Average inventory at hub = $(I_- - s + Q_d/2)$ and inventory cost at hub = $h\,(I_- - s + Q_d/2)$.

If $s \geq I_-$, Inventory during time period T2 would be equal to zero.

Thus, average inventory during time (T-T2) = $\{Q_d - (s - I_-)\}/2$ where T- T2 = $\{Q_d - (s- I_-)\}/D$

As the number of dispatch cycles in year = D/Q_d, the inventory carrying cost at the supply hub

$$= h(I_- - s + Q_d/2) \quad \text{if } s \le I_-,$$
$$h(Q_d - (s - I_-))^2 / 2Q_d \quad \text{if } s \ge I_-.$$

On the penalty cost for under-inventory, we have

Average under-inventory during T2 = s/2 and T2= s/Q_d

Penalty costs for under-inventory = $p_s s^2 / 2Q_d$

Likewise, if the maximum inventory at the hub, just after replenishment, is less than I^+, there would be no over-inventory. So, if $Q_d + I_- - s \le I^+$, penalty cost for over-inventory is zero. If $Q_d + I_- - s > I^+$, we would have over-inventory for time period T3. In this case, the average over-inventory for T3 = $(Q_d + I_- - s - I^+)/2$ and T3 = $(Q_d + I_- - s - I^+)/D$ and number of cycles in year = D/Q_d

So over-inventory penalty cost = $p_e (Q_d + I_- - s - I^+)^2 / 2Q_d$ To simplify notation, we define $\Delta I = I^+ - I_-$. So over-inventory cost = $p_e (Q_d - s - \Delta I)^2 / 2Q_d$

Hence, the penalty cost for over-inventory =

$$0 \quad \text{if } Q_d - s \le \Delta I,$$
$$p_e (Q_d - s - \Delta I)^2 / 2Q_d \quad \text{if } \Delta I \le Q_d - s.$$

Thus, the supplier faces the following optimisation problem (P):

(P) Min. $TC = A_p D/kQ_d + A_d D/Q_d + (k-1)Q_d h_p/2 + \{h(I_- - s + Q_d/2), \text{ if } s \le I_-, h(Q_d - (s - I_-))^2 / 2Q_d, \text{ if } I_- \le s\} + p_s s^2 / 2Q_d + \{0 \text{ if } Q_d - s \le \Delta I, p_e (Q_d - s - \Delta I)^2 / 2Q_d \text{ if } \Delta I \le Q_d - s\}$

s.t. $s \ge 0$, $Q_d \ge s$, k is a positive integer.

The entire problem is quite complicated as we have three decision variables and a discontinuous objective function involving two if-conditions. We now propose a methodology for solving problem P for a fixed k. We will use the term P(k) to denote problem P for a given value of k and call it problem P0.

5. Solution Methodology

In this section we will show the methodology for solving problem P0 for given k and suggest a methodology for finding optimum k.

Since we are only interested in getting insights and designing an algorithm for solving the same, we divide the entire region in to four regions, which collectively cover the entire feasible space (see Figure 3). The whole feasible space is divided into four regions (P1, P2, P3 and P4) and we define four corresponding sub-problems (P1, P2 P3 and P4) to remove the *if* conditions in the objective function.

