

**WORKING PAPER NO: 681**

**Wage Cyclicalities Across Time and Frequencies**

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Year of Publication – August 2023

# Wage Cyclicalilty Across Time and Frequencies

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July 2023

## Abstract

This paper documents how the cyclicalilty of real wages has evolved over time and across different frequencies in the US. We use individual level data from the CPS to construct composition bias corrected wage series at a quarterly frequency. Utilizing continuous wavelet tools, we find that the cyclicalilty of wages for all the workers as well as new hires has increased over time, and this increase is prevalent across all the frequencies. Further, the increase in cyclicalilty is primarily concentrated among the workers with less than college education. This decline in wage rigidity over the business cycle relates to broader structural changes in the labour market and has implications for labour search framework.

JEL codes: C49, E24, E32

Keywords: Wavelets, Wage cyclicalilty, Wage rigidity, Time-frequency analysis

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# 1. Introduction

Studying the behaviour of real wages over the business cycle is integral to understand the macroeconomic effects of business cycle fluctuations. Macroeconomists have long recognized this fact, and this has led to a large literature estimating the cyclicalities of real wages under various circumstances. [Dunlop \(1938\)](#) and [Tarshis \(1939\)](#) are some of the earliest studies that used aggregate data on wages and showed that the real wages are procyclical. Following this, a number of studies used aggregate data to estimate the behaviour of wages over business cycle and found mixed results. While [Bodkin \(1969\)](#) confirms the findings of the previous studies by showing that the wages are predominantly procyclical, [Chirinko \(1980\)](#) and [Neftci \(1978\)](#) find real wages to be countercyclical. Additionally, [Geary and Kennan \(1982\)](#) argue that there is no consistent relationship between wages and cyclical indicators, and [Sumner and Silver \(1989\)](#) find both procyclical and countercyclical patterns depending on the sample period.

Following the arguments of [Stockman \(1983\)](#), [Bils \(1985\)](#) and [Solon et al. \(1994\)](#) used individual level panel data on wages to find that real wages are substantially procyclical. They argued that, measuring wage cyclicity using aggregate wage data is misleading, as the cyclicity estimates suffer from composition bias. Aggregate data, by its way of construction, gives more weight to low-skilled workers during expansions than during recessions, thus introducing a countercyclical bias in the wage cyclicity estimates. This explains why some of the cyclicity estimates obtained from aggregate data were countercyclical and were also sensitive to the sample period. Following this, a large number of studies like [Shin \(1994\)](#), [Devereux \(2001\)](#), [Devereux and Hart \(2006\)](#), [Hart \(2006\)](#), [Carneiro et al. \(2012\)](#), and [Haefke et al. \(2013\)](#) use longitudinal data for keeping the composition of workers fixed over the business cycle, found that the real wages are procyclical and this finding is robust under different settings.

There has been a renewed interest in understanding the elasticity of wages over business cycle as the literature of search and matching models has resorted to wage rigidity as one of the ways to generate empirically consistent unemployment fluctuations. Studies like [Shimer \(2005\)](#) and [Costain and Reiter \(2008\)](#) showed that a standard labour search model generates too low a volatility in both unemployment and

vacancies compared to the data. Labeled as the unemployment volatility puzzle, [Hall \(2005\)](#) showed that introducing rigid wages in search models can help in increasing the volatilities thus making them empirically consistent. In addition, [Shimer \(2004\)](#) and [Pissarides \(2009\)](#) argued that the model's behaviour depends only on the wage rigidity of newly hired workers and not on the continuing workers. Following this, a number of studies like [Menzio \(2005\)](#), [Farmer and Hollenhorst \(2006\)](#), [Blanchard and Galí \(2007\)](#), [Hall and Milgrom \(2008\)](#), [Gertler and Trigari \(2009\)](#), [Shimer \(2010\)](#), [Michaillat \(2012\)](#), and [Christiano et al. \(2016\)](#) introduced some form of wage rigidity to make their models more consistent with the data.

We contribute to this literature on wage cyclicalities by studying how the cyclicalities of wages of all workers as well as new hires has evolved over time and across different frequencies in an integrated framework. Most of the existing studies use time domain techniques to measure wage cyclicalities and hence ignore any information regarding the frequency domain. Moreover, these analyses just measure the average cyclicalities of wages and they neither capture the evolution of cyclicalities over time nor distinguish the relationship between short-run, medium-run, or long-run horizons. In order to analyse the relationships over different time horizons, researchers have employed frequency domain methods like the Fourier transforms. Even though frequency domain methods are more commonly used for analysing aggregate business cycle behaviour, only a very few papers such as [Messina et al. \(2009\)](#), [Hart et al. \(2009\)](#), and [Marczak and Beissinger \(2013\)](#) employ spectral techniques to study real wage cyclicalities. But these frequency based methods completely ignore the temporal variation and hence cannot capture the changes in wage cyclicalities over time. Since our goal is to estimate the evolution of wage cyclicalities over time and across different frequencies, we make use of wavelet analysis to estimate the elasticity of wages over the business cycle.

Wavelet approach uses wavelet functions that are localized in both time and frequency domains as the basis function in contrast to sine or cosine functions used in spectral analysis. This enables wavelet techniques to capture the variations in the relationship both across time and across frequencies. In this paper, we employ continuous wavelet tools, namely, wavelet coherency, wavelet phase-difference, and wavelet gain

to analyse how the cyclicalities of wages has changed over time and at different business cycle frequencies. [Aguiar-Conraria and Soares \(2014\)](#) provides an accessible introduction to continuous wavelet tools and also surveys the literature that uses these techniques for answering a variety of questions. Wavelet coherency, the time-frequency analog of correlation, measures the magnitude of co-movement of wages with the business cycle at every time period and frequency, while the phase-difference gives us the direction of this relationship over time and frequency along with lead/lag of wages over the cyclical indicator. The aforementioned literature measures wage cyclicalities as the elasticity of wages with respect to a business cycle indicator, and so, all the papers use a regression setup to estimate cyclicalities. Hence, in order to estimate wage elasticity over time and frequency, we need a regression setup with both time-varying and frequency-varying regression coefficients. Wavelet gain employed in papers like [Mandler and Scharnagl \(2014\)](#), [Aguiar-Conraria et al. \(2018\)](#), and [Aguiar-Conraria et al. \(2020\)](#) provides us with such a framework, thus enabling us to estimate the changes in wage cyclicalities across time and frequencies. As mentioned before, even though there are a few papers applying spectral methods to study wage cyclicalities, [Marczak and Gómez \(2015\)](#) is the only paper that employs wavelet tools in this context. But they don't employ wavelet gain in order to estimate cyclicalities, which is the primary measure of cyclicalities in our study.

As pointed out by the earlier literature, it would be misleading to estimate wage cyclicalities using aggregate data. Hence, following [Haefke et al. \(2013\)](#), we use individual level data from Current Population Survey, Outgoing Rotation Group (CPS-ORG) over the period 1979 – 2019 to construct the quarterly wage series that is free of composition bias. Using individual level data from CPS enables us to control for the demographic characteristics of the workers, and hence takes care of the bias in cyclicalities estimates due to changes in composition of workers over the business cycle. Additionally, use of microdata from CPS allows us to identify and measure the wage cyclicalities of new hires separately, which is not a possibility when we use aggregate data. In sum, we employ wavelet tools on the wage series constructed from the CPS data to study the dynamics of wage cyclicalities of all workers as well as new hires across time and

frequencies.

Our paper makes three important contributions to the broad literature on wage cyclicality and also with respect to those using spectral and wavelet tools to measure cyclicality. One, all the studies using spectral and wavelet techniques including [Marczak and Gómez \(2015\)](#) use aggregate wage data to measure the co-movement of wages over the business cycle. As discussed before, this can potentially bias the estimates of wage cyclicality. We, on the other hand, construct aggregate wage series from individual level wage data from CPS after explicitly controlling for the demographic characteristics, thus accounting for the changes in worker composition over the business cycle. Two, in addition to all workers, we also explicitly consider the wage dynamics and cyclicality of new hires and this has important implications for developing and calibrating labour search and matching models. Three, we make a methodological contribution to this literature by introducing wavelet gain as a framework to estimate wage cyclicality across time and frequencies. This makes our cyclicality estimates consistent and comparable with the estimates of prior literature obtained using regression framework.

Our time-frequency analysis of wage cyclicality using wavelets delivers five key results. First, we find that the wages of all workers as well as new hires are predominantly procyclical over all the years and across all frequencies. This is consistent with other studies which use individual level data to estimate wage cyclicality. Second, we find wages of new hires are more responsive to business cycle conditions compared to continuing workers across the entire time-frequency domain. This is again in line with the prior studies like [Bils \(1985\)](#), [Solon et al. \(1994\)](#), and [Haefke et al. \(2013\)](#) who document similar patterns using time domain techniques. Third, the measured elasticity of wages is larger over the long-run horizon compared to the short-run frequencies. This could reflect the frictions in wage adjustment process that prevents instantaneous response of wages to changes in aggregate conditions. Fourth, the cyclicality of wages, both existing and new hires, has increased over time and this increase is prevalent across all the frequency regions. This finding relates to broader structural changes observed in the labour market including declining unionization and bargaining power of workers as documented in [Stansbury and Summers \(2020\)](#), and other studies such as [Farber](#)

et al. (2021) and Fortin et al. (2021). This finding also has implications for the calibration of labour search and matching models. Finally, this increase in cyclicalities is predominantly driven by the increase in wage cyclicalities of low-skilled workers while the cyclicalities of high-skilled wages do not show a consistent upward trend.

