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Disappointment, Calibration Anomaly and Risk Attitudes

Kanchan Mukherjee
Professor of OBHRM
Indian Institute of Management Bangalore
Bannerghatta Road, Bangalore 560076
kanchan.mukherjee@iimb.ac.in

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Abstract

According to the Rabin's (2000) calibration theorem, expected utility theory implies risk neutral behavior for lotteries with small to medium stakes. In such cases, even small degrees of risk aversion leads to implausibly high risk aversion over large stakes. However, laboratory experiments over modest stakes show significant utility curvatures that predict absurdly high risk aversion over large stakes. A modified utility approach using a Disappointment-Elation (D-E) framework is used to show that there is over estimation of the utility curvature when utilities are measured by various empirical methods leading to over estimation of risk aversion under expected utility theory.

A new disappointment based risk aversion parameter, measuring the difference in salience between disappointment and elation of an individual is derived and the tradeoff method of utility elicitation is used to develop an empirical measure of this parameter. Finally, this new risk aversion measure is extended to characterize disappointment based risk attitudes of groups.

Key words: disappointment, calibration theorem, utility theory, utility measurement, risk attitude

1. Introduction

Expected utility (EU) (von Neumann and Morgenstern 1944) is the normative model widely used by economists and decision theorists to model individual preferences under risk and uncertainty. Researchers have used the diminishing marginal utility of wealth concept to measure risk aversion of individuals by eliciting preferences between probability weighted gambles. The curvature of the utility function has been used as the measure of the degree of one's risk aversion (Pratt 1964). In addition, Arrow (1971) shows that a person who is an expected utility maximizer and has a differentiable utility function will always take a stake, however small, in a bet that has a positive expected value. Rabin (2000) points out that this limit result showing that expected utility maximizers are arbitrarily close to risk neutrality for arbitrarily small stakes actually extends to "quite sizable and economically important stakes", implying that people are approximately risk neutral when stakes are small to moderate. That expected utility fails to account for plausible degrees of risk aversion over modest stakes has been recognized earlier in the literature in various different contexts (Epstein 1992, Epstein and Zin 1990, Hansson 1988, Kandel and Stambaugh 1991, Loomes and Segal 1994, Segal and Spivak 1990). Rabin (2000) presents a theorem that calibrates a relationship between risk attitudes over small and large stakes and shows that "expected utility theory is manifestly not close to the right explanation of risk attitudes over modest stakes". The theorem is quite general assuming only an increasing and concave utility function.

As an example of the implication of expected utility, Rabin's calibration approach shows that, at any initial wealth level, a person with an increasing and concave utility function, who refuses a 50-50 gamble of losing $100 or gaining $110, will turn down a 50-50 bet of losing $1000 or an infinite sum of money.[1] Similarly, Hansson (1988) shows that a person who, at all initial wealth levels, has a certainty equivalent of $7 for a 50-50 gamble of gaining either $21 or nothing prefers a sure gain of $7 to any gamble where the probability of winning a positive amount of money is less than 0.4 irrespective of the magnitude of the

[1] The intuition behind the calibration theorem result is the following: Quoting from Camerer and Loewenstein (2004), "In expected utility theory, rejecting a (+$11, -$10) coin flip at wealth level W implies that the utility increase from the $11 gain is smaller than the total utility decrease from the $10 loss, meaning that the marginal utility of each dollar gained is at most 10/11 of the marginal utility of the 10th dollar lost. By concavity, this means that the marginal utility of the W+11th dollar as at most 10/11 the marginal utility of the W-10th dollar – a sharp 10 % drop in marginal utility for small change in overall wealth of $21. When the curvature of the utility function does not change unrealistically over ranges of wealth levels, this means the marginal utility plummets quickly as wealth increases – the marginal utility of W+$32 dollar(=W + 11 +21) can be at most (10/11)(10/11), which is around 5/6 of the marginal utility of the W-10th dollar. Every $21 decrease in wealth yields another 10% decline in marginal utility." This implies that the marginal utility of the W+500th dollar would be very small compared to the value of dollars that might be lost in a bet. Hence, under expected utility, very large gains in wealth would not tempt a person to risk a small loss of (say) $50, if he dislikes to losing $10 more than he likes gaining $11 at every level of wealth.

potential gain. Hence, seemingly realistic risk aversions over small stakes translate into absurdly high risk aversion over larger stakes. Consider, for example, a person with a logarithmic utility function. If his certainty equivalent for a 50-50 lottery of (+ \$10, + \$100) is \$45, then his certainty equivalent for a 50-50 lottery of (+ \$10, + \$1 million) is only about \$8000.

