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Abstract

We consider a retailer that offers multiple variants of two brands, all of which are substitutable, within a single product category. Due to supply disruptions, the retailer may not be able to always offer either or both brands. To mitigate potential loss in demand due to unavailability of one brand, the retailer may choose the product variety strategically and/or adjust the price of the available brand. The goal of this paper is to compare the relative impact of these two levers of demand management in the face of supply disruptions. To that end, we develop and analyze a model of a retailer buying from two brands (suppliers) subject to random supply disruptions. The retailer's customer demand depends on price and product variety, and their impact on customer choice is captured through the nested logit model. The model also takes into account fixed design and operational costs associated with product variety. Our analysis reveals that strategic choice of product variety yields most of the benefit; price is a largely ineffective lever. We also show that when the supply of one brand is disrupted, the optimal price for the other brand is lower than when both brands are available, provided the outside option is equally or less affected by the disruption. However, even if the outside option is affected more, this result may not necessarily reverse. Finally, even though responsive pricing does not improve the profit substantially, it may reduce safety stock requirements.

Keywords: supply chain management, nested logit model, supply disruptions, product variety, responsive pricing

1 Introduction

Supply chains have increasingly become global and complex over the last three decades. This complexity has made it difficult for businesses to control all the links in the supply chain. The result is greater likelihood of a supply disruption due to something going wrong somewhere in the supply chain (Snyder et al., 2016). Perhaps the first major instance of supply disruption in globalized supply chains occurred due to the US border shutdowns after 9/11. While Ford had to reduce production volumes at its assembly plants due to delays in delivery of components from Canada and Mexico, Toyota came within hours of shutting down its plant in Indiana (Sheffi, 2001). As

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another example, a tsunami caused by an undersea earthquake off the coast of Japan in March of 2011 and floods in Thailand eight months later caused massive disruptions in electronics and automotive industries (Bland and Kwong, 2011). More recently, COVID-19 has led to supply chain snarls throughout the world (Foldy, 2020).

A disruption in supply of products can easily cause massive losses for any firm, sometimes even leading to bankruptcy. The possibility of such disastrous consequences has motivated growing academic literature in recent years on identifying suitable supply chain strategies to manage disruptions (Ho et al., 2015; Snyder et al., 2016). Examples of such strategies are dual sourcing, safety inventory, demand management, and emergency suppliers. An alternative to using such strategies is the use of financial measures such as insurance. Although financial measures can compensate a firm for the loss of profit due to a disruption, they cannot compensate for the loss of competitive position due to stockouts (Tang, 2006). In contrast, supply chain strategies could accomplish both objectives: They may not only mitigate profit loss, but also prevent loss of competitive position. However, the literature is lopsided in that much of it has examined supply-side strategies based on inventory and suppliers with few studies focusing on demand management strategies. Our paper aims to redress this imbalance by deriving insights on two demand management strategies: product variety management and responsive pricing, to mitigate the effect of supply shortages. Both of these strategies mitigate demand loss by influencing customer choice behavior.

A recent academic study based on interviews of supply chain executives corroborates the value of our focus on product variety (Cohen et al., 2022). The strategy of responsive pricing is also rooted in practice. For example, in 2014 Chipotle faced shortage of beef due to drought in many parts of the US, while the supply of chicken was not affected (Valentine, 2014). The company changed the prices of steak burritos and chicken burritos by differential magnitudes that reflected the relative supply situation. It is common that a supply disruption changes the balance of supply and demand in the market, which leads firms to responsive pricing.

With this motivation, we consider a retailer that sells multiple product variants of two brands in a product category. Each brand is sourced from a different supplier, and both suppliers are prone to disruption. If a supplier is unable to supply the variants of its brand, the retailer can adjust the price of the other brand to lure customers to buy the available brand. In this way, responsive pricing could help the retailer alleviate the consequences of supply disruptions. An alternative to price adjustments, which are a tactical tool, is strategic selection of product variety, in which the retailer offers a balanced portfolio consisting of a suitable number of variants of both brands, after taking into account the possibility of disruption. With this strategy, the retailer can offer a reasonable amount of choice to customers even when one brand is unavailable; that is, through suitable design of its product variety, the retailer can mitigate demand loss.

To explore the relative impact of product variety management and responsive pricing, we develop a discrete-time model to maximize the expected profit of the retailer from the sale of the product over a finite planning horizon. At the beginning of the planning horizon, the retailer chooses the number of variants for each brand. While doing so, she also incurs product variety costs. Once the

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planning horizon starts, the retailer chooses the optimal price for each variant every period. The optimal price reflects the product variants that are available, because the variants of one or both brands may become unavailable from time to time due to supply disruptions.

Since the availability of variants may change over time, the choice available to customers may also vary accordingly. We model the selection of an available variant by a customer through the nested logit model (Anderson et al., 1992). This model gives us the expected demand for any product variant in any period, which depends on the price and variety (number of variants) of the brands that are available (not disrupted) in that period. Given the expected demand, the expected profit, which is the difference of gross profit and operational cost, can be computed for each period. This profit summed over all the periods in the planning horizon minus the product variety cost is the total expected profit of the retailer from the product.

We use the model to probe several research questions on the structure of optimal pricing and product variety decisions and how the possibility of supply disruptions influences these decisions. First, we examine the effect of supply disruptions on the optimal pricing policy through the following research question: *How does the optimal pricing policy when both suppliers are available compare to when one of them is unavailable?* We find that the optimal price for variants of an available brand decreases when the other brand is not available compared to when both brands are available, provided the disruption does not adversely affect the attractiveness of outside option in the nested logit model. The purpose of this price reduction is to attract customers who would have purchased a variant of the unavailable brand into purchasing a variant of the available brand. Interestingly, the converse of this result is not necessarily true: Even if the outside option is less attractive under disruption, the price of the available brand may still not increase. The reduction in available product variety may make it difficult for the retailer to take full advantage of the lower attractiveness of the outside option.

The result also implies that the optimal margin may not remain constant over time for a brand. In contrast, when suppliers are reliable, the optimal margin is not only constant over time but also across brands. This type of pricing policy, which we refer to as equal-margin policy, has been shown to be optimal in many contexts in the existing literature; see, for example, Hopp and Xu (2005) and Li and Huh (2011). The sub-optimality of this policy in the presence of supply disruptions is driven by differential number of product variants over time due to supply disruptions.

In a similar manner, we probe the selection of product variety. We seek to compare the total number of product variants when the suppliers are reliable to when they are not reliable. The purpose of this analysis is to establish whether or not the analogy with inventory management, in which random supply or demand forces a firm to keep extra stock in the form of safety stock, carries over to our context. The analogy will carry over if the total number of product variants is greater when suppliers are unreliable compared to when they are not reliable. Accordingly, our next research question is as follows: *Does supplier unreliability induce the retailer to ensure greater product variety and sell additional variants in the form of safety variety?* We examine this question computationally. We find that although the expansion of product variety is highly probable when suppliers are

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unreliable, there exist scenarios in which the variety may, in fact, reduce. Two parameters play an important role here, the attractiveness of outside option and customer heterogeneity. If the unavailability of one brand makes the outside option appear more attractive, then the total variety may decrease when suppliers are unreliable. The same outcome may also occur at low values of customer heterogeneity.

Having examined responsive pricing and strategic selection of product variety separately, we next compare the two strategies by probing the following research question: *Which of the two levers of demand management, responsive pricing or product variety, is more effective?* We explore this question computationally as well and find that the incremental profit improvement due to responsive pricing is relatively insignificant given strategic selection of product variety. Thus, product variety appears to be a more effective demand management lever than responsive pricing.

Apart from examining price and product variety, we aim to develop structural properties of the optimal profit function, especially with respect to product variety. We ask the following research question: *What is the relationship between product variety and expected profit? How does the optimal expected profit change with respect to model parameters?* We find that the expected profit over the planning horizon is jointly concave in the number of variants for each brand. This means that the marginal profit due to an additional variant of a brand decreases as the number of variants increases. Moreover, the expected profit is a submodular function of the number of variants for each brand. This property implies that greater variety in one brand reduces the marginal value of adding one more variant of the other brand, regardless of the quality or the propensity for disruption of each brand. We combine both results to also show that the optimal number of variants for a brand does not exceed the corresponding number when only that brand is offered. This means that supplier diversification leads to fewer variants of each brand compared to when only one brand is offered.