The rest of the paper is organized as follows. Section 2 describes the data and presents the regression estimates of wage cyclicalities. Section 3 explains the wavelet tools used in our analysis, while section 4 discusses the time-frequency results of wage cyclicalities. Section 5 talks about the implications of our findings, and section 6 concludes.

## 2. Data

It is well known from the studies like [Bils \(1985\)](#) and [Solon et al. \(1994\)](#) that estimates of wage cyclicalities obtained from aggregate data suffers from composition bias. During a recession, more low-skilled workers earning lower wages end up losing their jobs compared to high-skilled workers. Similarly, the opposite happens during an expansion. Thus, the composition of the workforce varies over the business cycle. This implies that, during a recession, the aggregate wages are constructed over the workforce with more high-skilled workers compared to an expansion, thus introducing a countercyclical bias in the estimates. In order to overcome this, we need to use individual level worker data to keep the composition fixed over the business cycle.

### 2.1 Individual Level Wage Data

Majority of studies in this literature use household level panel data to estimate the wage cyclicalities, thus ensuring the composition of workers is kept fixed over time. A number of studies like [Bils \(1985\)](#) and [Shin \(1994\)](#) use individual wage data from the National Longitudinal Survey of Youth (NLSY) while other studies like [Solon et al. \(1994\)](#) and [Devereux \(2001\)](#) use Panel Study of Income Dynamics (PSID) to estimate wage cyclicalities. Even though both NLSY and PSID keep the composition of workers fixed over the business cycle, they provide information on individual wages only at a yearly fre-

quency. Since we use wavelets in our analysis to study the changes in wage cyclicity, and wavelets are quite data demanding, it would be better if we have data at a higher frequency.

Hence, following [Haefke et al. \(2013\)](#), we use Current Population Survey, Outgoing Rotation Group (CPS-ORG) to obtain the individual level wage data. The major advantage of using CPS is, we will be able to construct wage series at a quarterly frequency, thus more suitable for the wavelet analysis. CPS-ORG is a monthly survey of US households that has been administered since 1979. [Haefke et al. \(2013\)](#) uses the CPS micro-data to construct quarterly wage series for the period of 1979-2006. We closely follow their methodology in constructing the quarterly wage data, and we extend the original data till 2019.<sup>1</sup>

The sample consists of both male and female non-supervisory workers in the private non-farm business sector, who are aged between 25 and 60. We measure wages as hourly earnings, obtained by dividing earnings by the usual hours worked. Further, the wages are deflated using the implicit deflator for private non-farm business sector. The panel structure of the CPS data is exploited to identify if the workers were newly hired. We match the workers with the preceding three monthly data files to identify new hires as those who were not working for at least one of the preceding three months.<sup>2,3</sup> In addition, the data also contains information on the demographic characteristics of the workers, along with the industry and occupation of their work. Finally, the data on aggregate labour productivity, measured as the output per hour in the non-farm business sector, is obtained from the BLS productivity and cost program.

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<sup>1</sup>We thank Thijs van Rens for providing us with some of the missing data files needed for generating our estimates.

<sup>2</sup>There is a discontinuity in matching in the third and fourth quarters of 1985 and 1995 due to changes in sample design. Therefore, we have missing values for those quarters.

<sup>3</sup>There is a possibility of misreporting of employment status in our data. When an employed worker reports being unemployed at any point in the survey, that worker would be identified as a new hire, resulting in our estimates of wage cyclicity of new hires being biased towards zero. Since we find, in line with the literature, that the wages of new hires are more procyclical, this bias will only work against our result.



## 2.2 Constructing the Wage Series

A number of papers in the literature measuring wage cyclicality including [Bils \(1985\)](#), [Solon et al. \(1994\)](#), and [Devereux \(2001\)](#) show that, controlling for the composition of workers is critical to get an unbiased estimate of cyclicality. Hence, we need to control for the observed and unobserved characteristics of individual workers over the business cycle. Let  $w_{it}$  be the wage earned by worker  $i$  at time  $t$ . Then, following [Haefke et al. \(2013\)](#), the individual wages can be modeled as

$$\log w_{it} = x_i' \beta + \log \hat{w}_{it}, \quad (1)$$

where  $x_i$  is the vector of individual characteristics that do not vary or change deterministically over time – education, gender, marital status, race, and a fourth order polynomial in experience, while  $\hat{w}_{it}$  captures the residual wage controlling for these factors. Even though these variables capture observed individual heterogeneity, they don't account for the individual fixed effects. Hence, majority of the papers in this literature take first difference of the wages to drop the fixed effects. However, doing this in our case will drop the wages of all the new hires from our analysis. Hence, just like [Haefke et al. \(2013\)](#), we work with wage levels controlling only for the observable factors, and not explicitly controlling for individual specific fixed effects. [Haefke et al. \(2013\)](#) shows that, controlling just for the observable factors works quite well in accounting for both observed and unobserved heterogeneity, taking care of the composition bias in the estimates of wage cyclicality. Thus, the residual  $\hat{w}_{it}$  denotes the worker wages corrected for the composition bias.

The residual wages are averaged over quarters for each subgroup, i.e., all workers and new hires. The wage index for subgroup  $j$ ,  $\hat{w}_{jt}$  is defined as follows:

$$\log \hat{w}_{jt} = \log w_{jt} - (x_{jt} - \bar{x}_j)' \beta, \quad (2)$$

where  $w_{jt}$  and  $x_{jt}$  refer to the average of the wages and the observables respectively for that subgroup of workers in quarter  $t$ , and  $\bar{x}_j$  is the overall subgroup average of the characteristics.

## 2.3 Wage Cyclicalilty

Wage cyclicalilty captures the response of wage to change in aggregate labour productivity, and is measured as the coefficient of regression of log real wage index on log real labour productivity. Wages that are flexible will have the regression coefficient close to one, while rigid wages will have the coefficient close to zero. We estimate this regression in first differences in order to avoid spurious correlation.

$$\Delta \log \hat{w}_{jt} = \alpha_j + \eta_j \Delta \log y_t + \epsilon_{jt}, \quad (3)$$

where  $\hat{w}_{jt}$  is the composition bias corrected real wage index and  $y_t$  the real labour productivity. In addition, we also include quarter dummies to control for the seasonality while estimating this regression. The estimates of wage cyclicalilty obtained from this regression are shown in table 1.

Table 1: Wage Cyclicalilty

	All Workers		New Hires	
	1979-2019	1984-2019	1979-2019	1984-2019
Wage cyclicalilty	0.14	0.21	0.52	0.79
Standard error	0.12	0.15	0.40	0.47
Quarters	157	138	157	138

*Note:* Wage cyclicalilty is measured as the coefficient of regression of log real wage index on log real labour productivity as shown in equation (3). In addition, the regression also includes quarter dummies to control for seasonality.

Our results are consistent with the broad literature on wage cyclicalilty and with [Haefke et al. \(2013\)](#) in particular. To be specific, we are able to generate the three major findings of [Haefke et al. \(2013\)](#). First, wages are procyclical with respect to aggregate labour productivity. Second, the wages of new hires respond much more to changes in productivity compared to the wages of all workers. Finally, comparing the estimates between 1979-2019 and 1984-2019, wages of both new hires and all workers are less elastic prior to 1984. This result provides an indication that wage rigidity might have

reduced since Great Moderation starting 1984. Even though this regression provides preliminary evidence, we next employ wavelet tools in order to carefully examine the evolution of wage rigidity over the entire time period.

### 3. Wavelets

Continuous wavelet transform is a powerful tool that uses frequency domain analyses to dig into changes in data both over time and across frequencies. Thus, wavelets combines the power of time-varying regressions and spectral analysis in a single integrated framework. This makes wavelets an ideal tool for studying the changes in wage rigidity both over time and across frequencies. [Aguiar-Conraria and Soares \(2014\)](#) provides a detailed introduction and survey of the continuous wavelet transform and the various wavelet tools that can be used for analysing the data. We start by using wavelet coherency and wavelet phase-difference to study the relationship between wages and productivity. A number of papers like [Aguiar-Conraria et al. \(2012a\)](#), [Aguiar-Conraria et al. \(2012b\)](#), and [Fankem and Mbesa \(2023\)](#) have also used these techniques under different contexts. In addition, we make use of wavelet gain, which is an analog of regression in a time frequency domain, to estimate the elasticity of wages across time and frequencies. [Aguiar-Conraria et al. \(2018\)](#) and [Fратиanni et al. \(2022\)](#) employ wavelet gain to estimate time-varying coefficients of the Taylor rule, while [Aguiar-Conraria et al. \(2020\)](#) uses this tool to discuss the changes in Okun's law over time. We now provide a brief introduction of the various wavelet tools we use in our analysis.

#### 3.1 Continuous Wavelet Transform

Fourier transform is a well known method to perform frequency domain analysis, where a time series is written as a combination of sine and cosine base functions. These sinusoidal base functions do not change over time and hence Fourier transforms cannot capture the changes in the spectral characteristics of a signal over time. In order to estimate the changes in a time series across both time and frequencies, we need a base function that changes over time. A wavelet  $\psi(t)$ , as the name signifies, represents a

small wave, oscillating around the time axis and loses its strength as it moves away from the centre. A wavelet transform decomposes the data in terms of time localized wavelets and hence enables us to capture the evolution of data in both time and frequency domains. The continuous wavelet transform of a series  $x(t)$  with respect to a given wavelet  $\psi(t)$  is given by the convolution

$$W_x(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{t - \tau}{s} \right) dt \quad (4)$$

where  $*$  denotes the complex conjugate.  $\tau$  is the translation parameter controlling the time location while  $s$  is the dilation parameter capturing the width of the wavelet  $\psi \left( \frac{t - \tau}{s} \right)$ . For  $|s| > 1$ , the function becomes broader thus leading to a lower frequency, and for  $|s| < 1$ , it becomes narrower corresponding to a higher frequency.