Most laboratory experiments, designed to elicit utility functions use small to medium stakes ranging from a few tens to a few hundred dollars to elicit responses which are then encoded into an utility function. The proliferation of utility functions with marked curvatures, mostly concave, implying significant risk aversion over modest stakes highlights the inability of expected utility to explain risk averse behavior over small to medium stakes. Loss aversion (Kahneman & Tversky, 1979), with its reference point framework and steeper sloping utility function in the loss domain, can explain modest scale risk aversion (Rabin, 2000). However, it cannot explain the anomaly when gambles are expressed only in the gain domain.

In this paper, I propose that the emotions of disappointment and elation play a key role in enhancing the curvatures of utility functions obtained through various utility measurement techniques. I consider the certainty equivalent, the probability equivalent, the lottery equivalent, and the tradeoff methods of utility elicitation and measurement and show that accounting for disappointment will make the curves flatter and more risk neutral, as implied by expected utility, thereby accounting for the anomaly described above. I do this for increasing and concave utility functions and conclude that the curvatures obtained in the utility functions over modest stakes are due to disappointment-aversion/elation-seeking tendencies of individuals. In addition, I introduce a disappointment based risk aversion parameter, δ that represents the differential salience of disappointment and elation in an individual and use the trade-off method to develop an empirical measure for this parameter. Using this empirical measure I develop a simple characterization of disappointment based risk attitude of groups and provide a methodology to compare the same across groups.

The structure of the paper is as follows. In the next section, I lay out the D-E framework along with the assumptions and the structure of the model. In section 3, I use the framework developed to show how it influences utility measurement. Section 4 is dedicated to the analysis and derivation of the results concerning the differential salience and this is followed by the conclusion.

2. The Disappointment-Elation Framework

The fundamental assumption in disappointment theory is that when an individual considers an uncertain *prospect*, he forms a *prior expectation* about the prospect. An uncertain prospect is a list of consequences with associated probabilities. Upon resolution of the uncertainty, in addition to the utility thus obtained from the realized outcome, the person experiences the emotions of disappointment or elation depending on whether the realized value of the outcome is respectively lower or higher than the prior expectation (Loomes and Sugden, 1986).

Let an *action* A_i denote the i[th] prospect, which is an n-tuple of state contingent consequences and p_j denote the probability of state j, where $0 < p_j \leq 1$ and $\sum_{j=1}^{n} p_j = 1$. Loomes and Sugden (1986) model disappointment as a differentiable real valued function $D(.)$ such that $D(c_{ij} - \overline{c_i}) \overset{>}{\underset{<}{=}} 0 \Leftrightarrow (c_{ij} - \overline{c_i}) \overset{>}{\underset{<}{=}} 0$, where c_{ij} denotes the *basic utility* of the outcome of the *j*th state in the *i*th action, and $\overline{c_i}$ denotes the expected basic utility $C(.)$ of the *i*th action, $\sum_{j=1}^{n} p_j c_{ij}$ and represents the prior expectation of the prospect A_i. Basic utility is the classical cardinal measure of utility devoid of any emotional content like disappointment or regret. The expected modified utility of the *i*th action is then represented as $E_i = \sum_{j=1}^{n} p_j (c_{ij} + D(c_{ij} - \overline{c_i}))$. An individual anticipates any disappointment or elation and seeks to maximize the expected modified utility. Hence, with ϕ, ≥ and ~ denoting strict preference, weak preference and indifference, $A_i \overset{\phi}{\underset{\pi}{\sim}} A_j \Leftrightarrow E_i \overset{>}{\underset{<}{=}} E_k$, for all A_i, A_k. They also assume that $D(c_{ij} - \overline{c_i})$ is a non-decreasing function, or $D'(c_{ij} - \overline{c_i}) \geq 0$, with $D(0) = 0$. The functional form of function $D(.)$ is assumed to be convex for all positive values of $c_{ij} - \overline{c_i}$ and concave for all negative values.

David Bell (1985) provides a simple model of disappointment, similar in spirit but more specific in formulation, where he assumes that the subject's preference over money and disappointment/elation is linear and additive. He represents the decision maker's utility as

"Total Utility = Economic Payoff + Psychological Satisfaction where psychological satisfaction is positive for elation and negative for disappointment." The total utility corresponds to Loomes & Sugden's modified utility described above.