We note that our demand-modeling framework also permits us to examine the impact of adoption of the responsive pricing strategy on the volatility of demand. Since the volatility of demand usually determines the safety stock requirement, this analysis sheds light on the effect of responsive pricing on the safety stock required. An interesting finding from this analysis is that responsive pricing leads to a reduction in the volatility of demand, which implies that the use of this strategy may also lead to lower safety stock requirement.

In some situations, it may be possible for the retailer to add additional variants of an available brand at short notice when the other brand is unavailable. We refer to this strategy as responsive variety strategy. To examine this strategy, we ask the following research questions: *Do the relationships explored in the previous research questions change when it is possible to modulate variety in response to a supply disruption? What is the profit improvement, if any, due to responsiveness in product variety?* We find that responsiveness in product variety does not alter the relationship between the expected profit and product variety stated above. Moreover, the responsive variety incrementally improves profit up to 8.6% for the range of parameters we consider. This strategy appears to be particularly effective when customers have heterogeneous preferences.

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The rest of the paper is organized as follows. We begin with a survey of relevant literature in the following section. In Section 3, we develop the model and provide a convenient list of notations. In Section 4, we analyze the model and present results answering the research questions above. A computational analysis of the pricing and variety decisions is presented in Section 5. In Section 6, we examine the responsive variety strategy. Finally, we conclude in Section 7. (Proofs are relegated to the Appendix.)

2 Literature Review

Since we derive insights regarding price and product variety in the face of supply disruptions, our work lies at the interface of operations and marketing. To position our paper, we first review the literature in the areas of supply chain disruptions and product variety.

The supply chain disruption literature falls within the broad category of supplier unreliability literature. When a supplier is unreliable, it may deliver nothing or it may deliver a quantity that is not equal to the quantity ordered. In the latter case, the quantity delivered may be lower than the quantity ordered due to yield uncertainty or random capacity. In our survey below, we have primarily summarized the literature in which the supplier is unable to deliver anything when it is disrupted, which is what we also presume. Readers who are interested in a comprehensive review of analytical models in this literature may refer to Snyder et al. (2016). For a broader review, which includes both analytical and qualitative models and considers full gamut of supply chain risks, readers may refer to Ho et al. (2015).

We classify the relevant supply chain disruption literature into three broad classes depending upon the mitigation strategy deployed. These classes correspond to inventory management, sourcing-related strategies such as dual sourcing, and demand management. Of these, the literature on inventory management is the oldest. The studies in this class examine optimal replenishment decisions given the possibility of supply disruptions. In general, the objective function lacks structure in such models, rendering optimal policy difficult to compute. Several papers have sidestepped this problem by deriving optimal parameters for a given policy. For example, Parlar and Perry (1996) identify parameters of a policy consisting of reorder point and order quantity in three scenarios in an Economic Order Quantity (EOQ) setting: a single supplier, two suppliers, and multiple suppliers. The supplier(s) in every scenario are prone to disruption. Similarly, Arreola-Risa and DeCroix (1998) identify optimal values of parameters in a (Q, R) policy when demand arrives as a Poisson process, and the up and down periods are exponentially distributed. In another paper, Parlar and Perry (1995) develop a model to determine three parameters of a policy in an EOQ setting: order quantity, reorder point when the supplier is available, and time until the next order when the supplier is unavailable (such orders remain backlogged until the supplier becomes available). Other studies in this vein include Snyder (2006), Parlar (1997), and Weiss and Rosenthal (1992).

We next discuss the literature on sourcing-related mitigation strategies. This literature can be broadly divided into two categories depending upon the research questions probed. The first

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category of papers concentrates on the number and identification of suppliers and allocation of orders among them. One example is Dada et al. (2007), who develop and analyze single-period models when suppliers are asymmetric in terms of procurement costs and yield distributions. They show that the optimal order quantity is greater when suppliers are unreliable compared to when they are not. Studies in the second category develop insights on sourcing strategies such as dual sourcing and contingent sourcing to mitigate supply disruptions. The primary objective of studies in this category is to shed light on when to deploy these strategies. Apart from sourcing-related strategies, such papers may also consider other strategies such as inventory, insurance, investment in supplier's processes to improve reliability, and doing nothing (or, passive acceptance). A few examples of studies in this category are as follows. Using an infinite-horizon planning model with one reliable and one unreliable supplier, Tomlin (2006) provides insights on when it is optimal to use the reliable supplier alone, dual sourcing, unreliable supplier with contingent sourcing, or unreliable supplier with inventory. Qi (2013) examines the possibility of waiting for the primary supplier, who is prone to disruptions, to recover from a disruption before an order is placed with a backup supplier, who is reliable. He shows that waiting is optimal when the primary supplier is disrupted often but recovers promptly. Tang et al. (2014) examine the role of incentives in inducing a supplier to improve its process reliability in a decentralized supply chain consisting of one buyer and one supplier. When the supplier delivers nothing upon disruption, they find that the buyer prefers to use subsidy to improve process reliability over order inflation. Chopra et al. (2007) show that bundling yield uncertainty (when quantity received is a random fraction of order quantity) and disruption (when nothing is delivered) in the presence of an unreliable and a reliable supplier leads a buyer to underutilize the reliable supplier and overutilize the other supplier.

In contrast to the literature on inventory and sourcing strategies, the literature on the use of demand management is rather limited. In fact, we are aware of only one study on this topic: Tomlin (2009) examines shifting of demand from one product to another in a newsvendor framework if the first product cannot be replenished due to a supply breakdown and compares its optimality to other strategies such as dual sourcing and contingent sourcing. He finds that demand shifting is not optimal to manage supply risk if dual sourcing from the same two suppliers is used for both products.

Although demand management through pricing has not been studied in the context of supply disruption, there do exist many papers on the use of this tool when suppliers deliver a random fraction of the order quantity. One example is Li et al. (2017), who consider two suppliers, one of which is reliable but expensive and the other one is unreliable. The unreliable supplier's capacity, hence the quantity delivered by it, is random. They compare responsive pricing, in which price is set after observing the quantity available to satisfy the demand, to dual sourcing as a strategy for managing supply uncertainty. They find that depending upon the cost parameters, the two strategies can be complements or substitutes. Two other examples of papers in which pricing strategy is analyzed in the presence of supply uncertainty are Tang and Yin (2007) and Dong et al. (2015).

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Since both product variety and price are tools to influence customer demand, this study contributes to the literature on managing disruptions using demand management strategies. Given the scant attention received by demand management strategies, we believe this study fills an important gap in the literature on supply chain disruptions.

We next position this study in the literature on product variety management and assortment planning. This literature is also vast and considers a diverse set of models and research questions. Apart from identifying product selection, studies in this stream may also consider other decisions such as price and order quantity. Further, these studies deploy a number of customer choice models such as the multinomial logit or MNL (van Ryzin and Mahajan, 1999), nested logit (Alptekinoğlu and Grasas, 2014), locational choice (Gaur and Honhon, 2006), probabilistic substitution (Smith and Agrawal, 2000), and preference-rank-based substitution (Honhon et al., 2012). Much of the literature assumes static substitution, in that the customers select a product from a given pool of products regardless of its availability. A key objective of these studies is to develop insights on the structure of the optimal assortment.

Given our focus on product variety and pricing decisions, we briefly review a few papers here in which either both price and assortment are optimized or only price is optimized given the assortment. Hopp and Xu (2005) use a Bayesian logit model to capture product substitution and study optimal length of product line and price vector in the presence of product modularity. They find that reducing product development cost through modular design leads to greater product variety. Li and Huh (2011) and Gallego and Wang (2014) examine optimal price vector for a given assortment when product substitution is modeled using the nested logit model. Aydin and Porteus (2008) analyze the order quantity and price decisions for a given assortment under a general price-demand curve. For an excellent review of assortment planning literature, see Kök et al. (2015).

Overall, this study lies at the interface of two large streams of literature on assortment planning/product variety management and supply chain disruptions. Ours is the first study that examines the design of assortment from the strategic perspective of supply chain disruptions.

3 Model

We consider a retailer that sells multiple variants of two branded products. Each brand is sourced from a different supplier. The brands may differ in quality, though product variants for a given brand are of the same quality and are horizontally differentiated. From a modeling perspective, therefore, we assume that all the variants of a brand offer the same expected utility to a customer at identical prices; the selection of a variant by a customer is driven by personal taste. However, due to the quality difference, any two variants of different brands may differ in their expected utility to a customer even when they have identical prices.