In order to study the dynamics of real wages over the business cycle, we need to have a complex-valued wavelet, as the corresponding wavelet transform will contain information on both amplitude and phase, where amplitude measures the magnitude while phase captures the direction of the cyclical relationship. Even though there exists a number of different wavelet functions such as Daubechies, Haar, Mexican Hat, etc., Morlet wavelet has been the most popular complex-valued wavelet, and almost all studies in Economics applying continuous wavelet transform such as [Aguiar-Conraria et al. \(2018\)](#), [Aguiar-Conraria et al. \(2020\)](#), and [Fankem and Mbesa \(2023\)](#) use this wavelet function. The popularity of the Morlet wavelet is due to the following properties as summarized by [Aguiar-Conraria et al. \(2012b\)](#) and [Aguiar-Conraria and Soares \(2014\)](#). First, for a given wavelet, the relationship between the wavelet scale and frequency need not be straightforward. So, the literature proposes three meaningful ways to convert scales to frequencies – peak frequency, energy frequency, and central instantaneous frequency. In the case of Morlet wavelet, all these measures are equivalent, thus enabling a consistent conversion of scales to frequencies. Second, Morlet wavelet attains the lowest uncertainty in the time-frequency plane, thus achieving the best possible joint time-frequency concentration. Third, for a given time-frequency concentration, this wavelet achieves the optimal balance between time and frequency localisation. Fourth, it's an analytic wavelet under convenient parametrization, and

analytic wavelets are useful for analysing cyclical variables as the wavelet transform can resolve the instantaneous amplitude and phase for each value of time and scale  $(\tau, s)$ . Formally, the Morlet wavelet is given by

$$\psi(t) = \pi^{-\frac{1}{4}} e^{\omega_0 i t} e^{-\frac{t^2}{2}}. \quad (5)$$

We use Morlet wavelet with  $\omega_0 = 6$  for our analysis. With this value of  $\omega_0$ , there exists an almost exact inverse relationship between wavelet scales and frequencies, thus enabling us to move between scales and frequencies seamlessly.

### 3.2 Wavelet Coherency and Phase Difference

Wavelet power spectrum measures the variance distribution of the time series  $x$  across time and frequencies, and is given by

$$(WPS)_x = W_x W_x^* = |W_x|^2. \quad (6)$$

In a time series analysis, we use covariance and correlation to study relationships between two variables. Similarly, in the context of wavelets, we define cross-wavelet power and coherency to study these relationships both across time and frequency. The cross-wavelet power between two series  $y(t)$  and  $x(t)$  is defined as the absolute value of the cross-wavelet transform, given by  $W_{yx} = W_y W_x^*$ . Similar to correlation, complex wavelet coherency between  $y$  and  $x$  is obtained by normalizing the cross-wavelet power with the square root of the wavelet powers of  $x$  and  $y$ , and is given by

$$\varrho_{yx} = \frac{S(W_{yx})}{[S(|W_y|^2)S(|W_x|^2)]^{1/2}}, \quad (7)$$

where  $S$  is a smoothing function across time and frequency. Denoting the smoothed cross-wavelet transform as  $S_{yx}$  and the square root of the smoothed wavelet power of  $x$  as  $\sigma_x = \sqrt{S(|W_x|^2)} = \sqrt{S_{xx}}$ , the complex wavelet coherency can be written as

$$\varrho_{yx} = \frac{S_{yx}}{\sigma_y \sigma_x} \quad (8)$$

Complex valued coherency can be represented in a polar form as  $\varrho_{yx} = |\varrho_{yx}|e^{i\phi_{yx}}$ . The absolute value of the complex wavelet coherency, denoted by  $R_{yx} = |\varrho_{yx}|$ , captures the magnitude of the relationship and is referred to as the wavelet coherency. Similarly, the angle of the complex coherency,  $\phi_{yx}$ , represents the phase-difference between the two series. The wavelet phase-difference  $\phi_{yx}$  provides information on the direction of the relationship across time and frequency, and also the relative leads and lags of the two series.

If the computed phase-difference is zero, then both the series exactly coincide with each other at the given frequency. If  $\phi_{yx} \in (0, \frac{\pi}{2})$ , then both the series move in the same direction (in phase), but  $y$  leads  $x$ . If  $\phi_{yx} \in (-\frac{\pi}{2}, 0)$ , then  $x$  leads  $y$ . Similarly, a phase-difference of  $\pi$  or  $-\pi$  indicates an anti-phase relationship, with  $x$  leading if  $\phi_{yx} \in (\frac{\pi}{2}, \pi)$  and  $y$  leading if  $\phi_{yx} \in (-\pi, -\frac{\pi}{2})$ .

### 3.3 Wavelet Gain

In order to answer how the elasticity of wage has changed over time and frequency, we need a regression setup with its coefficients depending both on time and frequency. Wavelet gain, as given by [Mandler and Scharnagl \(2014\)](#) and [Aguilar-Conraria et al. \(2018\)](#), provides an analog of the regression framework across time and frequencies. With this tool, we will be able to estimate the wage cyclicalities that are both time-varying and frequency-varying. The complex wavelet gain of  $y$  on  $x$ , denoted by  $\mathcal{G}_{yx}$ , is given by  $\mathcal{G}_{yx} = \frac{S_{yx}}{S_{xx}} = \varrho_{yx} \frac{\sigma_y}{\sigma_x}$ . Wavelet gain,  $G_{yx}$  is defined as the modulus of the complex wavelet gain

$$G_{yx} = \frac{|S_{yx}|}{S_{xx}} = R_{yx} \frac{\sigma_y}{\sigma_x}. \quad (9)$$

The wavelet gain can be interpreted as the absolute value of the regression coefficient of  $y$  on  $x$  at a given moment in time and a specific frequency. Thus, wavelet gain gives only the magnitude of the regression coefficient, while the sign of the coefficient can be obtained from the phase-difference  $\phi_{yx}$ .

## 4. Results

We now use the wavelet tools discussed so far – wavelet coherency, phase-difference, and wavelet gain, to analyse the relationship between wages and productivity, and how the elasticity of wages behaves across time and at different frequencies.<sup>4</sup> Wavelet coherency gives the magnitude of correlation between real wages and productivity over time and at different frequencies, while the phase-difference gives the direction of the relationship. Wavelet gain captures the absolute value of the coefficient of regression between wages and productivity, and hence measures the cyclicity of wages at different time and frequencies. We analyse the wage cyclicity of all the workers and new hires separately as the wages of new hires behave differently over the business cycles.

### 4.1 All Workers

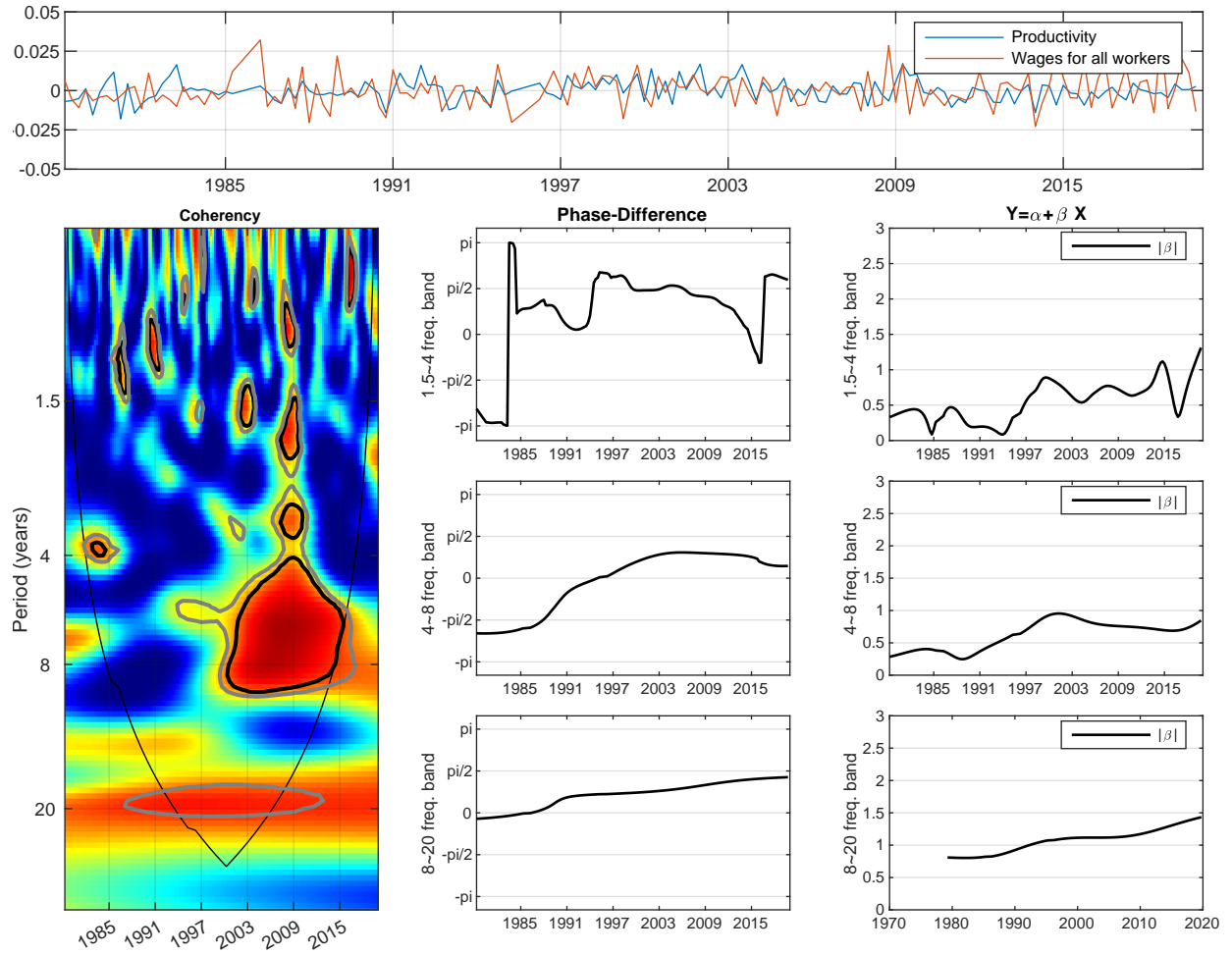
The results from the wavelet analysis for all the workers are summarized in figure 1. Following Aguiar-Conraria et al. (2018), we analyse the relationship over a wide range of frequencies. We present results for coherency, phase-difference, and wavelet gain for three frequency intervals, 1.5 ~ 4 years (short-run), 4 ~ 8 years (medium-run), and 8 ~ 20 years (long-run).