Bell models disappointment as a linear function of the difference between the *reference point* and the losing outcome where the reference point is the expected value of the lottery and corresponds to the prior expectation of Loomes & Sugden. Hence for a gamble yielding \$x with probability p and \$y with probability (1-p), denoted by (x, p; y), where x is weakly preferred to y, a person's disappointment is given by

$$\text{Disappointment} = q(px + (1-p)y - y) = qp(x - y)$$

where $q \geq 0$ is a "constant reflecting the degree to which a unit of disappointment affects the decision maker." Similarly, elation is given by

$$\text{Elation} = k(x - px - (1-p)y) = k(1-p)(x - y)$$

where $k \geq 0$ is a "constant reflecting the degree to which a unit of elation affects the decision maker."

I adopt Bell's (1985) simple formulation and rely on the following additional assumptions for the derivation of the results in the rest of the paper. I have used the terms lottery and gamble interchangeably in what follows.

Assumption 1: Disappointment is a stronger emotion than elation for risk averse individuals. An individual is risk averse when he strictly prefers a sure outcome equal to the expected value of a gamble to the gamble itself. This is quite intuitive and consistent with the assumptions of Prospect theory (Kahneman and Tversky, 1979). A risk averse person is more sensitive to potential losses than he is to potential gains in a gamble. The assumption then follows directly since losses lead to disappointment and gains lead to elation. I shall assume the expected value of the lottery as the natural reference point or prior expectation from which the individual evaluates the outcomes of the lottery. A risk averse individual accepts a lower sure thing in return for a lottery with higher expected value, precisely because the potential loss measured from the said reference point and its accompanying disutility weighs heavier on his mind than his emotional state of elation from the corresponding gains. Hence, in keeping with the loss aversion assumption of prospect theory, I assume that the fundamentally different emotions of Disappointment and Elation are represented by different functions, with the former operating in the loss domain, i.e., the domain of outcomes lower than the reference point, and the latter operating in the gain domain, i.e., the domain of outcomes higher than the reference point. Bell (1985) shows that a person affected more by disappointment than by elation will have a concave utility function even when constant marginal value for money is assumed.

Disappointment and elation are denoted by differentiable functions $D(.)$ and by $E(.)$ respectively. I assume that $D(.) < 0$ for $(x_{ij} - \overline{x_i}) < 0$ and 0 for $(x_{ij} - \overline{x_i}) \geq 0$ and likewise $E(.) > 0$ for $(x_{ij} - \overline{x_i}) > 0$ and 0 for $(x_{ij} - \overline{x_i}) \leq 0$, where x_{ij} denotes the outcome of the *j*th state in the *i*th action, and $\overline{x_i}$ denotes the expected value of the *i*th action, $\sum_{j=1}^{n} p_j x_{ij}$ and represents the prior expectation of the ith prospect. These functions are non-decreasing in their arguments, or $D^{'}(.), E^{'}(.) \geq 0$. In addition, the differential salience is modeled by $D^{'}(-x) > E^{'}(x) \forall x$, which implies $|D(-x)| > |E(x)| \forall x$. The expected modified utility of a lottery now becomes $\sum_{j=1}^{n} p_j \{x_{ij} + D(x_{ij} - \overline{x_i}) + E(x_{ij} - \overline{x_i})\}$.

Assumption 2: I qualify Bell's (1985) assumption of linearity of the disappointment and elation functions with the assumption that they are *approximately linear over modest stakes*, thereby retaining generality over large stakes. This is a natural consequence of the calibration theorem discussed above for the case of the utility function. As small curvatures of the utility function over modest stakes lead to absurdly high risk aversion over large stakes, so will small curvatures of a disappointment or elation function over modest amounts translate to implausible intensities of these emotions over large amounts. Hence, for example, if we assume that the elation function is concave with a logarithmic curve, a person winning \$500 million dollars above expectation is only 5 times more elated than a person winning \$50 above expectation. On the other hand, if we assume that it is convex with a power function, $E(x) = x^{1.5}$, a person winning \$10,000 dollars above expectation is almost 3000 times happier than a person winning \$50 above expectation. However difficult it is to imagine a quantification of an emotion, the above figures do suggest an implausibly absurd multiplication of well being under a non-linear elation function. Similar implausible ratios can be demonstrated in the disappointment domain as well, suggesting that the linear assumption is appropriate for modest stake gambles. In the analyses that follows, I assume $E(x) = kx$ and $D(x) = qx$, where $q > k$.

It will prove helpful to model the differential salience of the two emotions of an individual by $\delta = q - k$, where $\delta > 0$. Intuitively, δ is a measure of the importance or proneness of an individual to disappointment relative to the emotion of elation. It will become clear in the analysis that follows that it is this difference of the two emotional drives that influence choice behavior rather than the strong or weak presence of any one emotion. Hence, the fact that an individual feels intense euphoria on winning tells us nothing about his likely decisions when faced with uncertainty, unless we also know how badly he reacts to losses. Under the model presented here, high values of q and/or k denotes a highly emotional person. However, what influences decision making is the *difference* in the intensities of the two emotions.