The retailer faces two decisions: the variety or the number of product variants for each brand and the price for each variant. These decisions are determined by maximizing the expected profit from the sale of all the variants over a planning horizon equal to the life cycle of the product. Let

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the number of periods in the planning horizon be denoted by T. While the number of product variants is optimized at the beginning of the planning horizon (an assumption we relax in Section 6), the price is optimized every period. Thus, product variety is a strategic decision, and price is a tactical decision. This decision hierarchy reflects the fixed costs associated with each decision.

Both suppliers are unreliable and are prone to disruption. When a supplier is disrupted in any given period, the retailer is not able to offer any variant from the associated brand in that period. Further, consistent with the decision hierarchy discussed above, in the event of a disruption for one brand, the retailer is unable to add more product variants from the available brand. This may occur if suppliers incur significant fixed costs to design each variant and qualify the product and its required components. Potential variation in the number of available product variants over time is the reason that prices are optimized every period. The optimal prices, thus, reflect the available product variety.

3.1 Supply Model

In each period, each supplier can be in one of the two states: 1 or 0. When a supplier or brand is in state 1, it is *available*, meaning it can supply all the variants of that brand.[1] In state 0, however, it cannot supply any of the product variants.

We model suppliers' availability as a discrete-time Markov chain with four states, 00, 01, 10, and 11, where a state ij corresponds to supplier 1 being in state i and supplier 2 being in state j. (Note that it is possible that the supplier availability is correlated.) This type of availability model has been prevalent in the literature; as an example, see Yang and Babich (2015). A more detailed discussion of the suppliers' availability model, including the Markov chain assumption, is available in Subsection 8.8 in the Appendix.

3.2 Demand Model

The demand for each product variant in a period depends on the number of product variants available for each brand as well as the price of each variant. Let the number of product variants offered for brand k be denoted by n_k ($k = 1, 2$). (See Table 1 for a list of notations.) Since all the variants of a brand have the same quality, it is optimal to set identical prices for them. Let P_k^{ij} be the price for variants of brand k when suppliers are in state ij, provided brand k is available in state ij. Note that we have not indexed prices by time. The reason is that the price for variants of a brand remains the same in any two periods in which the Markov chain is in the same state.

Both price and the number of product variants for each brand are inputs to a customer choice model, which determines the demand for each product variant every period. We use the nested logit model to represent customer choice behavior (Ben-Akiva and Lerman, 1985; Anderson et al., 1992). To illustrate the application of this model to our context, consider a period in which both brands are available. According to the nested logit framework, each customer first either selects

[1]Throughout this study, we use supplier and brand interchangeably, because each brand has a unique supplier.

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a brand k with probability Q_k^{11} or chooses not to purchase any variant. If she selects one of the brands, then she next chooses a variant corresponding to the selected brand. Let the probability of selection of variant m of brand k be denoted by $q_{m|k}^{11}$. This conditional probability is given by $q_{m|k}^{11} = e^{\frac{a_k - P_k^{11}}{\mu}} \Big/ n_k e^{\frac{a_k - P_k^{11}}{\mu}} = 1/n_k, \quad k = 1, 2$, where a_k is the expected utility of a variant of brand k, and μ is a parameter that tracks the variability around this expected utility (capturing customer heterogeneity). It controls the rate of within-brand substitutions; larger μ means smaller rate of within-brand substitution.

To obtain the unconditional probability of selection of a variant, $q_{m|k}^{11}$ must be multiplied with Q_k^{11}. This latter probability is equal to

$$Q_k^{11} = \frac{\left(n_k e^{\frac{a_k - P_k^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + \left(n_1 e^{\frac{a_1 - P_1^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}} + \left(n_2 e^{\frac{a_2 - P_2^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}}},$$

where γ tracks how differentiated the two brands are (larger γ implying more differentiated "nests"). It controls the rate of between-brand substitutions; larger γ means smaller rate of between-brand substitution. Note that Q_k^{11} may also be interpreted as the market share of brand k. Moreover, in the above expression, $a_k - P_k^{11}$ is the expected net utility from any variant of brand k, and ω^{11} is the expected utility of the no-purchase or outside option in state 11. The linearity of expected net utility in price is a common assumption in the operations management literature on assortment planning (see, for example, Li and Huh (2011) and Gallego and Wang (2014)). The parameter a_k can be interpreted as the quality of brand k and ω^{11} as a composite measure of the attractiveness of all competing retailer product varieties in state 11. Without loss of generality, we assume that $a_1 \geq a_2$.

We assume that the price sensitivity of demand is the same for both brands. The assumption follows from the pool of customers for the two brands being the same and is without loss of generality; all of our analytical results continue to hold even when the price-sensitivity of demand depends on the brand. For simplicity, we set the price-sensitivity of demand equal to one. Additionally, for consistency with utility maximization, we impose $\gamma > \mu$, which is a standard assumption (Anderson et al., 1992, pp. 47-48). The relationship $\gamma > \mu$ implies that variants for the same brand are more alike than variants across the two brands.

Putting everything together (and as noted above), the probability of a customer selecting variant m belonging to brand k is $q_{mk}^{11} = q_{m|k}^{11} \times Q_k^{11}$. Note that the probability of a customer not choosing any variant offered is equal to $1 - \sum_{k=1}^{2} \sum_{m=1}^{n_k} q_{mk}^{11} = 1 - Q_1^{11} - Q_2^{11}$.

We next develop an expression for the probability of selection of a variant in a period when only brand 1 is available. Observe that brand 2, when available, may be sold by many retailers, some of which can be competitors of the retailer. This means that its unavailability may affect the attractiveness of their product variety as well. Since this attractiveness is reflected in the utility of

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outside option, this parameter may now change. We therefore define a new parameter ω^{10} as the utility of outside option in state 10.

Depending upon the competitive landscape for the product, ω^{10} can be either less than or greater than ω^{11}. For example, if supplier 2 is exclusive to the retailer, the unavailability of its brand would not affect other retailers. For a customer, in fact, the attractiveness of the competing retailers' product varieties should increase in such a scenario, which would imply ω^{10} to be larger than ω^{11}. In contrast, if brand 2 is the only brand (or a leading brand) at one or more competing retailers, then its unavailability may lead to a reduced utility of outside option (i.e., $\omega^{10} \leq \omega^{11}$). Since either of ω^{10} and ω^{11} can be greater, we do not assume any specific order between them.

Given that only brand 1 is available, the probability that a customer purchases the variant m of brand 1 is equal to $q_{m1}^{10} = q_{m|1}^{10} \times Q_1^{10}$, where $q_{m|1}^{10} = e^{\frac{a_1 - P_1^{10}}{\mu}} \Big/ n_1 e^{\frac{a_1 - P_1^{10}}{\mu}} = \frac{1}{n_1}$ is the conditional probability that a customer chooses variant m given n_1 choices, and $Q_1^{10} = \left(n_1 e^{\frac{a_1 - P_1^{10}}{\mu}}\right)^{\frac{\mu}{\gamma}} \Big/ \left(e^{\frac{\omega^{10}}{\gamma}} + \left(n_1 e^{\frac{a_1 - P_1^{10}}{\mu}}\right)^{\frac{\mu}{\gamma}}\right)$ is the probability that a customer chooses to buy a variant of brand 1 instead of walking away. Similarly, when only brand 2 is available, the probability that a customer purchases variant m (of brand 2) is equal to $q_{m2}^{01} = q_{m|2}^{01} \times Q_2^{01}$, where $q_{m|2}^{01} = \frac{1}{n_2}$ and $Q_2^{01} = \left(n_2 e^{\frac{a_2 - P_2^{01}}{\mu}}\right)^{\frac{\mu}{\gamma}} \Big/ \left(e^{\frac{\omega^{01}}{\gamma}} + \left(n_2 e^{\frac{a_2 - P_2^{01}}{\mu}}\right)^{\frac{\mu}{\gamma}}\right)$. As above, ω^{01}, the utility of outside option, could be smaller or larger than ω^{11}.