The wavelet coherency shows that the regions of high coherency are sparsely distributed at the shorter end of the business cycle. Among the long-run cycles, in particular the upper end of the spectrum, we find that the coherency is consistently high throughout the entire sample period. Interestingly, at the business cycle frequencies of 4 ~ 8 years, we do not find much correlation in the earlier part of our sample. But post 2000, the coherency has become very strong as seen by the emergence of dark red regions in the coherency plot that are statistically significant. This shows that the relationship between real wages and productivity has strengthened over time.

Even though the coherency gives us a measure of the magnitude of the correlation, we need to pair it up with the phase-difference to understand the direction of the cor-

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<sup>4</sup>The wavelet results are generated using ASToolbox v.2018 available at <https://sites.google.com/site/aguiarconraria/wavelets-and-economics/the-astoolbox>. Aguiar-Conraria and Soares (2014) provides a detailed description of this toolbox.



**Figure 1: All workers:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.

relation and also the lead/lag structure between the variables. The phase difference at the short-run frequencies varies considerably over time. Despite that, wages and productivity are positively related for most of the sample, except for occasional negative relationship at the start and end of the sample. At longer frequencies, the phase dif-



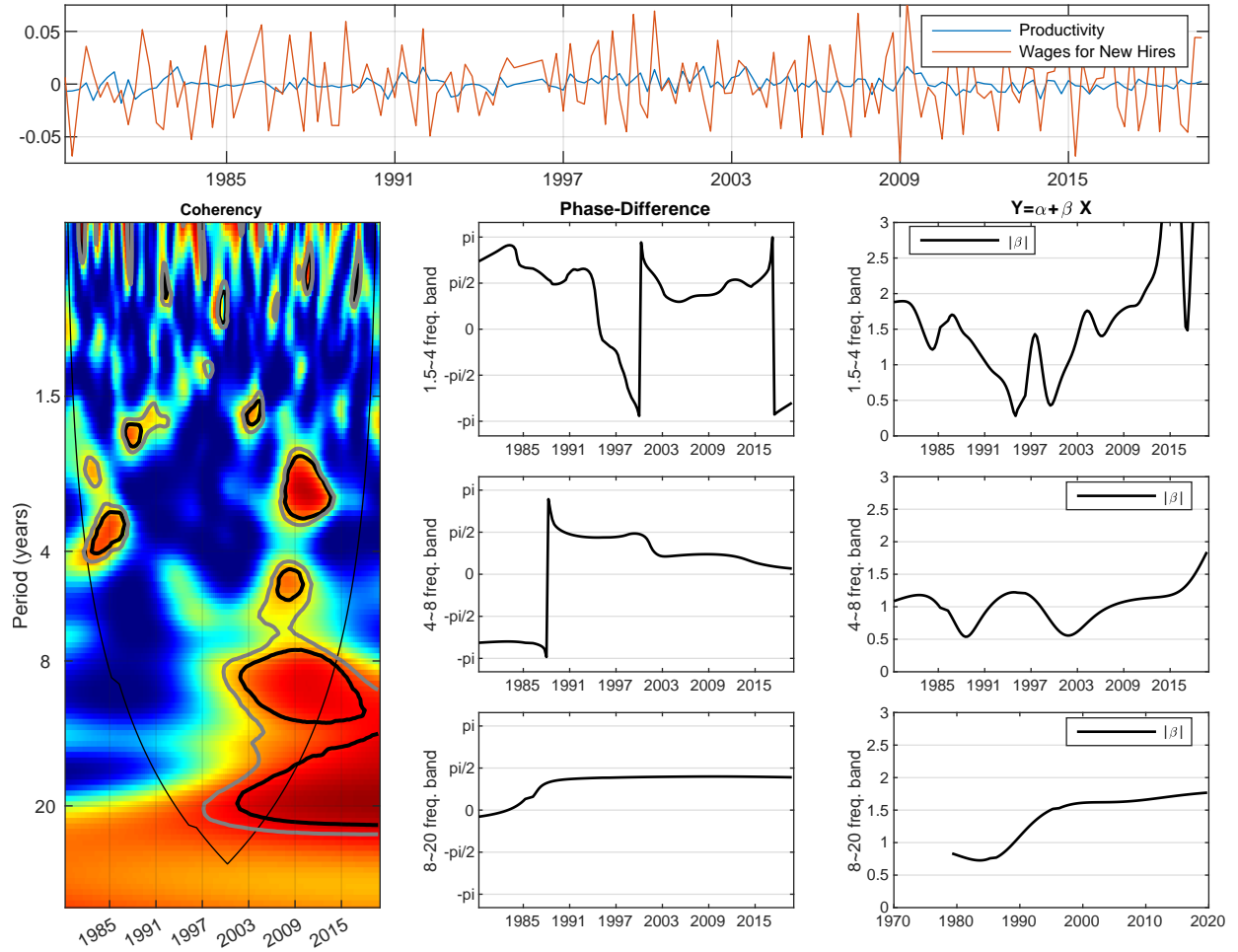
ferences are considerably more stable, with wages moving positively with productivity most of the time at both business cycle frequency and in the long run. Additionally, we find that, predominantly wages lead productivity in our sample, except during the early part of the sample where productivity leads wages. This is consistent with the findings of [Marczak and Gómez \(2015\)](#), who using aggregate data also finds that the real wages lead the business cycle in the US.

We next turn towards the main focus of our paper, wage cyclicalities, as measured by the wavelet gain. We find that there is considerable variation in our estimates at high frequencies, while the trend becomes smoother at lower frequencies. Comparing the average cyclicalities across frequency bands, we find that the magnitude of cyclicalities is higher at longer frequencies compared to shorter ones. This indicates that wages take time to adjust, and the procyclical relationship gets stronger when we look at longer time horizons. Importantly, within each frequency interval, we find that the wage elasticity shows an increasing trend over time. Thus, wages have become more flexible over time with respect to changes in productivity, and this pattern holds across all the frequencies.

## 4.2 New Hires

Figure 2 summarizes the results for new hires. Even for the new hires, the regions of high coherency are scarcely distributed at shorter frequencies. Just like for all the workers, the coherency is consistently high at the long-run frequencies throughout the sample period. And post 2000, the correlation becomes stronger with increase in the spread of high coherency regions that are statistically significant as seen from the coherency plot. Similar to our previous results, the phase-differences have significant variations at the short-run frequency region. But the wages are predominantly procyclical at both medium-run and long-run frequencies, with wages leading productivity most of the time.

We earlier established that wages of new hires respond more to productivity changes compared to all the workers. Consistent with this finding, we find that the wavelet gain of new hires is also larger than that of all workers across all the frequency intervals.



**Figure 2: New hires:** Log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.

Similar to the previous case, we find that there is substantial variation of cyclicity over time at higher frequencies. And more importantly, just like the case of all workers, we find that the wage cyclicity of new hires have also increased over time and this increase is seen across all the frequency intervals. This analysis shows that the wages

of all workers as well as new hires have become more procyclical over time.

### 4.3 Cyclicalities across Gender and Skills

We now document the evolution of wage elasticity separately across gender and skills. Following the majority of literature, we classify workers with a college degree or more as ‘high-skilled’, while those with less than a college degree as ‘low-skilled’. The results of the wavelet analysis for the different groups are provided in appendix A. The broad results continue to hold, i.e., wages are procyclical across the various subgroups of workers, and the wages are more flexible in the longer frequencies compared to the shorter ones. Interestingly, we find that, the increase in wage elasticity over time is predominantly driven by the low-skilled workers. Comparing figures A1 and A2, we can clearly see that there is an increasing trend in elasticity of wages among low-skilled men, while the elasticity doesn’t increase among high-skilled men. And, we find a similar pattern among female workers as can be seen in figures A3 and A4, with the increase in wage cyclicalities more prominent among low-skilled women. Appendix A also contains the results for new hires separately, where again we find that the increase in cyclicalities is more pronounced among low-skilled workers.