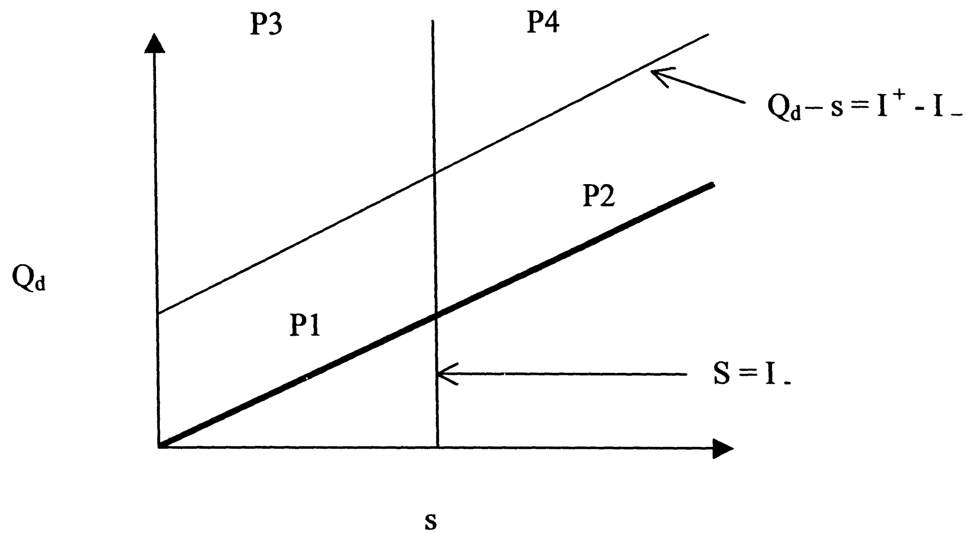

Figure 3: Graphical Representation of Areas

For example, region P1 contains the sub problem P1 to be solved explicitly i.e. Minimise Production costs + Dispatch cost + Carrying costs at the hub + Inventory carrying costs at supplier's warehouse+ Penalty cost for under-inventory. Hence,

Minimize $TC = A_p D/kQ_d + A_d D/Q_d + (k\text{-}1)Q_d h_p/2 + h\,(I_- - s + Q_d/2). + p_s s^2 /2Q_d$
s.t. $s \leq I\text{-}$ and $Qd - s \leq \Delta I$.

In Problem P1, we have no backorders and no over-inventory situation. Similar expressions can be written for Problems P2, P3 and P4. Each problem is a convex function in Q_d and s and we can find an easy way of solving each of the regional problem. We will show certain results, which will help us in getting the optimal solution for P0 once we have solutions for the four sub-problems. Let j be the index denoting the type of problem. It will take value 0 for the main problem and will take values 1 to 4 for the sub-problems.

Proposition 3: $TC(P0(Q_d, s), k) = TC(Pj(Q_d, s), k)$ if (Q_d, s) is feasible in j=1,2,3,4.

For a feasible (Q_d, s) in any region, the value of TC for the given pair of variables for the sub-problem is identical to the value of TC for the same pair of variables in the main problem. This is because of the way the regions are defined.

Proposition 4: $TC^*(P0, k) = \min_j (TC^*(Pj, k)$.

Once the solution is found in a specific region j, automatically all the additional constraints that are imposed are satisfied for problem P0. So, (Q_d, s) which is optimal in P0 would be feasible in Pj. Let Q_d^*, s^* and TC^* represent respective values for the optimal solution for problem P0. TC(Pj $((Q_d, s), k) = TC^*$. We only need to show that any other feasible point in region j, other than (Q_d^*, s^*) cannot have a value which is less than TC^*. Suppose we have a point in region j which has a value of $TC^a < TC^*$. Let Q_d^a, s^a and TC^a reflect respective values for point a where $TC^a < TC^*$, then TC $(P0(Q_d^a, s^a)) < TC^*$ which means (Q_d^*, s^*) is not an optimal solution to P0 which is a contradiction. Hence, we cannot find a point in region j with a lower value of TC than point (Q_d^*, s^*). So the optimal solution to problem P0 is also an optimal solution to problem Pj if the optimal point of problem P0 lies in region j. Using the same logic, we can show that the optimum value of TC for the other three regions would not be smaller than TC^*. Conversely, if we have a region k which has the lowest value of TC among all four regions, then the optimal solution to region k would also be the optimal solution to the problem P0.

We now present a methodology for solving the four regional problems. Once we have found a solution to the four regional problems, the region that has the least value of TC would yield the optimal solution for P0.

We will describe two situations In Problems P1 and P2, we assume that the upper limit on inventory is not a constraint and in Problems P3 and P4, we take a case where the upper limit on inventory is a binding constraint. In situation 1 (Problem P1) $I_- \geq s^*$ that is optimal reorder point is positive or no backorders. In P2, we take a case where $s^* \geq I_-$ that means r_d^* is negative and all the demand orders which cannot be met from inventory at the hub will be backordered. Since inventory-carrying costs have different expressions under both the cases, they are defined as separate problems P1 and P2. For a given value of s, only one of the situations would be valid.