3. Influence of Disappointment/Elation in Utility Measurement

There are several ways to measure utility of an individual in the laboratory. The most widely used are certainty equivalent, probability equivalent, lottery equivalent and tradeoff methods of utility elicitation. The important thing to note is that the analyses that follow are not affected by any procedural shortcomings of the measurement methods. The analyses show that whatever the procedural defects, every method overestimates the curvature of the utility function giving rise to over estimated risk aversion for modest stakes. Formally stated:

Proposition 1: *Under the disappointment-elation framework outlined in Section 2 above and the assumption of concavity of the utility function, the certainty equivalent, probability equivalent, lottery equivalent and tradeoff methods of utility elicitation result in the overestimation of the curvature of the utility functions. Consequently, the degree of risk aversion is also over estimated.*

In what follows, I shall consider each utility elicitation technique, provide a brief description of each technique and demonstrate the results separately. The utility measured by the expected utility equation is denoted by $U(x)$ and the utility resulting from the modified equations in the D-E model is denoted by $M(x)$.

3.1 Certainty Equivalent Method

I shall denote by (x, p) the lottery that gives \$$x$ with probability p and zero with probability $1-p$ and use this simple gamble for simplicity of exposition. The results hold true for any lottery.

In the certainty equivalent (CE) method of utility elicitation, subjects are asked to indicate the sure amount, called the certainty equivalent that would make them indifferent to the gamble (x, p). The experiment is designed to construct the individuals' utility function in the interval zero to \$$x$. The utilities of zero and \$x are normalized at zero and 1 respectively. Hence, if \$$x_e$ is the certainty equivalent elicited from the individual, expected utility equivalence gives us $U(x_e) = p$. Thus, the repeated elicitation of several CE responses in an iterated manner using the method above gives us several points on the utility curve, which can then be found by finding the best fit regression line explaining the obtained responses.

As an example, suppose we want to construct the utility function of a person, A, between zero and \$100 at his current level of wealth. The first step is to elicit the sure amount that will make A indifferent to the gamble *(100, p)*. Here p is a real number in (0, 1) and represents the probability of winning \$100. Using $p = 0.5$, assume A expresses indifference between \$40 for sure and the lottery *(100, 0.5)*. Setting $U(0) = 0$ and $U(100) = 1$, we obtain $U(40) = 0.5$ by equating the expected utilities of the gamble and the sure amount. The setting of the scale from 0 to 1 is purely for procedural reasons and poses no problem as expected utility functions are unique up to positive affine transformations. The next step is to elicit A's certainty equivalent to the lottery *(40, 0.5)*. An elicitation of \$18 implies $U(18) = 0.5^2 = 0.25$ as is easily verified by equating the two expected utilities. Hence, by successive elicitations using the previous response as an element in the choice, we get a series of coordinates in x X U space which can then be used to estimate the utility function by obtaining the best fit curve passing through these points.

Now consider the utility of x_e, the elicited certainty equivalent of lottery *(x, p)*, using the D-E framework described in Section 2 above;

$$
\begin{aligned}
M(x_e) &= p + (1-p)D(-px) + pE(x - px) \\
&= U(x_e) - p(1-p)x(q-k) \\
&= U(x_e) - \delta p(1-p)x
\end{aligned}
$$

Remembering the assumption $\delta > 0$ gives us $M(x_e) < U(x_e)$. Hence, the modified utility is lower than the utility measure used to obtain the individual's utility function. This pushes the measured coordinates of certainty equivalents vertically down thereby reducing the curvature of the resulting modified utility function and consequently the measure of risk aversion of the individual.

Note that the modified utility is the same as the expected utility for people having equal propensity for both disappointment and elation, i.e., $\delta = 0$. Hence, the D-E framework collapses to the EU framework when $\delta = 0$ showing that EU is a special case of the D-E framework. This is true for all the methods discussed within the D-E framework in this paper.