### 4.4 Discussion

Our analysis of wage cyclicalities using wavelets yields five key results. First, we find that the wage cyclicalities of all workers as well as new hires is consistently procyclical across time and frequencies. This is in contrast to papers like Neftci (1978), Hart et al. (2009), and Marczak and Gómez (2015) who find either countercyclical or acyclical patterns using aggregate data. Our study once again shows the importance of controlling for individual level characteristics while estimating wage cyclicalities, as argued by papers like Bils (1985) and Solon (1992). Second, in line with the previous studies like Shin (1994) and Haefke et al. (2013), we also find that the wages of new hires to be more cyclical than those of continuing workers. But by using wavelet analysis, we further show that this pattern is prevalent over time and across frequencies. Third, we find

that the magnitude of wage cyclicalities is higher in the long-run compared to short-run frequencies. This could indicate the presence of frictions that hinders wage adjustment in the short-run, but generates a stronger response of wages in the long-run. Fourth, estimates of wavelet gain show that the wage cyclicalities of all workers as well as new hires has increased over time and is seen across all frequencies. Finally, this increase in cyclicalities is predominantly driven by the wages of low-skilled workers. The overall increase in wage cyclicalities and its differential trend among low-skilled and high-skilled workers could reflect the underlying structural changes in the labour market, and also has implications for the labour search and matching models.

## 5. Implications

We now discuss how our findings relate to the broader changes in the labour market and also its implications for labour search and matching models.

### 5.1 Great Moderation and Structural Changes

Using wavelet analysis, we find that the cyclicalities of wages has increased over time for both new hires and all workers, and this has implications for understanding the Great Moderation. [Galí and Gambetti \(2009\)](#) and [Stirolh \(2009\)](#) showed that the volatility and correlations of various labour market variables had a marked decline post 1984. Studies like [Champagne and Kurmann \(2013\)](#) and [Haefke et al. \(2013\)](#) show that, in contrast to a number of macroeconomic variables, the volatility of real wages increased during this time. Using our analysis, we find that the wage cyclicalities have also increased over time for all the workers as well as for new hires. This finding thus strengthens the argument proposed by [Champagne and Kurmann \(2013\)](#) and [Nucci and Riggi \(2013\)](#), that an increase in flexibility of wages predominantly due to the emergence of performance-pay contracts led to the Great Moderation.

This increase in wage elasticity that we document is also consistent with the declining worker power hypothesis put forth by [Stansbury and Summers \(2020\)](#). The bargaining power that enabled workers to bargain for long-term wage contracts, which helped

them smooth their wages over the business cycle has been declining considerably. This in turn could lead to higher variability in wages over the business cycle. Additionally, we find that the increase in wage elasticity is concentrated among the low-skilled workers. This adds strength to the previous argument, as the decline in unionization rates was significantly larger among the low-skilled workers. Using data from Current Population Survey, Outgoing Rotation Group (CPS-ORG), [Stansbury and Summers \(2020\)](#) show that the union coverage rate for non-college-educated workers declined from 19% in 1984 to 8% in 2019, while the decline among college-educated workers is more modest during the same period, with coverage rate going down 8% to 6%. Our argument is also consistent with the findings of [Messina et al. \(2009\)](#), who using data on OECD countries shows that, countries with stronger unions tend to have less procyclical wages. Our analysis establishes a similar relationship between unions and wage dynamics in the US, with declining unionization and increasing wage cyclicalities over time.

## 5.2 Labour Search and Unemployment Volatility Puzzle

The estimates of wage elasticity also has implications for the model fit of canonical search and matching models. [Shimer \(2005\)](#) and [Costain and Reiter \(2008\)](#) documented that the benchmark search and matching model generates very low volatility in unemployment and vacancies compared to the data. [Hall \(2005\)](#) by introducing equilibrium wage stickiness in the model in place of Nash bargaining showed that, this modification significantly increases the volatility in both vacancies and unemployment. Relatedly, [Shimer \(2004\)](#) and [Pissarides \(2009\)](#) showed that, job creation and unemployment in the model is influenced by the wage behaviour of newly hired workers, and not affected by that of existing employment relationships. This implies, in order to improve the fit of the model, the wages of new hires should not respond much over the business cycle. Due to these findings, a large number of studies followed suit and introduced some form of wage stickiness in their models to match the data ([Menzio \(2005\)](#); [Farmer and Hollenhorst \(2006\)](#); [Moen and Rosen \(2006\)](#); [Braun et al. \(2006\)](#); [Blanchard and Galí \(2007\)](#); [Hall and Milgrom \(2008\)](#); [Gertler and Trigari \(2009\)](#); [Kennan \(2010\)](#);

Shimer (2010); Michaillat (2012); Christiano et al. (2016)).

Even though wage stickiness was one of the widely used solutions to improve the model fit, a number of studies like [Bils \(1985\)](#), [Solon et al. \(1994\)](#), [Devereux \(2001\)](#), and importantly [Haefke et al. \(2013\)](#) document that real wages of all workers, controlling for composition bias are quite procyclical, and wages of new hires respond even more over the business cycle compared to the wages of continuing workers. These studies show that, assuming rigid wages in a search model, particularly for new hires, is not empirically valid. In this paper, we further document that the cyclicality of wages for all the workers and new hires has actually increased over time. Thus, with our finding, the assumption of wage rigidity in a search model in order to solve the unemployment volatility puzzle has become even more untenable.

## 6. Conclusion

We employ continuous time wavelet tools to analyse how wage cyclicalities in the US have evolved over time and across different frequencies. We use individual level wage data from CPS-ORG to construct aggregate wage series after controlling for the changes in worker composition. Applying wavelet analysis, we find that, the wage cyclicalities of all workers as well as new hires have increased over time, and this increase is prevalent across all the frequency intervals. Also, the increase in cyclicalities is concentrated among the low-skilled workers who had the largest decline in their union membership over the last four decades. This suggests that the increase in wage cyclicalities might be an artefact of declining worker power and could reflect broader structural changes in the labour market. More research is needed to establish the underlying factors driving this increase in wage cyclicalities.