Problem P1

$$\text{Total Costs} = (A_p/k + A_d)\, D/ Q_d + h\,(I_- - s + Q_d/2) + h_p\, Q_d(k-1)/2 + p_s s^2 /2\, Q_d \qquad (1)$$

For a given value of k, the optimal value of (Q_d, s) would be at $\partial TC/ \partial Q_d = 0$ and $\partial TC/ \partial s = 0$.

$$\partial TC/ \partial Q_d = h/2 + + h_p (k-1)/2 - \{ (A_p/k + A_d)\, D + p_s s^2 /2 \}/ Q_d^2 = 0$$

$$\partial TC/ \partial s = p_s s / Q_d = 0$$

yielding

$$s^* = Q_d^*\, h/p_s \text{ and } Q_d^* = \sqrt{2(A_p/k + A_d)\, D\, p_s/ \{(h\, p_s - h^2) + h_p (k-1)\, p_s\}} \qquad (2)$$

It is interesting to see that the expression for s does not contain k explicitly. For k=1, equation (2) reduces to $Q_d^* = \sqrt{2(A_p + A_d)\, D\, p_s / (h\, p_s - h^2)}$ and $s^* = Q_d^*\, h/p_s$.

This requires $p_s > h$, otherwise the above equation would be violated. If p_s=h, it would make sense for the supplier to backlog the entire replenishment at least till I_-. This also apparent from eqn (2) where s^* would take a value greater than Q_d^* which is not possible. s/Qd may be a ratio which organizations might like to fix as a policy variable. It captures a percent of the time during which the supplier would have inventory, which is less than the minimum stipulated amount I-. Normally the hub operator would like to fix this ratio at 0.1 or 0.05 so he must exact a penalty, which is substantially larger than the cost of holding.

The sufficiency conditions for the above optimality are as follows.

$\partial TC^2/\partial Q_d^2 (Q_d^*, s^*) > 0$, $\partial TC^2/\partial s^2 (Q_d^*, s^*) > 0$,

and $\{\partial TC^2/\partial Q_d^2 (Q_d^*, s^*)\}\ \{\partial TC^2/\partial s^2 (Q_d^*, s^*)\} - \{\partial TC^2/\partial Q_d s (Q_d^*, s^*)\}^2 > 0$.

In the above case we have assumed that $I_- \geq s^*$ that is s^* is never more than the minimum inventory. If the expression shown below is less than or equal to I_- and maximum inventory $(Q_d - s) < I^+$ then we use Problem P1. If $s^* = h\sqrt{2(A_p + A_d)D/(h(1 - h/p_s)/p_s} > I_-$, we use Problem P2.

Should Q_d *, s* violate the constraints and lie outside region P1, we find the closest point which is in the region by using whichever is the relevant constraint. Since TC is convex, the above methodology will result into an optimal solution for a constrained problem. For example if $s^* > I_-$, the optimum solution to the constrained problem would be at $s^* = I_-$. This can be easily solved using Lagrangian relaxation. Refer appendix 1 for detailed analysis of problems P2, P3 and P4.

6. Algorithm

So far, we have solved Problem P0 for a value of k. Now we will find k* so that we can solve for Problem P. Only the production cost and carrying cost at the hub are functions of k, so we keep the other terms as they are and use the earlier expressions.

$TC = A_p D/kQ_d + (k-1)Q_d h_p/2$ + (Dispatch cost + Carrying costs at hub + Penalty costs for under- and over-inventory).

TC is convex with respect to k and TC is likely to behave as shown in Figure 4. If we treat k as real, we can find k* which would be either [k] or [k]-1 where [k] is the smallest integer that is larger than k. Since k is integral, we now provide a simple method of finding k*, namely that of enumeration. We start with k = 1 and find TC*(PO, k) such that TC*(P0, k*+1) ≥ TC*(P0, k*).