Decision Variables	
n_k	Number of product variants offered for brand k
P_k^{ij}	Price for variants of brand k in state ij
Q_k^{ij}	Market share for one variant of brand k in state ij
Parameters and Functions	
c_k	Unit purchasing cost for variants of brand k
ω^{ij}	Utility of outside option in state ij
μ	Degree of customer heterogeneity
γ	Measure of brand disparity
ρ	Failure correlation of suppliers
N_t	Market size in period t
T	Length of product lifecycle/planning horizon
F_k	Fixed cost per variant of brand k for being included in the assortment
$f(n)$	Operational cost incurred in a period in which a total of n product variants are offered
Z_t^{ij}	Expected profit in period t given product variety and prices when suppliers are in state ij
π^{ij}	Limiting probability of Markov chain that describes supplier availability being in state ij

Table 1: Summary of Notation

3.3 Expected Profit

The expected profit over the planning horizon comprises three major components. The first component encapsulates fixed costs associated with including the product variants of each brand in the assortment. Examples of costs captured by this component are cost of creation of shelf space, which includes the cost of accessories, for effective display of inventory of a variant in the store; modification of sales register to include variants; and setting up of processes for procurement and

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display of each variant in the store. These costs are incurred prior to launch of the product in the store(s) and are assumed to be proportional to the number of variants for a brand. Let F_k be the unit fixed cost per variant of brand k. The total fixed cost is thus equal to $F_1 n_1 + F_2 n_2$.

The second component captures operational costs. It is well-understood in academia and practice that higher product variety increases operational costs (Kök et al., 2015); these costs may arise from increased handling effort and greater risk of excess inventory (van Ryzin and Mahajan, 1999). In every period in the planning horizon, we model such costs as $f(n)$, where $f(\cdot)$ is a convex and increasing function, and n is the total number of product variants offered in the period. Specifically, n is equal to n_1+n_2 in state 11, n_1 in state 10, and n_2 in state 01. We do not model inventory-related costs explicitly since replenishment decisions are beyond our scope of product-strategic decisions. Moreover, an explicit modeling of such decisions makes the model intractable. For the same reason, many recent studies on assortment planning and product variety management also do not model inventory costs; see, for example, Li and Huh (2011) and Davis et al. (2014).

The third component captures gross profit from the sales of the product. Consider first a period in which both brands are available. Let c_k be the unit purchasing cost for one unit of a variant of brand k. The profit per customer is equal to

$$\begin{aligned}
\sum_{m=1}^{n_1}(P_1^{11}-c_1)q_{m1}^{11}+\sum_{m=1}^{n_2}(P_2^{11}-c_2)q_{m2}^{11} &= (P_1^{11}-c_1)\sum_{m=1}^{n_1}(q_{m|1}^{11}Q_1^{11})+(P_2^{11}-c_2)\sum_{m=1}^{n_2}(q_{m|2}^{11}Q_2^{11}) \\
&= (P_1^{11}-c_1)\sum_{m=1}^{n_1}\left(\frac{1}{n_1}Q_1^{11}\right)+(P_2^{11}-c_2)\sum_{m=1}^{n_2}\left(\frac{1}{n_2}Q_2^{11}\right) \\
&= (P_1^{11}-c_1)Q_1^{11}+(P_2^{11}-c_2)Q_2^{11}. \qquad (3.1)
\end{aligned}$$

Let N_t be a random variable that denotes the market size in period t. We assume that the market size is independent of the customers' brand and product variant choice process outlined above. Multiplying the profit per customer in (3.1) with $E(N_t)$ and subtracting operational costs from it gives us the total expected profit earned during period t, Z_t^{11}, which is equal to

$$\max_{P_1^{11},P_2^{11}\geq 0} Z_t^{11}(n_1,n_2,P_1^{11},P_2^{11}) = E(N_t)\left\{(P_1^{11}-c_1)Q_1^{11}+(P_2^{11}-c_2)Q_2^{11}\right\}-f(n_1+n_2). \qquad (3.2)$$

Consider now a period in which only brand 1 is available. As above, the profit per customer is equal to $(P_1^{10}-c_1)Q_1^{10}$. Therefore, the total expected profit earned during the period, $Z_t^{10} = \max_{P_1^{10}\geq 0} Z_t^{10}(n_1,P_1^{10}) = E(N_t)(P_1^{10}-c_1)Q_1^{10}-f(n_1)$. In a similar manner, the expected profit when only brand 2 is available, $Z_t^{01} = \max_{P_2^{01}\geq 0} Z_t^{01}(n_2,P_2^{01}) = E(N_t)(P_2^{01}-c_2)Q_2^{01}-f(n_2)$. When none of the suppliers is able to deliver the product (in state 00), the retailer may incur a loss (denoted by Z^{00}, a non-positive constant), which we assume to be independent of the variety and pricing decisions.

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Bringing all the components together, the total expected profit over the planning horizon is equal to

$$
\begin{aligned}
Z_r^* &= \max_{n_1,n_2\geq 0}\left\{-F_1n_1 - F_2n_2 + \sum_{t=1}^{T} E\left\{\mathbf{1}(X_1^t=1,X_2^t=1)\max_{P_1^{11},P_2^{11}\geq 0} Z_t^{11}(n_1,n_2,P_1^{11},P_2^{11})\right.\right.\\
&+ \mathbf{1}(X_1^t=1,X_2^t=0)\max_{P_1^{10}\geq 0} Z_t^{10}(n_1,P_1^{10}) + \mathbf{1}(X_1^t=0,X_2^t=1)\max_{P_2^{01}\geq 0} Z_t^{01}(n_2,P_2^{01})\\
&+ \left.\left.\mathbf{1}(X_1^t=0,X_2^t=0)Z^{00}\right\}\right\},
\end{aligned}
$$

where $X_k^t \in \{0,1\}$ is a random variable that indicates the availability of brand k in period t and $\mathbf{1}(\cdot)$ is an indicator function.[2] Observe that we assume n_1 and n_2 take non-negative real values and not non-negative integer values. This simplification keeps the notation and analysis simple. Furthermore, this simplification has practically no impact on the insights we derive.

We assume that at the time of product launch as well as throughout the product lifecycle, the Markov chain that governs supplier availability is in steady state, i.e., the probability $P(X_1^0 = i, X_2^0 = j) = \pi^{ij}$, where π^{ij} is the limiting probability. We also assume that the number of customer arrivals N_t is independent of the evolution of supply disruptions. Using these assumptions, the optimal expected profit over the planning horizon is equal to

$$
\begin{aligned}
Z_r^* &= \max_{n_1,n_2\geq 0}\left\{-F_1n_1 - F_2n_2 + \sum_{t=1}^{T}\left\{\pi^{11}\max_{P_1^{11},P_2^{11}\geq 0} Z_t^{11}(n_1,n_2,P_1^{11},P_2^{11}) + \pi^{10}\max_{P_1^{10}\geq 0} Z_t^{10}(n_1,P_1^{10})\right.\right.\\
&\left.\left.+\ \pi^{01}\max_{P_2^{01}\geq 0} Z_t^{01}(n_2,P_2^{01}) + \pi^{00}Z^{00}\right\}\right\}. \qquad (3.3)
\end{aligned}
$$

We refer to this model as *responsive pricing (RP) model*. Let $G(n_1,n_2)$ denotes the maximand for the outer maximization problem; that is,

$$
\begin{aligned}
G(n_1,n_2) &= -F_1n_1 - F_2n_2 + \sum_{t=1}^{T}\left\{\pi^{11}\max_{P_1^{11},P_2^{11}\geq 0} Z_t^{11}(n_1,n_2,P_1^{11},P_2^{11}) + \pi^{10}\max_{P_1^{10}\geq 0} Z_t^{10}(n_1,P_1^{10})\right.\\
&\left.+\ \pi^{01}\max_{P_2^{01}\geq 0} Z_t^{01}(n_2,P_2^{01})\right\}. \qquad (3.4)
\end{aligned}
$$

Further, let $(n_1^*, n_2^*) = \arg\max_{n_1,n_2\geq 0} G(n_1,n_2)$.

4 Analysis

In this section, we analytically derive insights on the optimal decisions (on price and variety) as well as the optimal expected profit.

[2] Since price(s) remain(s) the same during any period in a given state, the three price maximization problems can be consolidated into a single price-maximization problem, which can be defined outside the summation over t.

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4.1 Optimal Price

We first consider the inner optimization problem for state 11 in Eq. (3.3), in which prices are the decision variables, and present two basic results that are adapted from Li and Huh (2011). In the first result, the expected gross profit Z_t^{11} is shown to be a concave function of market shares (Q_1^{11}, Q_2^{11}). To derive this result, the prices (P_1^{11}, P_2^{11}) need to be expressed in terms of the market shares. Accordingly, we observe that

$$\frac{\left(n_1 e^{\left(\frac{a_1 - P_1^{11}}{\mu}\right)}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}} = \frac{Q_1^{11}}{1 - Q_1^{11} - Q_2^{11}}.$$

Taking log of both sides and rearranging terms, we get

$$P_1^{11} = a_1 + \mu \ln n_1 - \omega^{11} + \gamma \left\{ \ln(1 - Q_1^{11} - Q_2^{11}) - \ln(Q_1^{11}) \right\}. \tag{4.5}$$

Similarly, P_2^{11} can be expressed in terms of (Q_1^{11}, Q_2^{11}). It can be easily seen that P_1^{11} and P_2^{11} are injective (one-to-one) functions of (Q_1^{11}, Q_2^{11}). Thus, Z_t^{11} can be equivalently stated as a function of $(n_1, n_2, Q_1^{11}, Q_2^{11})$.