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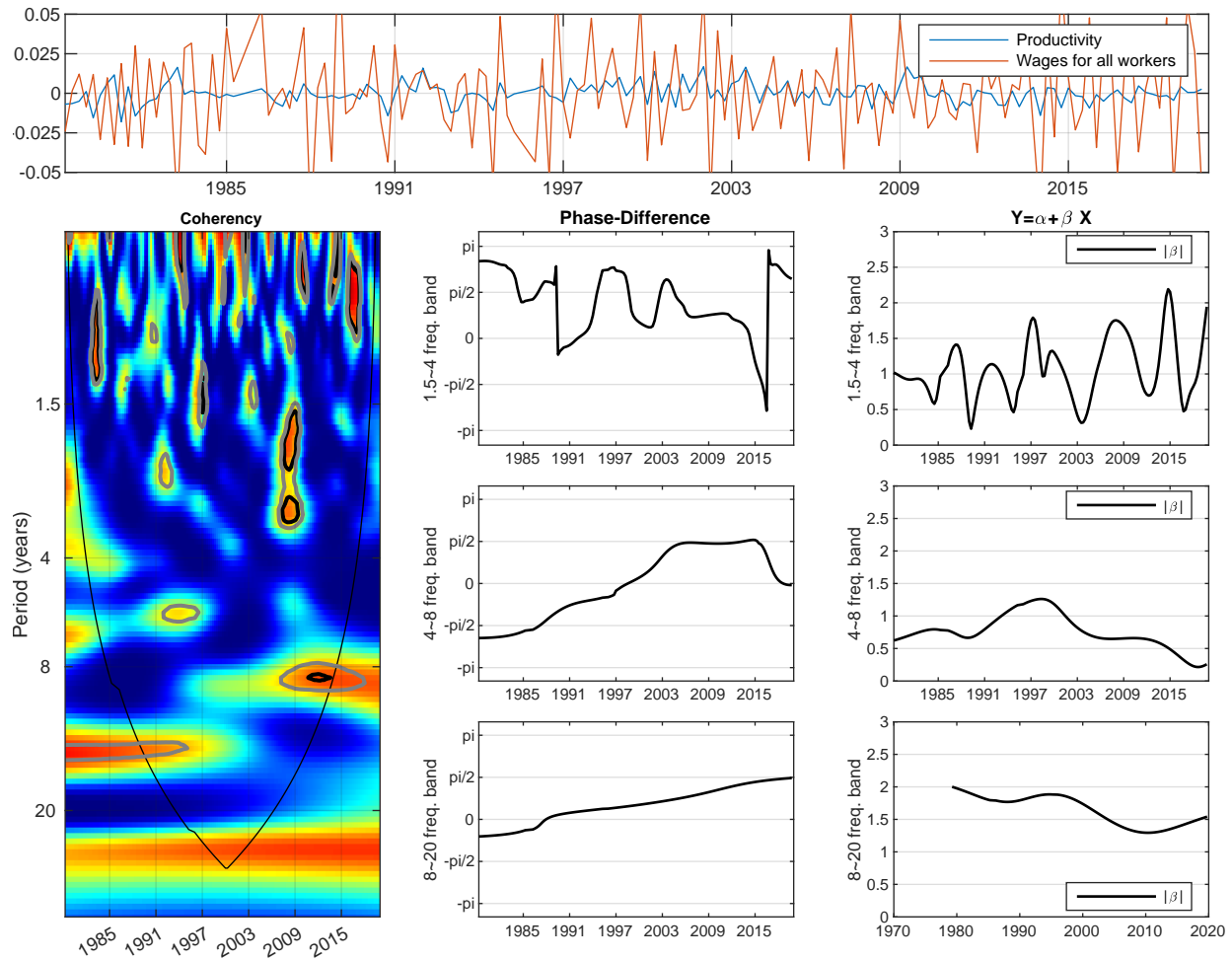
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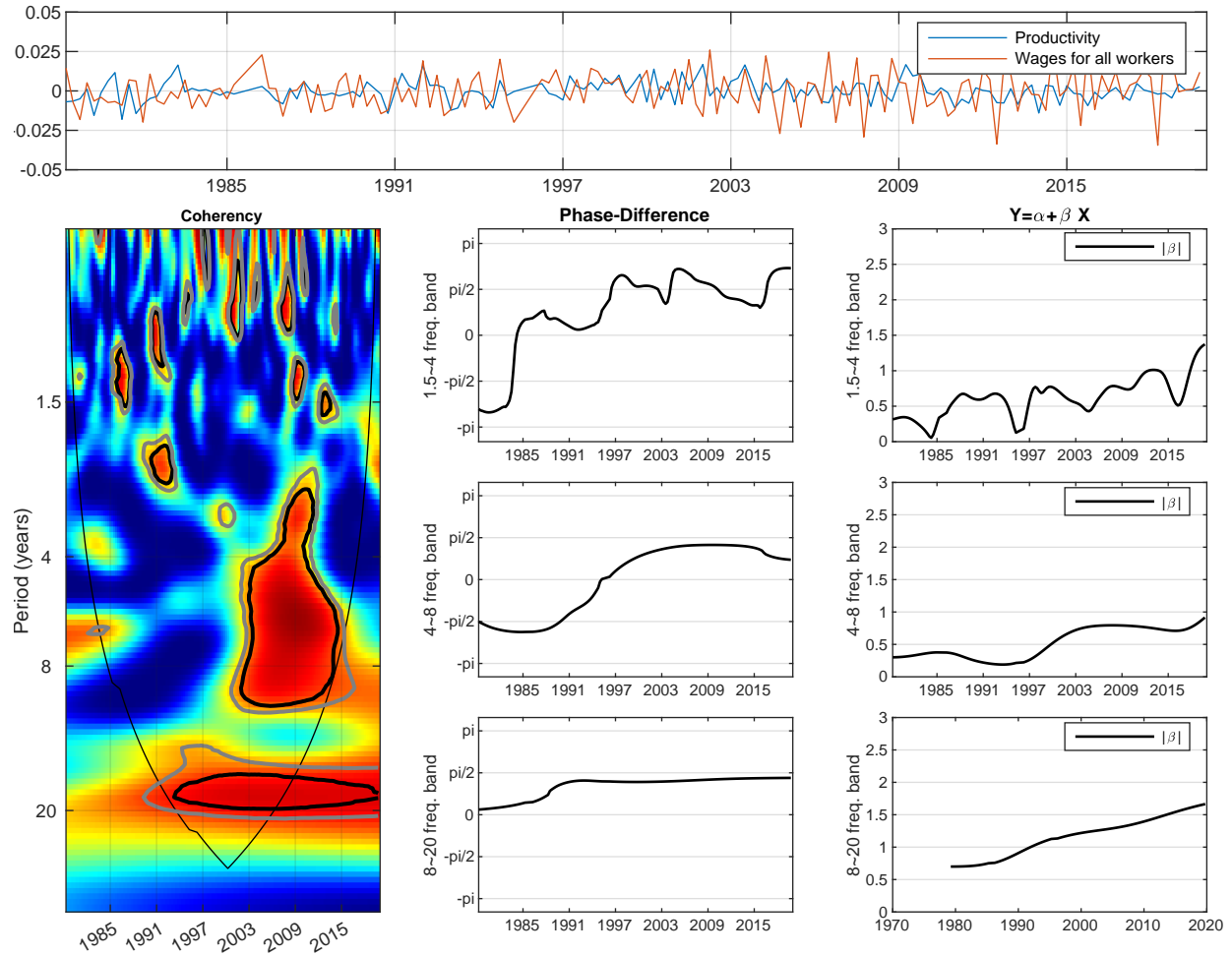
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# Online Appendix

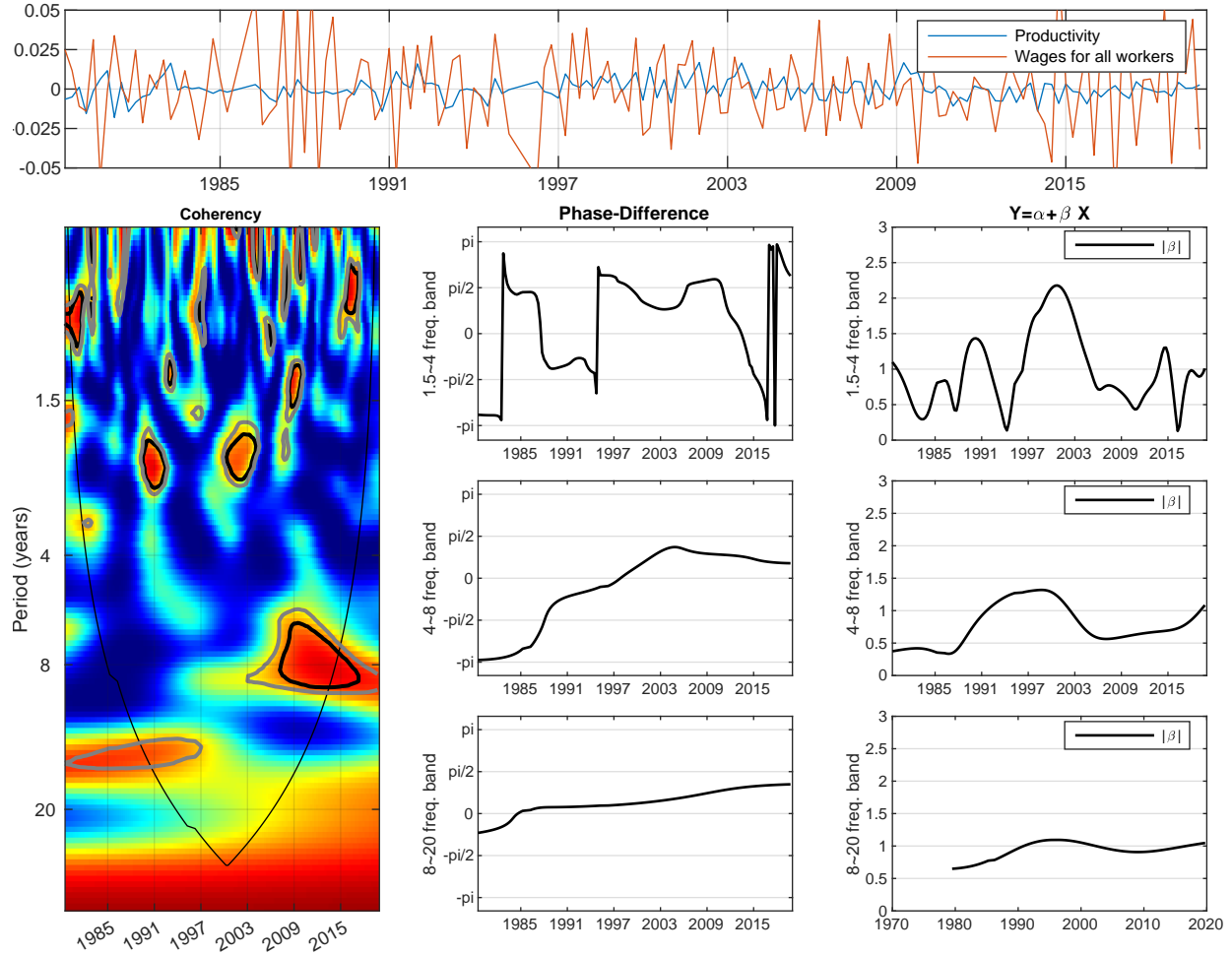
## A. Cyclicalty across Gender and Skills