3.2 Probability Equivalent Method

In the probability equivalent (PE) method of utility elicitation, subjects are asked to indicate the probability p that would make them indifferent between a given sure outcome, x_s, and the gamble (x, p). In this method the sure thing is fixed and the probability p is elicited. The same normalization as in the CE method gives $U(x_s) = p$. For example, if we were to elicit A's utility using this method, the first step would be to elicit the probability p that will make A indifferent between (say) \$50 for sure and the gamble $(100, p)$. A response of 0.6 implies $U(50) = 0.6$, having set $U(0) = 0$ and $U(100) = 1$. In the next step, elicit p' that will make A indifferent between (say) \$25 and $(50, p')$. Say the response is $p' = 0.7$. Equating expected utilities yields $U(25) = 0.6 * 0.7 = 0.42$. In this way every elicitation provides us with an additional coordinate which are then used to obtain the utility curve.

The modified utility analysis is identical to the CE case yielding the same results.

3.3 Lottery Equivalent Method

In the lottery equivalent (LE) method, the subject is asked to specify the value of probability p^* that makes him indifferent between the two lotteries (x^*, p^*) and (x, p), (McCord and Neufville, 1986). Setting $U(0) = 0$ and $U(x^*) = 1$, we get

$$U(x) = p^*/p$$

Here, we are interested in finding the utility function between 0 and x^* and have $x < x^*$ and $p^* < p$. This is called the LE probability (LEP) method of utility elicitation. A variant of this method, called the LE outcome (LEO) method fixes the values of x^*, p^* and p and elicits x, (Delquié, 1993).

Using the previous example, lets construct A's utility function using the LEP method. The first step is to elicit the probability p^* that will make A indifferent between the lotteries $(100, p^*)$ and $(50, 0.5)$. A response of $p^* = 0.3$ implies $U(50) = 0.3/0.5 = 0.6$, having set, as before, $U(0) = 0$ and $U(100) = 1$. The next step is to elicit the probability that will make A indifferent between the gambles $(50, p^{**})$ and $(25, 0.5)$. A response of $p^{**} = 0.35$ will imply $U(25) = U(50)*0.35/0.5 = 0.42$. This process of iteration yields coordinates on the utility function which can then be used to draw the best fit curve and obtain A's utility profile.

On the other hand, the modified utility of x is obtained by equating the modified utilities of the two lotteries. Setting $M(0) = 0$ and $M(x^*) = 1$ and using $U(x) = p^*/p$ we get;

$$M(x) = U(x) - \delta[\frac{(1-p^*)p^*x^* - (1-p)px}{p}] \quad 3.1$$

For Risk Averse individuals, we have

(i) $\delta > 0$

(ii) $U(x) = \frac{p^*}{p} > U_N(x) = \frac{x}{x^*} \Rightarrow p^*x^* > px$, where $U_N(x)$ is the utility of x had the individual been risk neutral.

The conditions above along with the fact that *(1-p*) > (1-p)* imply that the D-E correction term $\delta[\frac{(1-p^*)p^*x^* - (1-p)px}{p}]$ is positive. Hence, $M(x) < U(x)$, allowing us to conclude that the coordinates in the D-E framework will be lower than those obtained under EU, thereby leading to a flatter utility curve and a lower measured value of risk aversion.

3.4 Tradeoff Method

In the Tradeoff (TO) method two lotteries, $(X_1, p; r)$ and $(X_0, p; R)$ are compared (Wakker and Deneffe, 1996). Here, we have the 'reference outcomes' $R > r$ such that $X_1 > X_0$. X_1 is elicited by fixing all the other parameters such that the person is indifferent between the two lotteries. In the next step, X_2 is elicited to make the subject indifferent between the lotteries, $(X_2, p; r)$ and $(X_1, p; R)$. Equating the expected utilities of the two indifferences and some algebra yields

$$U(X_2) - U(X_1) = U(X_1) - U(X_0).$$

Setting $U(X_0) = 0$, we get

$$U(X_2) = 2U(X_1).$$

By induction, if X_j is elicited such that the respondent is indifferent between $(X_j, p; r)$ and $(X_{j-1}, p; R)$, then in combination with the other indifferences, we get

$$U(X_j) = jU(X_1)$$

By setting $U(X_1) = 1/n$, where n is the index of the last outcome, we get n-1 points of the utility function between $U(X_0) = 0$ and $U(X_n) = 1$, thereby allowing us to estimate the individual's utility function, where

$$U(X_j) = j/n \quad 3.2$$

Hence, we get a series of $ values equally spaced on the utility dimension.