In the second result, optimality of the equal-margin policy in state 11 is established. As per the policy, regardless of the purchasing costs and qualities of the two brands, the retailer will set the price in such a way to extract the same margin out of the two brands. The optimal margin can be expressed in terms of Lambert's W function (also called Omega function), which is a multi-valued function that satisfies $z = W(z)e^{W(z)}$. Further, the function is single-valued and increasing when $z \geq 0$.

We state both results in the following lemma. Define (P_1^{11*}, P_2^{11*}) and (Q_1^{11*}, Q_2^{11*}) as the optimal price and market share vectors given the number of product variants (n_1, n_2). Let $r_1^{11*} = P_1^{11*} - c_1$ and $r_2^{11*} = P_2^{11*} - c_2$ be the optimal margins for brands 1 and 2, respectively, in state 11.

Lemma 1. *(Adapted from Li and Huh (2011))*

1. The expected profit per period in state 11*,* $Z_t^{11}(n_1, n_2, Q_1^{11}, Q_2^{11})$*, is jointly concave in* (Q_1^{11}, Q_2^{11}) *for any given* (n_1, n_2)*.*

2. The optimal pricing policy for state 11 is an equal-margin policy that sets both margins $r_1^{11*} = P_1^{11*} - c_1$ *and* $r_2^{11*} = P_2^{11*} - c_2$ *equal to* $r^{11*} = \Pi^* + \gamma$*, where*

$$\Pi^* \quad = \quad \gamma \cdot W \left(\frac{\sum_{k=1}^{2} \left(n_k e^{\frac{a_k - c_k - \gamma}{\mu}} \right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}} \right). \tag{4.6}$$

Observe that the equal-margin policy would be optimal every period if suppliers were perfectly reliable. This policy (alternatively called constant-markup pricing) has also been shown to be

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optimal in various other settings; see, for example, Anderson and de Palma (1992), Besanko et al. (1998), and Hopp and Xu (2005).

Using the same approach as in part 2 of the above lemma, we next characterize the optimal pricing policy for states 10 and 01 in the following proposition. As before, let P_k^{ij*} be the optimal price for brand k given n_k variants $(k = 1, 2)$ in state ij. Let $r^{10*} = P_1^{10*} - c_1$ and $r^{01*} = P_2^{01*} - c_2$ be the optimal margins in state 10 and 01, respectively.

Proposition 2. *1. For a given n_1,*

$$P_1^{10*} = \gamma W \left(\frac{\left(n_1 e^{\frac{a_1 - c_1 - \gamma}{\mu}} \right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}} \right) + c_1 + \gamma.$$

2. For a given n_2,

$$P_2^{01*} = \gamma W \left(\frac{\left(n_2 e^{\frac{a_2 - c_2 - \gamma}{\mu}} \right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{01}}{\gamma}}} \right) + c_2 + \gamma.$$

Since $P_1^{10*} \neq P_1^{11*}$ and $P_2^{01*} \neq P_2^{11*}$, the optimal margin for a brand may change over time. Clearly, the equal-margin policy is not optimal when suppliers are not reliable. The sub-optimality of this policy is primarily caused by the variation in product variety over time. Another contributing factor is the change in the utility of outside option over time, though the equal-margin policy is not necessarily optimal even when the utility of outside option is the same in different states of the Markov chain; that is, even when $\omega^{11} = \omega^{10} = \omega^{01}$. In the following proposition, we explore how supply disruptions influence optimal prices by comparing them in different states.

Proposition 3. *1. For any given n_1 and n_2, if $\omega^{10} \geq \omega^{11}$, then $P_1^{10*} \leq P_1^{11*}$. Similarly, if $\omega^{01} \geq \omega^{11}$, then $P_2^{01*} \leq P_2^{11*}$.*

2. The optimal margins $r^{11*} = P_1^{11*} - c_1 = P_2^{11*} - c_2$, $r^{10*} = P_1^{10*} - c_1$, and $r^{01*} = P_2^{01*} - c_2$ (and thus optimal prices) are decreasing in ω^{11}, ω^{10}, and ω^{01}, respectively.

Part 1 indicates that if the utility of outside option remains unchanged or increases when one brand becomes unavailable, it is optimal to decreases the price of the available brand. (Throughout this study, we use increases/decreases and less/greater than in a weak sense; otherwise, we add *strictly* while describing it.) This also means that all else being equal, the optimal price for the available brand reduces in the face of a supply disruption. The purpose of this price reduction is to induce some of the customers who would have bought the unavailable brand and who may now choose an outside option to purchase a variant of the available brand.

The converse of the first part, however, is not necessarily true. Even if ω^{10} (ω^{01}) is less than ω^{11}, the price for brand 1 (brand 2) in state 10 (state 01) may still be lower than in state 11. The reason lies in the greater product variety in state 11. Due to greater variety, the probability of

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purchase of at least one variant of brand 1 (brand 2) in state 11 may be higher than in state 10 (state 01) even if the outside option is less attractive in state 10 (state 01) compared to state 11. This enables the retailer to charge a higher price for brand 1 (brand 2) in state 11 compared to state 10 (state 01). More precisely, if $\omega^{11} \geq \omega^{10}$, P_1^{11*} may still exceed P_1^{10*} when n_2 is sufficiently large. The large value of n_2 will buoy P_1^{11*} but will not affect P_1^{10*}. We illustrate this observation in the following example.

Example 4. *Let $\omega^{11} = 6$ and $\omega^{10} = 5.5$. The remaining parameters are as given in Table 2. The optimal price for brand 1 in states 11 and 10 are equal to 8.75 and 8.68, respectively.*

Even though a reduction in the utility of outside option in state 10 may not lead to a greater price in state 10 (compared to state 11), the price in state 10 (for brand 1) nonetheless increases. This is implied by part 2 of Proposition 3. By setting a higher price, the retailer is taking advantage of the lower attractiveness of the outside option.

In the following proposition, we relate the optimal margins in different states to additional model parameters. In general, the margins increase as the relevant number of product variants or quality increases and decrease as the unit purchasing cost increases. The margins also increase as customer heterogeneity (μ) increases.

Proposition 5. *Both r^{11*} and r^{10*} increase as n_1 or a_1 increases or as c_1 decreases. Similarly, both r^{11*} and r^{01*} increase as n_2 or a_2 increases or as c_2 decreases. All three margins increase as μ increases, provided the number of variants offered for the available brand is at least one.*

As the product variety increases, the probability of a customer purchasing a product increases. The retailer takes advantage of this by raising the price, which results in a higher margin. The effects of c_k and a_k are similarly intuitive. Finally, as customer heterogeneity increases, the likelihood of larger *ex-post* utilities increases, resulting in greater willingness-to-pay. Once again, the retailer takes advantage of this and increases the price, which leads to a greater margin.[3]

4.2 Optimal Variety and Expected Profit

We now focus on the outer optimization problem in Eq. (3.3) in which product variety is the decision variable. Along the way, we also develop properties of the expected profit function.

We begin by showing in the following theorem that the expected profit over the planning horizon $G(n_1, n_2)$ (Eq. 3.4) is jointly concave in the numbers of product variants for the two brand. The concavity implies that the marginal benefit of an additional variant of a brand reduces as the number of variants for that brand increases. This function is also submodular in the numbers of product variants for both brands, (n_1, n_2). The submodularity implies that the variants of the two brands are substitutable; that is, greater number of variants of one brand reduces the marginal benefit of an additional variant of the other brand. Observe that product variants are substitutable even

[3]Observe that greater heterogeneity also implies greater likelihood of smaller *ex-post* utilities. However, this does not affect the outcome since such customers would not have purchased the product in the first place.

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though the qualities of the brands and reliabilities of the suppliers may differ significantly. Both concavity and submodularity are also useful in searching for the optimal values of n_1 and n_2 in the computational experiments in Section 5.