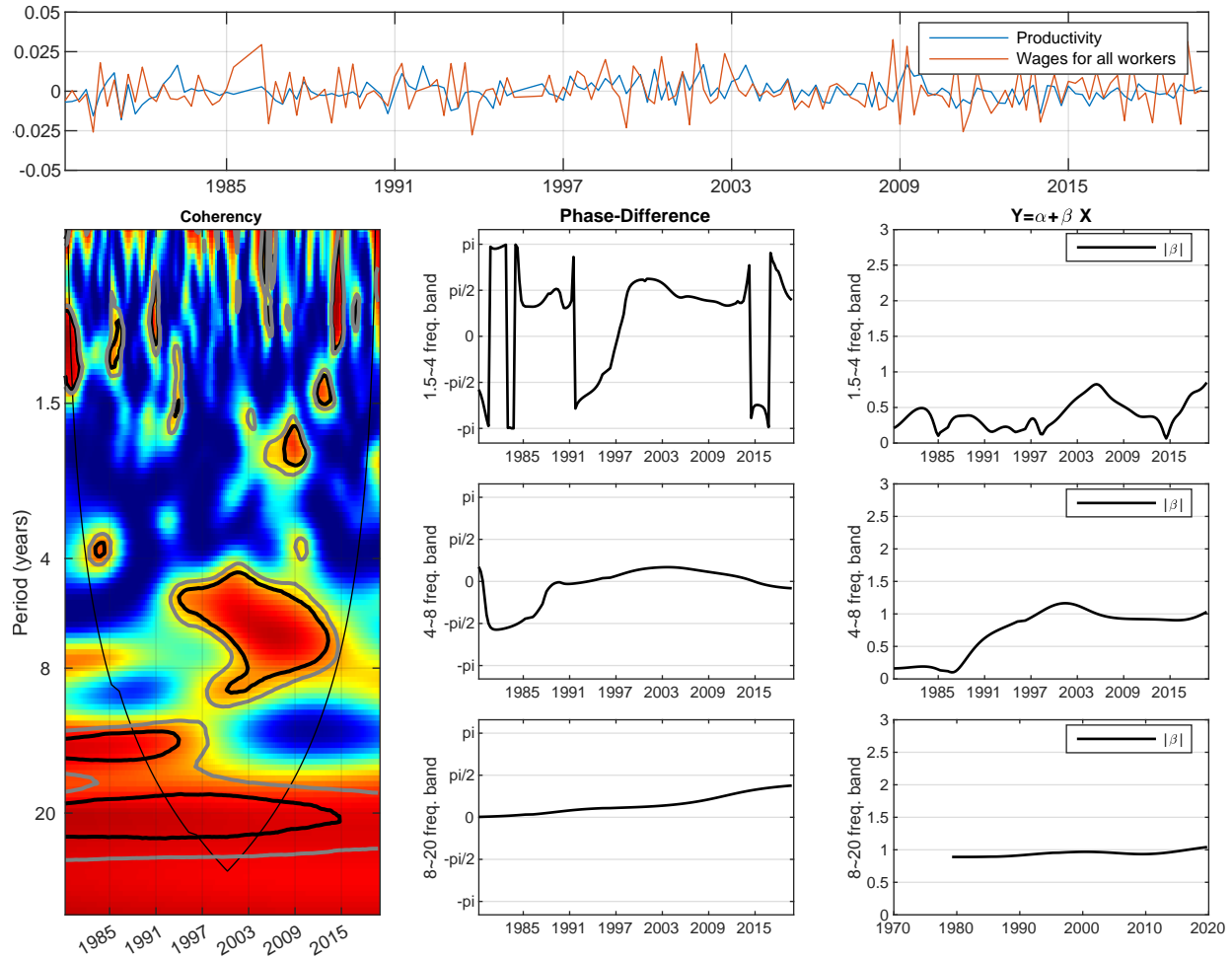
**Figure A1: High-skilled male, All workers:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherence between wages and productivity (left panel). Warm colours indicate high coherence, and cold colours indicate low coherence. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.



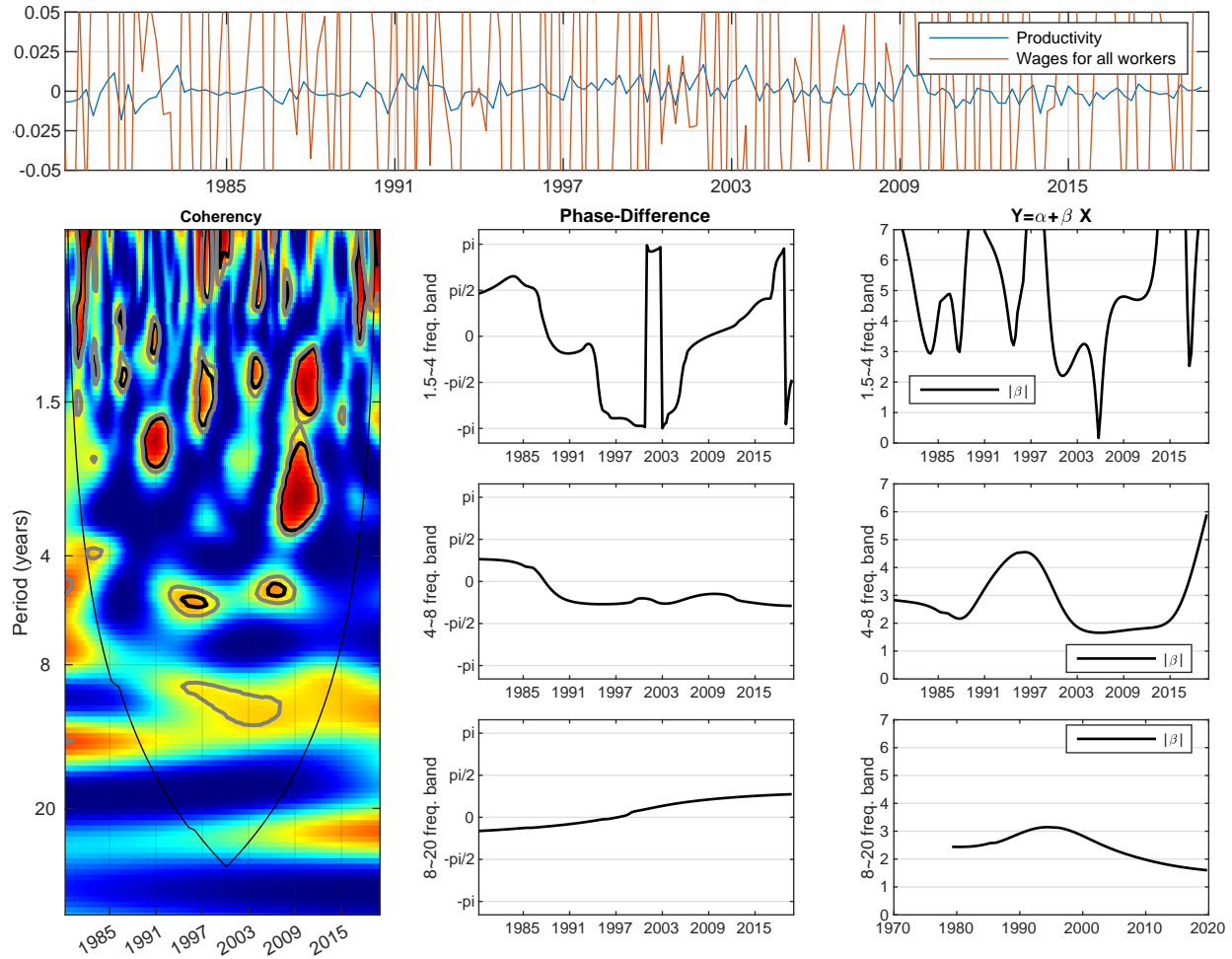
**Figure A2: Low-skilled male, All workers:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherence between wages and productivity (left panel). Warm colours indicate high coherence, and cold colours indicate low coherence. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.