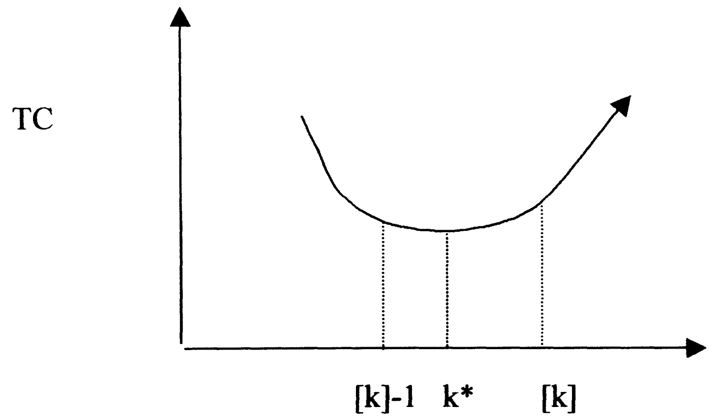

Figure 4: Behaviour of TC w.r.t k

The algorithm is provided as follows.

10 Let k=1, cost*=M (M >> 0)

20 Solve P1(k), P2(k), P3(k) and P4(k)

30 Choose j such that TC (Pj, k) has least cost among all four sub-problems.

40 If TC(j, k) < cost*, then k*=k, where cost* = TC(j, k), Q_d* = Q_d (j, k), s* = s (j, k)

else goto 60

50 k= k+1

60 stop.

Numerical Example

Suppose $A_d = 20$, $A_p = 720$, $h = 10$, $h_d = 5$, $D = 4000$ (demand/week = 80) .

Table 1: Computation of Q_d, s, k and TC for different supply hub parameters

I_-	I^+	P_s	P_e	Q_d	S	TC	k	Optimal sub problem
80	160	10	10	559	372	6523	2	P4
80	160	20	10	367	183	7332	3	P4
80	160	20	20	353	196	7442	3	P4
80	160	40	10	279	93	7909	4	P4
80	160	40	20	265	102	8071	4	P4
80	160	80	10	263	50	8370	5	P3
80	160	80	20	238	55	8732	6	P3
80	240	10	10	596	334	6407	2	P4
80	240	20	10	378	169	7168	3	P4
80	240	20	20	371	174	7192	3	P4
80	240	40	10	351	104	7649	3	P4
80	240	40	20	283	87	7724	4	P4
80	240	80	10	326	55	8076	4	P3
80	240	80	20	297	57	8352	5	P3
160	240	10	10	604	382	6941	2	P4
160	240	20	10	378	209	7968	3	P4
160	240	20	20	366	220	8048	3	P4
160	240	40	10	335	118	8823	4	P4
160	240	40	20	294	121	9140	5	P4
160	240	80	10	263	50	9170	5	P4
160	240	80	20	238	55	9132	6	P4
160	320	10	10	423	292	7150	3	P2
160	320	20	10	518	259	7776	2	P4
160	320	20	20	499	268	7847	2	P4
160	320	40	10	346	107	8615	4	P4
160	320	40	20	314	104	8891	5	P4
160	320	80	10	326	54	8876	4	P4
160	320	80	20	297	57	9152	5	P4

We can see what happens to the optimal cost and other values with respect to changes in I_-, I^+, p_s and p_e. These are the parameters in the control of the hub. In the above example minimum inventory (I_-) has been kept as one or two weeks of demand and maximum inventory (I^+) has been varied from two to four weeks of demand. Similarly penalties for under stocking (P_s/h varies from one to eight) and over stocking (P_e/h varies from one to two) have been varied to see its impact on over all performance. Sensitivity analysis of the kind carried out in this section would help supplier and supply hub to set up supply hub parameters in a way which would mange concerns of both parties. A_d and A_p are essentially in the hands of the supplier. Similar results can be derived for other base values for the supplier parameters.

7. Concluding remarks

This paper has attempted to provide a model of hub management under deterministic conditions for demand. The insights gained have implications for hub operators and suppliers. We now reiterate those insights. First, the hub operator can ensure the smooth operations of the hub by manipulating the penalty costs for over and under stocking, the inventory levels to be set for minimum and maximum inventory. Second, the supplier can manage the flow of inventory by ensuring the right mix of the production setup cost and the dispatching set up cost. Through this, it is hoped that hub operators can now use the results to better manage the cost allocation mechanism for joint inventory management in the supply chain. One way of doing so is to use appropriate cost ratios to drive hub performance, for example, s/Q_d. Future research will focus on the case when demand is stochastic.