Theorem 6. *The expected profit for responsive pricing over the planning horizon $G(n_1, n_2)$ is jointly concave in n_1 and n_2 for $n_1 \geq 0$ and $n_2 \geq 0$. Furthermore, $G(n_1, n_2)$ is submodular in (n_1, n_2).*

Another implication of the above theorem is that the optimal number of variants of a brand cannot exceed the corresponding value when variants of only that brand are sold. Thus, the optimal number of variants of brand k when only brand k is sold provides an upper bound on the optimal number of variants of brand k when both brands are sold, n_k^*. This bound is useful while searching for the optimal solution since we need to search for n_k^* only between 0 and this bound. We state this observation formally in the following theorem.

Theorem 7. *Define $n_1^{\#} := \arg\max_{n_1 \geq 0} G(n_1, 0)$ and $n_2^{\#} := \arg\max_{n_2 \geq 0} G(0, n_2)$. Then, $n_1^* \leq n_1^{\#}$ and $n_2^* \leq n_2^{\#}$.*

In the following proposition, we show that the optimal expected profit Z_r^* decreases as any of the outside option utilities increases. The proposition also illustrates relationship between Z_r^* and unit costs c_k, expected utilities a_k, and the degree of customer heterogeneity μ.

Proposition 8. *The optimal expected profit Z_r^* is a decreasing function of c_1, c_2, ω^{11}, ω^{10}, and ω^{01} and an increasing function of a_1 and a_2. Furthermore, Z_r^* also increases with μ, provided the optimal number of variants for each brand included in the assortment is at least one.*

Observe that the (increasing or decreasing) relationship between the optimal expected profit and a model parameter is the same as that of the parameter with the optimal margins (Propositions 3 and 5). We note that these relationships also hold for the expected profit per period in different states; that is, for $Z_t^{11}(n_1, n_2, P_1^{11*}, P_2^{11*}), Z_t^{10}(n_1, P_1^{10*})$, and $Z_t^{01}(n_2, P_2^{01*})$.

Finally, we explore the role of brand and market parameters in supplier diversification. Since supplier diversification is likely to be an automatic choice when suppliers are unreliable, we assume that they are perfectly reliable for this analysis. Brand parameters, quality (a_k) and unit cost (c_k), have a clear role in this decision since an increase in a_k or a reduction in c_k for brand k should lead to a greater number of variants for that brand.

Proposition 9. *Suppose that $F_1 = F_2 > 0$, and let $a_1 - c_1 > a_2 - c_2$. The retailer will offer strictly greater number of variants of brand 1 than brand 2, that is, $n_1^* > n_2^* > 0$. The outcomes regarding brand 1 and brand 2 reverse when $a_1 - c_1 < a_2 - c_2$. Finally, $n_1^* = n_2^* > 0$ when $a_1 - c_1 = a_2 - c_2$.*

The proposition shows that it is always optimal to offer both brands, though the brand with greater quality-cost differential dominates the other brand in terms of the number of product variants. Another observation based on this result is that the impact of quality and unit cost parameters operates through their difference.

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5 Computational Experiments

Since a closed-form solution for the optimal number of product variants for each brand does not appear possible, the rest of our analysis uses extensive numerical experiments. We first describe our experimental setup (Subsection 5.1) and then present three types of experiments. In Subsection 5.2, we explore the effect of supply disruption risk on the total number of variants. This analysis is important since increasing product variety complicates the management of operations. We also examine how the optimal product variety and its allocation among the two brands vary with respect to a few critical model parameters. In Subsection 5.3, we estimate the incremental profit improvements due to strategic product variety and responsive pricing. Finally, in Subsection 5.4, we develop insights on the effect of responsive pricing on optimal safety stock required.

5.1 Experimental Setup

To understand the effect of risk of supply disruption on the total number of product variants, we compare the total number of product variants over both brands offered in the responsive pricing model to another scenario in which there is no risk of supply disruption. In this scenario, both suppliers are always available, so the expected profit in every period is equal to $\max_{P_1,P_2\geq 0} Z_t^{11}(n_1,n_2,P_1,P_2)$, where Z_t^{11} is as defined in Eq. (3.2). We refer to the corresponding model, which is an auxiliary model, as *perfect supply (PS) model*. Let Z_p^* be the optimal expected profit over the planning horizon for this model. Then,

$$Z_p^* = \max_{n_1,n_2\geq 0} \left\{ -F_1 n_1 - F_2 n_2 + \sum_{t=1}^{T} \max_{P_1,P_2\geq 0} Z_t^{11}(n_1,n_2,P_1,P_2) \right\}. \tag{5.7}$$

To estimate the incremental profit improvements due to the product variety and responsive pricing strategies, we consider another auxiliary model in which not only the product variety but also the corresponding prices are determined at the beginning of planning horizon. The prices then remain unchanged throughout the planning horizon. Let Z_s^* be the optimal expected profit for this auxiliary model, which we refer to as *static pricing (SP) model*. Then,

$$Z_s^* \quad = \quad \max_{n_1,n_2\geq 0} Z_s(n_1,n_2) \tag{5.8}$$

where

$$\begin{aligned} Z_s(n_1,n_2) \quad &= \quad \left\{ -F_1 n_1 - F_2 n_2 + \max_{P_1,P_2\geq 0} \sum_{t=1}^{T} \left\{ \pi^{11} Z_t^{11}(n_1,n_2,P_1,P_2) + \pi^{10} Z_t^{10}(n_1,P_1) \right.\right. \\ &+ \quad \left.\left. \pi^{01} Z_t^{01}(n_2,P_2) + \pi^{00} Z^{00} \right\} \right\}. \end{aligned} \tag{5.9}$$

Note that periodic expected profits only depend on the state (and not on t). To determine the incremental profit improvement due to strategic selection of product variety, we compare the optimal

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profit in the static pricing model with the profit in the static pricing model in which the product variety is the same as the perfect supply model. Although prices are reoptimized in the latter scenario, they remain unchanged throughout the planning horizon in a manner similar to the perfect supply model. Thus, the profit improvement could be primarily attributed to the strategic selection of product variety.

To compute the incremental profit improvement due to responsive pricing, we compare the optimal profits in the static and responsive pricing models. Although both prices and the number of product variants are optimized in the responsive pricing model, the profit difference between the static and responsive pricing models can be primarily attributed to responsive pricing since the difference in product variety between the two models is small (see Subsection 5.2.1).

Default values of the parameters and the functional form for f used in the experiments are provided in Table 2. In all the sensitivity analyses that we report in this section, we vary one parameter while keeping other parameter values fixed to their default values.

F_1	F_2	$f(n)$	π^{11}	π^{10}	π^{01}	$\omega = \omega^{11}$	ω^{10}	ω^{01}	μ	γ	N	T	c_1	c_2	a_1	a_2
5000	5000	$25n^2$	0.3	0.4	0.3	6	6	6	2	2.5	1500	150	6	4	7	5

Table 2: Parameters and Functional Forms for Numerical Experiments

5.2 Optimal Variety under Supply Disruption

In this subsection, we report outcomes of three sets of experiments. The objective for the first set is to determine the extent to which responsive pricing results in changes in product variety (Subsection 5.2.1) - that is, we explore how the distribution of product variants across the two brands and their total count change with the practice of responsive pricing. In the second set of experiments, we examine whether product variety is always greater when there is risk of supply disruption compared to when suppliers are perfectly reliable (Subsection 5.2.2). The third set of experiments are aimed at understanding how model parameters such as correlation between supplier failures and customer heterogeneity affect the optimal number of product variants (Subsection 5.2.3).

5.2.1 Responsive Pricing and Product Variety

To explore the impact of responsive pricing on product variety, we identify and compare the optimal number of product variants for each brand with and without responsive pricing. For the scenario with responsive pricing, these values are computed using the RP model defined in Eq. (3.3); and for the scenario without responsive pricing, these values are computed using the model defined in Eq. (5.8). We find that the impact of responsive pricing is very small on both the brand-level and the total variety. The relative change in the number of variants for either brand and the total variety due to responsive pricing is usually less than 1%. Sample data on the optimal number of product variants for each brand for both scenarios is available in Table 4 in the Appendix.

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5.2.2 Supplier Unreliability and Product Variety

To understand whether the risk of supply disruption results in greater product variety, we compare the total number of product variants (over both brands) with and without the risk of disruption. For the scenario with the risk of disruption, we use the responsive pricing model to identify the optimal variety (Eq. 3.3). For the scenario in which there is no risk of supply disruption, we use the PS model (Eq. 5.7) to identify the optimal variety.

The experiments indicate that the possibility of supply disruptions may not always result in greater variety (Figure 1). This observation is contrary to our initial intuition since we expected that consideration of risk of supply disruption should always result in redundancy in the form of additional product variants.