I assume $0 \leq r < R < X_{j-1} < X_j$ (Wakker and Deneffe, 1996, p 1139, Abdellaoui, 2000) in the analysis of the modified utility that follows. *(Detailed working is shown in the Appendix)*. Starting with the lotteries *(X_1, p; r)* and *(X_0, p; R)*, using the equations under the D-E framework and simplifying the terms we obtain the modified utilities

$$M(X_2) = 2M(X_1) + (1-p)\ \delta\ (X_2 - 2X_1 + X_0) \quad 3.3$$

Extending this result to the lotteries (X_j, p; r) and (X_{j-1}, p; R), we obtain

$$M(X_j) = jM(X_1) + (1-p)\ \delta\ (X_j - jX_1 + (j-1)X_0) \quad 3.4$$

Hence, for the terminal elicitation we have for X_n

$$M(X_n) = nM(X_1) + (1-p)\ \delta\ (X_n - nX_1 + (n-1)X_0) \quad 3.5$$

Normalizing, $U(X_n) = 1$, we obtain

$$M(X_1) = n^{-1}(1 - (1-p)\ \delta\ (X_n - nX_1 + (n-1)X_0) \quad 3.6$$

Substituting the expression for $M(X_1)$ and $U(X_j)$ into the equation for $M(X_j)$ we obtain

$$M(X_j) = U(X_j) - (1-P)\ \delta\ \ (j/n(X_n - X_0) - (X_j - X_0)) \quad 3.7$$

A concave utility function implies $X_n - X_{n-1} > \ldots > X_j - X_{j-1} > \ldots > X_1 - X_0$ which in turn implies that

$$j/n(X_n - X_0) - (X_j - X_0) > 0 \quad 3.8$$

In addition, $\delta > 0$ implies $M(X_j) < U(X_j)$ leading us to the conclusion that the utility curvature is overestimated.

Similar to all the methods considered above the modified utility collapses to expected utility when there is zero differential salience between disappointment and elation, i.e., $\delta = 0$. Hence, the model used generalizes EU by incorporating the effects of disappointment and elation into the decision framework.

4. The Differential Salience δ

4.1 The disappointment based risk aversion, δ

I have used only the tradeoff method in the analysis of δ in this section as this method has certain advantages over the other methods (Wakker and Deneffe, 1996, p 1139, Abdellaoui, 2000, Bleichrodt, Pinto and Wakker, 2001). The TO method is free of the certainty effect, where a certain outcome is weighed more heavily than an uncertain outcome of the same magnitude. Both, the CE and PE methods use a certain outcome for utility elicitation and suffer from this bias. The certainty effect shifts the empirically observed utility function up and to the left (McCord & Neufville, 1986). The TO method is also free from reference point effects since the same reference outcomes r and R are used in all the elicitations. In addition, there is no response mode bias in this method. Response mode bias refers to the different utility curves obtained depending on whether the elicited dimension is x or p. This leads to systematic differences in the utility curves obtained by the CE and PE methods for the same individual. The reader can refer to Hershey & Shoemaker (1985) for a detailed discussion of the inconsistencies between the two methods. Also, the response mode bias leads to the non-equivalence of the results obtained in the LEO and LEP methods as reported by Delquié (1993). The reader can refer to Wakker and Deneffe (1996) for a detailed exposition of the advantages of the TO method over the other methods.

Since the TO method is least affected by different biases, it seems most appropriate to base the analysis of δ on this method.

Proposition 2: *Based on the TO method of utility elicitation and Assumptions 1 and 2 below, the differential salience of an individual can be estimated by the expression* $\delta = (1 - p)^{-1}(X_n - X_0)^{-1}$, *where p is the probability used in the lotteries,* X_0 *is the starting amount and* X_n *is the final elicited amount in the TO method.*

Assumption 1: The modified utility curve of an individual is linear over modest stakes.

Assumption 2: The curvature in the utility function obtained in the TO elicitation method is entirely due to the disappointment-elation differential salience.

Proof: Assumption 1 allows us to write the modified utility as

$$M(X_j) = (X_j - X_0)/(X_n - X_0) \qquad 4.1$$

Assumption 2 allows us to equate 4.1 with the result obtained for the modified utility 3.7 above and repeated below

$$M(X_j) = j/n - (1-p)\ \delta\ (j/n(X_n - X_0) - (X_j - X_0))$$

This yields the following result for the differential salience δ

$$\delta = (1 - p)^{-1}(X_n - X_0)^{-1} \qquad \text{QED} \qquad 4.2$$

This result indicates that relative disappointment should be greater when the probability p of the favorable outcome is higher. This result is intuitive and has empirical support in the literature. Dijk and Pligt (1997) conducted a series of experiments and reported that "disappointment after not obtaining a desired outcome is more intense when the probability of obtaining this outcome was higher". Further intuition is provided after the following discussion on comparison of the differential salience between subjects.