**Figure A3: High-skilled female, All workers:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.

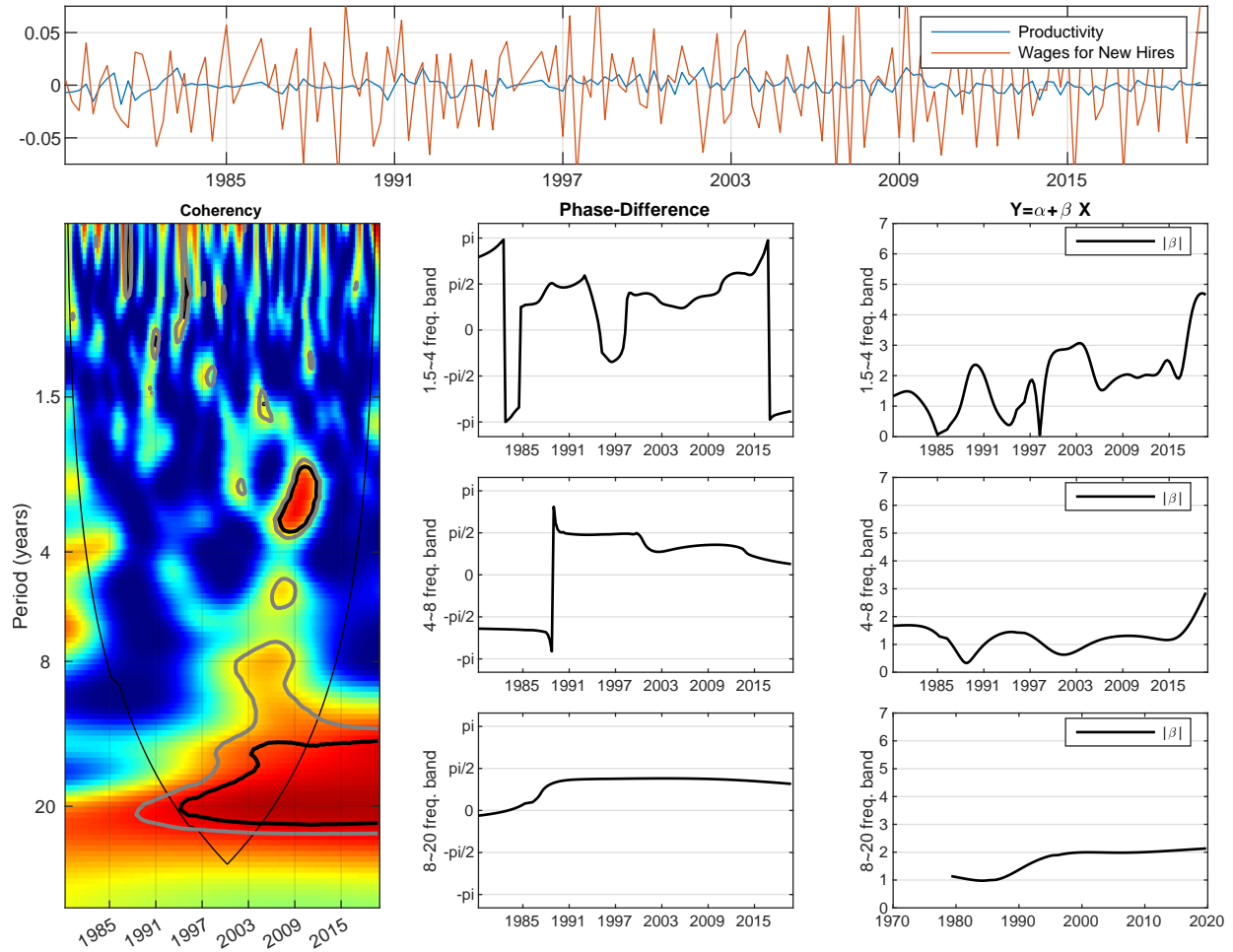


**Figure A4: Low-skilled female, All workers:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.

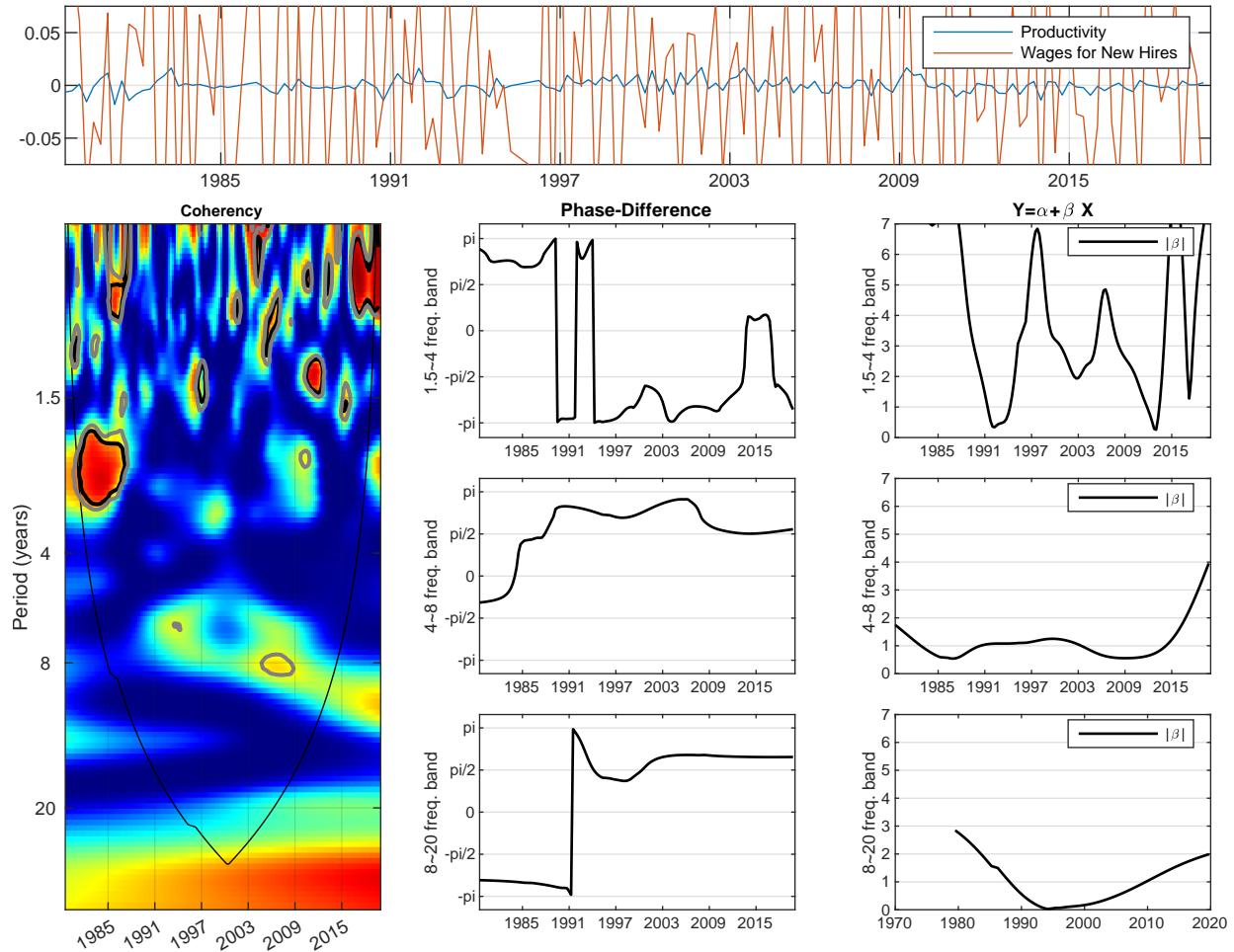


**Figure A5: High-skilled male, New hires:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.

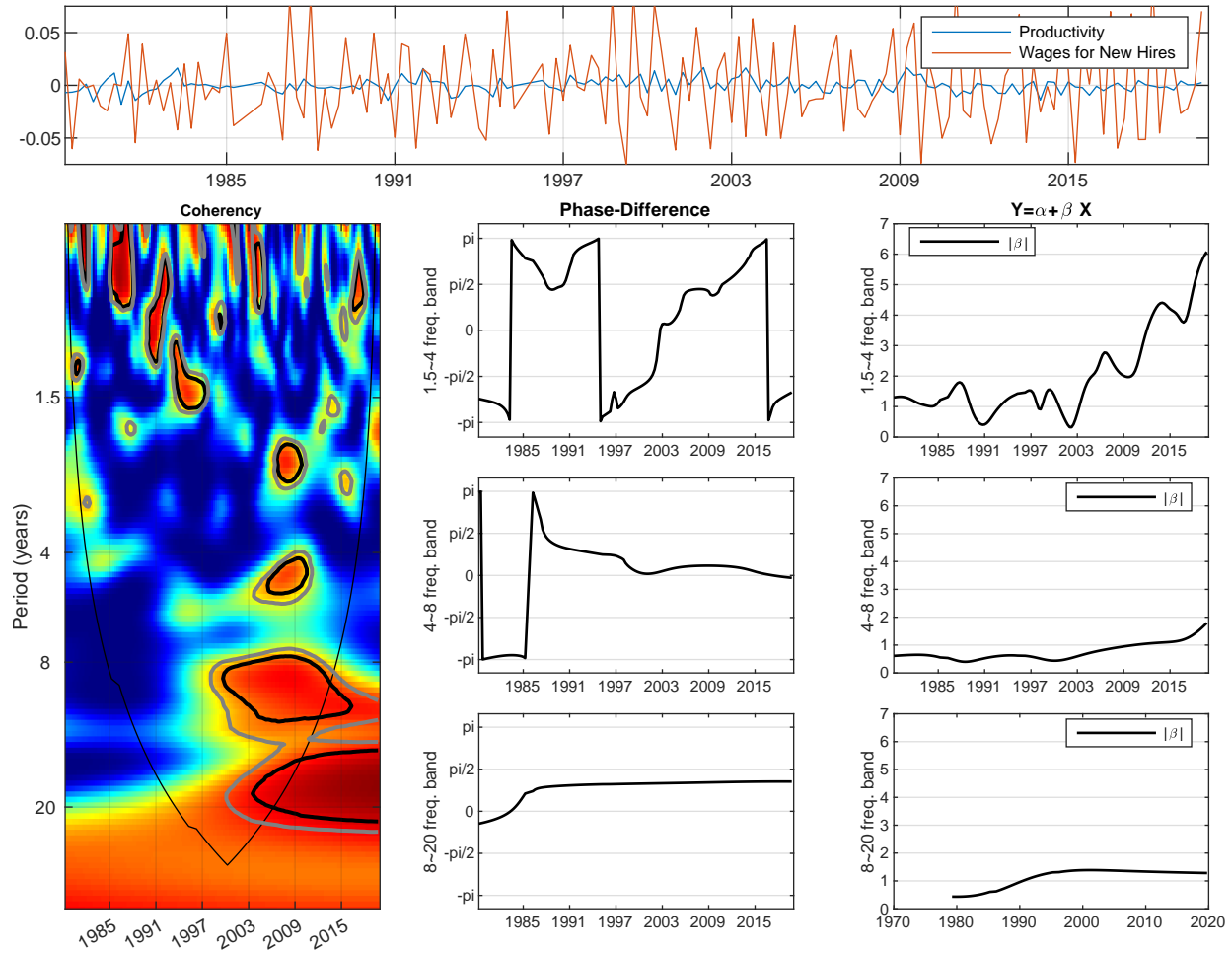




**Figure A6: Low-skilled male, New hires:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.



**Figure A7: High-skilled female, New hires:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.



**Figure A8: Low-skilled female, New hires:** Change in log real wages and log real labour productivity between 1979-2019 (top). Wavelet coherency between wages and productivity (left panel). Warm colours indicate high coherency, and cold colours indicate low coherency. Black and grey contours indicate statistical significance at 5% and 10% respectively. The parabolic line is the cone of influence that highlights the regions affected by edge effects. In the centre are the phase differences, and on the right are the wavelet gains for the three frequency intervals.