Three parameters, ω^{10}, ω^{01}, and μ, play a crucial role in determining whether the risk of disruption leads to increased variety. The roles of ω^{01} and ω^{10} are relatively straightforward. When one or both the parameters are smaller than ω^{11}, then the probability of selection of the available brand gets a boost when only one brand is available, and so we are likely to see greater variety when supply could be disrupted. Conversely, an increase in either or both of the two parameters is likely to generate an opposite result. (See Figures 1(a) and 1(b).) However, even when $\omega^{10} = \omega^{01} = \omega^{11}$, both outcomes are possible depending upon the value of μ. In Figure 1(c), we observe that the variety could reduce in the presence of supply disruptions for small values of μ. Conversely, for high values of μ, the variety is greater in the presence of supply disruptions. The heterogeneity of customers naturally favors greater product variety. However, it appears to induce a greater change when only one brand may be available occasionally (considering the possibility of supply disruptions) compared to when both brands are always available.

5.2.3 Sensitivity Analysis on Product Variety

In this subsection, we describe how the optimal number of product variants for each brand varies with respect to four parameters in our base (responsive pricing or RP) model: μ (range: 1-3), ω^{10} (range: 0-15), ρ (failure correlation of suppliers, range: -1 to -0.1), and γ (range: 2.5-7). (The relationships with respect to ω^{01} are similar to ω^{10}.) A graphical representation of these relationships is shown in Figure 2. A summary of observations from the figure is as follows.

1. As the customer base becomes more heterogeneous (as μ increases), the retailer increases the number of product variants of each brand (Figure 2(a)).[4] The heterogeneity of customer base implies that customers are more likely to have high *ex-post* utilities for the two brands, resulting in greater willingness to buy. To take advantage of this, the retailer offers additional variants and accordingly charges a higher price (Figure 3(a)).

[4]The reason for n_1 increasing more than n_2 with respect to μ in Figure 2(a) is that $\pi^{10} > \pi^{01}$. Since brand 1 is available greater fraction of time ($\pi^{11} + \pi^{10}$) compared to brand 2 ($\pi^{11} + \pi^{01}$), it is more profitable to carry a higher number of variants for it as customer heterogeneity increases. Observe that the two brands are equivalent otherwise as $\omega^{10} = \omega^{01}, F_1 = F_2$, and $a_1 - c_1 = a_2 - c_2$.

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2. As the utility of outside option when only brand 1 is available, ω^{10}, decreases, the number of variants of brand 1 increases (Figure 2(b)). At the same time, to keep the fixed costs under control, the retailer reduces the number of product variants of brand 2. A consequence of greater number of variants of brand 1 is that it is optimal to charge a higher price for the brand (Figure 3(b)).

3. The number of variants for brands 1 and 2 increases and decreases, respectively, as the failure correlation increases (Figure 2(c)), in the sense of becoming less negative.[5] (Throughout Section 5, an increase in correlation means correlation becomes less negative.) Before we explain the underlying mechanics, we note that different values of correlation are obtained by varying π^{11} and π^{01} such that their sum is equal to 0.75. (The values of π^{10} and π^{00} remain fixed at 0.25 and 0, respectively. See Subsection 8.9 in the Appendix for more details.) Further, correlation increases with π^{11}. This increase in correlation is synchronous with a reduction in the value of π^{01}, which is the probability that only brand 2 is available, which explains the reduction in the number of variants of brand 2. At the same time, the probability of brand 1 being available increases ($\pi^{10} + \pi^{11}$), which leads to a greater number of variants for the brand.

Observe that the roles of suppliers 1 and 2 can be interchanged while obtaining correlation values (i.e., fix $\pi^{01} = 0.25$ and $\pi^{11} = 0.75 - \pi^{10}$). In that case, the number of variants of brands 1 and 2 will decrease and increase, respectively, as correlation increases.

Overall, greater failure correlation first leads to greater supplier diversification in the form of similar number of product variants for both brands ($\rho \in (-1, -0.34)$). Specifically, for $\rho = -0.34$, $n_1^* = n_2^*$. Subsequently, as correlation increases further, the diversification appears to reduce with greater divergence ($n_1^* > n_2^*$) in the number of product variants for the two brands.

4. The number of variants of both brands increases as brand disparity γ increases (Figure 2(d)). An increase in brand disparity means the variants in one brand are less substitutable for the variants in the other brand. Clearly, with reduced substitutability, the retailer should offer more variants of each brand, which is reflected in the figure.

5.3 Relative Value Added by Strategic Product Variety and Responsive Pricing

In this subsection, we compare incremental profit improvements due to strategic selection of product variety and responsive pricing. To obtain the incremental profit improvement due to strategic selection of product variety, we first determine the benefit of realigning product variety when suppliers are unreliable. We estimate this benefit as $Z_s^* - Z_s(n_1^{PS}, n_2^{PS})$, where Z_s^* and Z_s are as defined in Eq. (5.8) and Eq. (5.9), respectively; and $n_k^{PS}, k = 1, 2$ is the optimal number of product variants for brand k in the perfect supply model (Eq. 5.7). Subsequently, we divide this value by $Z_s(n_1^{PS}, n_2^{PS})$ to obtain the profit improvement in percentage; that is,

$$\text{PIP (Product Variety)} = \frac{Z_s^* - Z_s(n_1^{PS}, n_2^{PS})}{Z_s(n_1^{PS}, n_2^{PS})} \times 100, \tag{5.10}$$

[5] We set π^{00} to zero in the experiments since varying π^{00} would have resulted in a confounding effect on the optimal product variety. Fixing π^{00} to zero, however, ensures that positive correlation values cannot be attained. This is why the range of correlation values considered in the figure is narrower than $[-1, 1]$.

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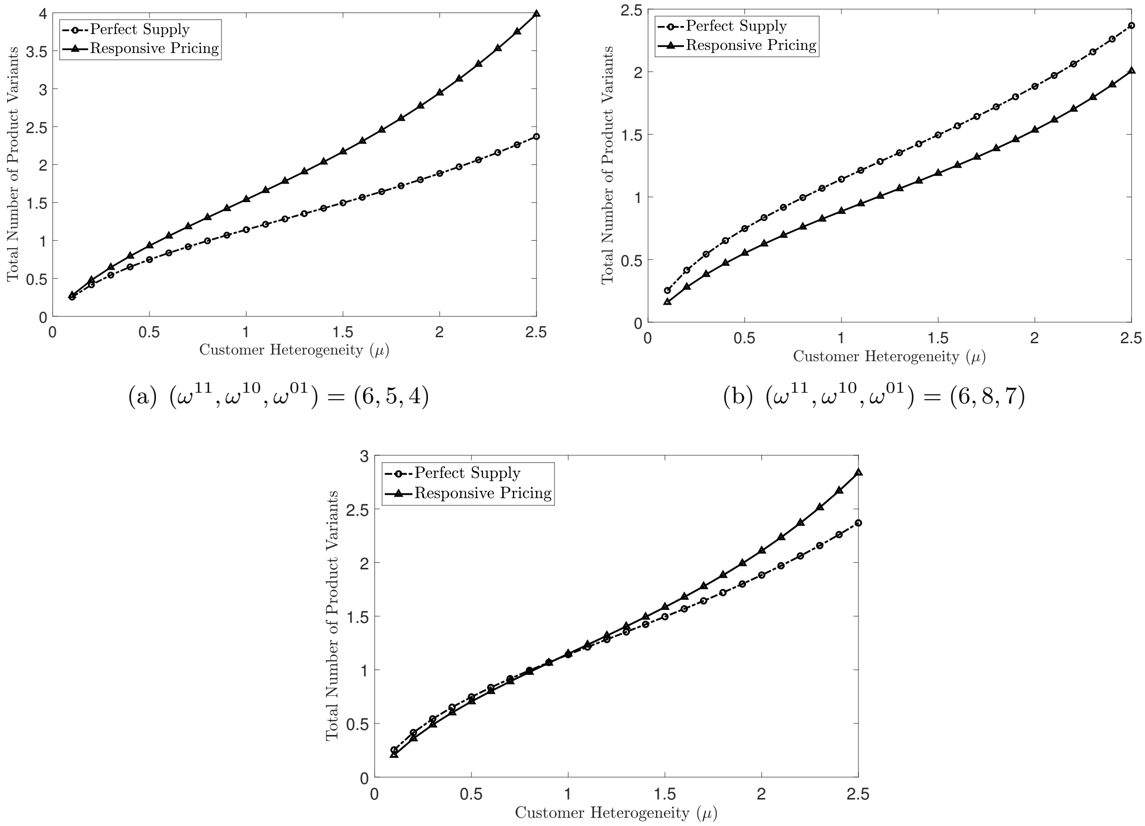