Result 4.2 for δ gives us a convenient and parsimonious way to compare the differential salience of disappointment and elation between subjects. This result can also be interpreted as a disappointment based risk attitude measure in the same spirit as the exposition in Bell (1985). He shows that a relative aversion to disappointment over elation causes risk averse behavior by the decision maker even under assumption of constant marginal value for money. Hence, using this result we can obtain a measure of risk aversion in the absence of a curvature in the utility function.

We can operationalize the measurements in the following way: we fix *p, r, R* and X_0 for the elicitations in the TO method such that all the lotteries in the measurement process involve modest stakes. Next we decide n, the number of elicitations. The important consideration here is the trade off between making the process long and tiresome for the subject by extracting too many elicitations and obtaining insignificant results by recording too few elicitations. An optimal n can be obtained by the experimenter from an initial pilot phase in the process. The δ values of two subjects can be compared through the ratio

$$\frac{\delta_1}{\delta_2} = \frac{(X_n - X_0)_2}{(X_n - X_0)_1} \qquad 4.3$$

The above relationship is intuitive and consistent with the features of the framework presented in this paper. To see this consider the two lotteries L: $(X_1, 0.5; 0)$ and M: $(100, 0.5; 50)$. We get these lotteries by setting $p = 0.5$, $R = 50$, $r = 0$ and $X_0 = 100$ in the lotteries used in the first step of the trade-off method explained in the previous section. The subject is asked to state the value of X_1 that will make her indifferent between the two lotteries. Note that based on our assumption of the reference point being the expected value of the lotteries, both disappointment and elation in lottery *L* will be based on the value $0.5X_1$ which is the difference between the two possible outcomes and the expected value. Hence, an individual A who is more sensitive to disappointment relative to elation compared to individual B will specify a lower value of X_1 than person B. The result obtained for the ratio of differential salience (4.3) represents this behavior.

4.2 Characterization of Group Risk Attitude

Proposition 3: *The risk aversion measure based on the differential salience* δ *, can be used to characterize the risk attitudes of groups and serve as a population segmentation variable.*

Risk aversion, the behavior induced by diminishing marginal utility, is popularly assessed by the Arrow-Pratt measure (Pratt 1964, Arrow 1971) which is the negative of the ratio of the second derivative to the first derivative of the utility function, $-U''(x)/U'(x)$, where x is the level of wealth at which the risk aversion is evaluated. Now, as per the implications of expected utility theory, if utility curves are almost linear (due to risk neutrality) for small to medium stake lotteries, the risk aversion at any wealth level should not be sensitive to small changes in wealth. Hence, in a lab exercise to obtain utility curves, if the stakes in the lotteries are small compared to the wealth level of a subject, then we should be able to obtain a single measure of risk aversion for that subject at his current level of wealth, not sensitive within the range of the stakes used. Using the Arrow-Pratt measure on the concave utility curves obtained by the various utility elicitation methods does not accomplish this as it is sensitive to the changing curvature of the utility function. On the other hand, the disappointment base risk aversion measure δ obtained from the modified utility framework (equation 4.2) is not a derivative based measure and is insensitive to changing curvatures of the utility function. In fact, it is derived under the assumption that utility curves are linear over small stakes. Hence, as long as the stakes used in the utility elicitation process are small compared to the wealth level, the single δ measure of risk aversion provides us with a reading of the subject's risk attitude that is more stable and locally insensitive to changing wealth levels.

Since we have a measure of risk aversion which is locally unchanging, we can use result 4.3 to obtain the distribution of the disappointment based risk attitude measure for a sample of individuals leading to a characterization of the risk attitudes of a group. In a stable environment, under the assumption that the distribution of real wealth of a homogenous group, for e.g. lawyers, doctors or professors, is constant over time, we can use standardized tests to make inter group comparisons of risk attitudes as well. This can be done by using the trade-off method to obtain the values of $(X_n - X_0)_i$ for all the members i in the sample. The experimental parameters r, R and n should be chosen so that the obtained value of $(X_n - X_0)_i$ is small compared to the wealth level of the individual. In order to define a scale we can pick a reference point, $(X_n - X_0)_k$ and compare the empirically obtained values $(X_n - X_0)_i$ with this reference point. The choice of the reference point is arbitrary but it is desirable to fix a point whose magnitude is in the vicinity of commonly observed responses. We then obtain the ratio of differential salience of individual i relative to the chosen reference outcome. Let the ratios be denoted by

$$\lambda_i = \frac{\delta_i}{\delta_k} = \frac{(X_n - X_0)_k}{(X_n - X_0)_i} \qquad 4.4$$

Hence, higher the value of λ_i, more risk averse the individual. In this way, having obtained the values of λ_i's for all members in the sample, we can characterize the risk attitude of the group by the first and second moments of the obtained data, i.e., $\lambda_X = \frac{1}{K}\sum_i \lambda_i$ and $\sigma_X = \frac{1}{K-1}\sum_i (\lambda_i - \lambda_X)^2$ for population *X* and sample size *K*.