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Appendix 1

Problem P2

$$\text{Total Cost } = (A_p/k + A_d)D/Q_d + h(Q_d - (s- L))^2/2Q_d + h_pQ_d(k - 1)/2 + p_s s^2/2Q_d \qquad (3)$$

For a given value of k, the optimal value of (Q_d, s) would be at $\partial TC/\partial Q_d = 0$ and $\partial TC/\partial s = 0$.

This yields

$s^* = h(Q_d + I_-)/(h + p_s)$

and

$$Q_d = \sqrt{\{2(A_p/k + A_d)D(p_s + h) + hp_sI_-^2\}/\{hp_s + h_p(k - 1)(p_s + h)\}} \qquad (4)$$

For k=1, this would simplify to $Q_d = \sqrt{\{2(A_p + A_d)D(p_s + h) + hp_sI_-^2\}/hp_s}$.

We note that this is very similar to the expression for a standard backorder case with Q_d replaced by $Q_d + I_-$. With $I_- = 0$ and $k = 1$, Q_d is EOQ with $Q_d = \sqrt{\{2(A_p + A_d)D(p_s + h)\}/hp_s}$.

Again, if Q_d^*, s^* violate the constraints (solution lies outside region P2), we can find the closest point which is in the region using whichever is the relevant constraint. Since TC is convex, optimum solution exists for a constrained problem. For example if $s^* < I_-$ optimum solution to the constrained problem at $s^* = I_-$.

Problem P3

Total cost

$$= (A_p/k + A_d)D/Q_d + h(I_- - s + Q_d/2) + h_pQ_d(k - 1)/2 + p_ss^2/2Q_d + p_e(Q_d - s - \Delta I)^2/2Q_d \qquad (5)$$

For a given value of k, the optimal value of (Q_d, s) would be at $\partial TC/\partial Q_d = 0$ and $\partial TC/\partial s = 0$.

This yields

$s^* = \{Q_d(h + p_e) - p_e\Delta I\}/(p_s + p_e)$

and

$$Q_d^* = \sqrt{\{2(A_p/k + A_d)D(p_s + p_e) + p_sp_e\Delta I^2\}/\{(h(p_s - h - p_e) + p_sp_e + h_p(k - 1)(p_s + p_e)\}} \qquad (6)$$

Of course, if there is no penalty for over-inventory i.e. $p_e = 0$, the expression for s^* and Q_d^* reduces to (2) which is equivalent to problem P1.

In this case, the supplier faces an interesting trade-off between incurring penalty for excess inventory and penalty for under-inventory. It is interesting to note that for period T3 the supplier incurs a penalty cost at the rate of p_e and during T2 he incurs a penalty cost at $p_s - h$.

Again, when Q_d^*, s^* violate the constraints (solution lies outside region P3) we employ the same approach as described in problem P2.

Problem P4

Total cost

$$= (A_p/k + A_d)D/Q_d + h(Q_d - (s - I_-))^2/2Q_d + h_pQ_d(k - 1)/2 + p_s s^2/2Q_d + p_e(Q_d - s - \Delta I)^2/2Q_d \quad (7)$$

For a given value of k, the optimal value of (Q_d, s) would be at $\partial TC/\partial Q_d = 0$ and $\partial TC/\partial s = 0$.

This yields

$$s^* = \{Q_d(h + p_e) - (p_e\Delta I - hI_-)\}/(h + p_s + p_e) \quad (8)$$

The expression $h(s - I_-)^2/2 + p_s s^2/2$ in (7) which involves s can be simplified after substituting the value of s using (8) to give $\{h^2Q_d^2 + hp_sI_-^2\}/2(p_s + h)$.

After some algebraic manipulations, we get

$$Q_d^* = \sqrt{\{2(A_p/k + A_d)D(h + p_s + p_e) + hI_-^2(p_e + p_s) + \Delta I^2(h + p_s) + 2hp_e\Delta II_-\}/\{(p_s(h + p_e) + h_p(k - 1)(h + p_s + p_e)\}} \quad (9)$$

With no penalty for over-inventory i.e. $p_e = 0$, the expression for s^* and Q_d^* reduces to (4) which is equivalent to problem P2.

Again, when Q_d^*, s^* violate the constraints (solution lies outside region P4) we employ the same approach as described in problem P2.