As an example, consider the exercise of comparing the risk attitudes of corporate managers, group *X* and university professors, group *Y*. The first step is to set the experimental variables defined by the six-tuple $(r, R, p, X_0, n, (X_n - X_0)_k)$. All these variables have been defined above. The values of *r, R* and *n* should be such that the elicited responses are small compared to overall wealth levels. The next step is to use the trade-off method to elicit X_n from samples of corporate managers and university professors. Going through with the methodology described above we obtain λ_X , λ_Y , σ_X and σ_Y . We can then use standard statistical methods like z-tests to compare the risk attitudes of the two samples. How the group risk attitude measures depend on the experimental parameters is beyond the scope of this paper and I leave it to future research.

Alternatively, the differential salience can be used as a segmenting variable. A population can be segmented based on the degree of risk aversion of the individuals as measured by δ . This parameter can also be used in correlational analysis to explain more complex behaviors. A more complete analysis of the various applications stemming from this idea will be pursued in future research.

6. Conclusion

I have addressed the anomaly inherent in applying the classical utility theory to modest stake gambles. The curvatures of utility functions obtained in laboratory experiments using small to medium stakes predict absurdly high risk aversion over large stakes. A fundamental implication of the utility theory is that people should exhibit very close to risk neutral attitudes towards uncertain outcomes. So, how can we explain this anomaly?

I propose a modified utility approach drawing from the disappointment-elation models first proposed by Loomes and Sugden (1986) and Bell (1985). The assumptions on which the results of this paper rests are that risk averse people are more susceptible to disappointment than to elation, and the D-E functions are linear in the outcomes over small to medium stakes for the same reasons why the utility function should be.

Using the modified model I show that there is over estimation of the curvatures when utilities are measured by the certainty equivalent, probability equivalent, lottery equivalent and tradeoff methods of utility measurement. In addition, the results in the modified utility approach depend on δ, the differential salience of disappointment and elation. The parameter δ measures the relative degree to which a person is affected by disappointment vis-à-vis elation, and can serve to be an alternative measure of risk aversion under the assumption that the curvature in the elicited utility function is entirely due to the disappointment-elation differential salience. The advantage of this risk aversion measure is that it is not based on derivatives and is unchanging for small changes in wealth levels. The tradeoff method can be used for empirical measurement of δ for individuals, which can then be compared across individuals leading to a characterization of group risk attitudes. This characterization can be easily used to compare risk attitudes of different population segments, which in itself has useful applications in areas such a customized marketing, insurance product design and policy formulation, among others. A limitation of this study is the assumption that the concave curvature of the utility function measured empirically is entirely due to the Disappointment-Elation effect. The exploration of other emotions playing a role in this is a subject for additional research.

Risk aversion is a concept measured and understood by the external manifestation of human behavior in response to a decision situation with uncertain outcomes. But what are the core emotions and tendencies of the human mind that drive this behavior? It is my conjecture that disappointment and elation are two examples of such emotions and ongoing research will lead to better understanding of how the intricacies of the human mind influence our risk attitudes.

Appendix

Derivation of Equation 3.3

Let the lotteries be: A: $(X_1, P; r)$ and B: $(X_0, P; R)$

$E[A] = X_1P + (1-P)r$

$E[B] = X_0P + (1-P)R$

$U(A) = PU(X_1) + (1 - P)U(r) - q[PX_1 + (1-P)r - r](1-P) + k[X_1 - PX_1 - (1-P)r]P$

$= PU(X_1) + (1 - P)U(r) - (1-P)P\delta(X_1 - r)$

Similarly,

$U(B) = PU(X_0) + (1 - P)U(R) - (1-P)P\delta(X_0 - R)$

$U(A) = U(B)$ implies

$U(X_1) - U(X_0) = (1-P)\delta(X_1 - r - X_0 + R) + (1-P)[(U(R) - U(r)]/P$ A.1

Similarly,

$U(X_2) - U(X_1) = (1-P)\delta(X_2 - r - X_1 + R) + (1-P)[(U(R) - U(r)]/P$ A.2

Solving A.1 and A.2 and setting $U(X_0) = 0$ results in Equation 3.3

$U(X_2) = 2U(X_1) + (1-P)\delta(X_2 - 2X_1 + X_0)$.

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