(a) $(\omega^{11}, \omega^{10}, \omega^{01}) = (6, 5, 4)$

(b) $(\omega^{11}, \omega^{10}, \omega^{01}) = (6, 8, 7)$

(c) $(\omega^{11}, \omega^{10}, \omega^{01}) = (6, 6, 6)$

Figure 1: Impact of Customer Heterogeneity (μ) and Utilities of Outside Option in States 10 (ω^{10}) and 01 (ω^{01}) on Total Product Variety

where PIP stands for percent improvement in profit. Similarly, to determine the percent profit improvement due to responsive pricing, we first estimate the benefit of responsive pricing as $Z_r^* - Z_s^*$, where Z_r^* and Z_s^* are as defined in Eq. (3.3) and Eq. (5.8), respectively. Subsequently, we divide this value by Z_s^* to obtain the profit improvement in percentage; that is,

$$\text{PIP (Responsive Pricing)} = \frac{Z_r^* - Z_s^*}{Z_s^*} \times 100. \tag{5.11}$$

Additionally, we examine how both the increments vary as a function of μ, ω^{10}, ρ, and γ. The range for each parameter remains the same as in the previous subsection.

The outcomes from this set of experiments are presented in Figure 5. A summary of insights is as follows.

1. The incremental profit due to strategic selection of product variety is significantly more than that of responsive pricing. (Observe that the scales for the two PIPs are different in Figure 5.) In fact, the ratio of the PIPs due to the two strategies could be as high as three orders of magnitude. This means that the residual value added by responsive pricing after product variety redesign is

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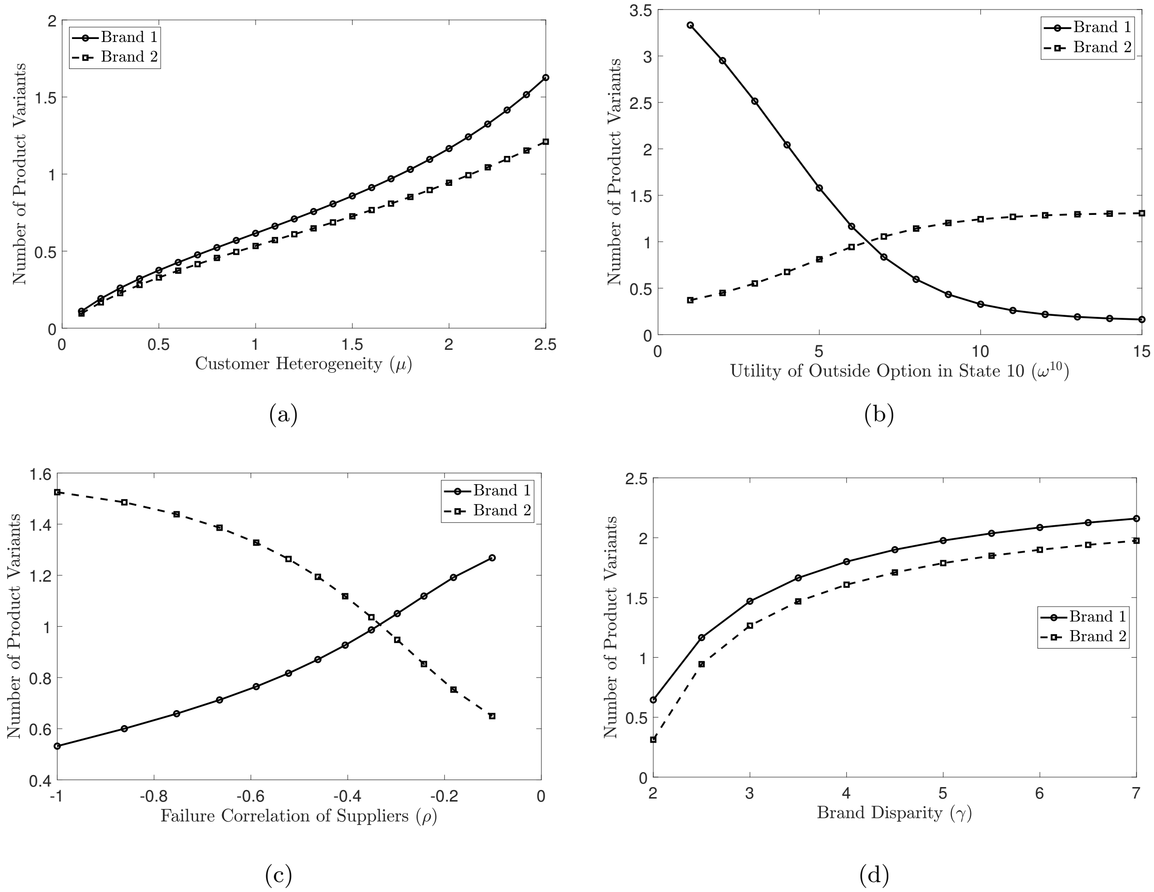

Figure 2: Number of Variants for Each Brand in the Responsive Pricing Scenario as a Function of Model Parameters

small. Thus, we can conclude that product variety redesign is a more effective lever to manage supply disruptions than responsive pricing. The shapes of the PIP curves for product variety are primarily determined by how different the optimal product variety is for the static pricing versus perfect supply models. Similarly, the magnitude of price difference between static and responsive pricing scenarios in different states primarily determines shapes of the PIP curves corresponding to the responsive pricing strategy. In what follows, these observations are used to explain the shapes of the PIP curves.

2. As the customer heterogeneity increases, the PIP due to strategic selection of product variety generally increases (Figure 5(a)). The PIP curve mirrors the difference in the number of product variants between the static pricing and perfect supply models. (See, for example, Figure 1(c). The number of variants for each brand is nearly identical in the static and responsive pricing models.) Since the difference in the number of product variants increases between the static pricing and perfect supply scenarios with μ, the percent profit improvement also increases with μ. Similarly, the benefit of responsive pricing increases with μ. As we saw in the previous subsection, in general, higher customer heterogeneity causes the retailer to increase the number of variants for both brands.

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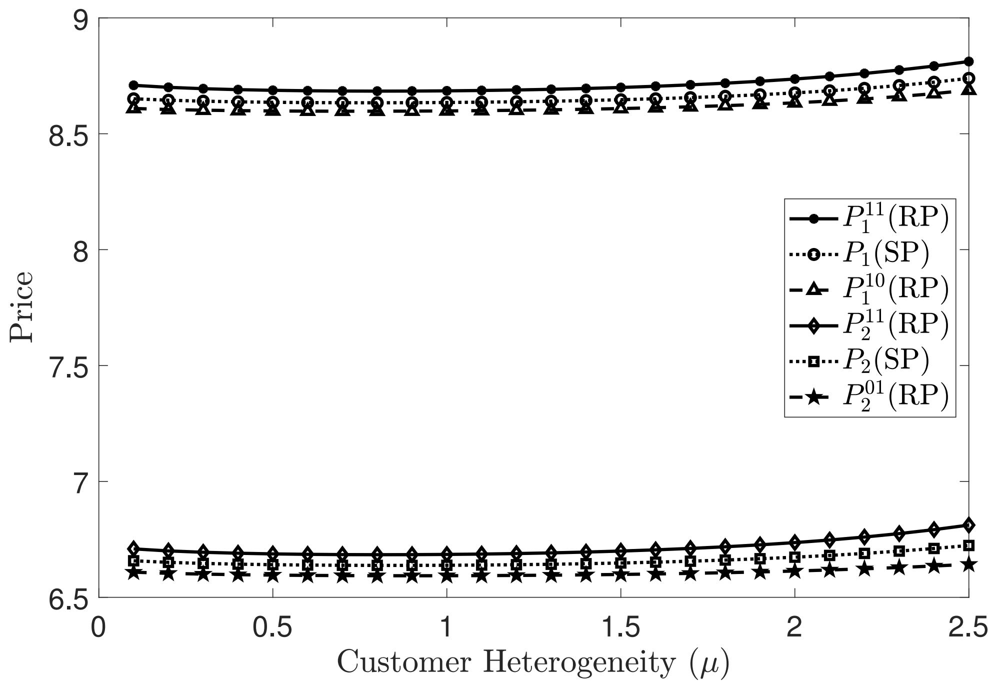
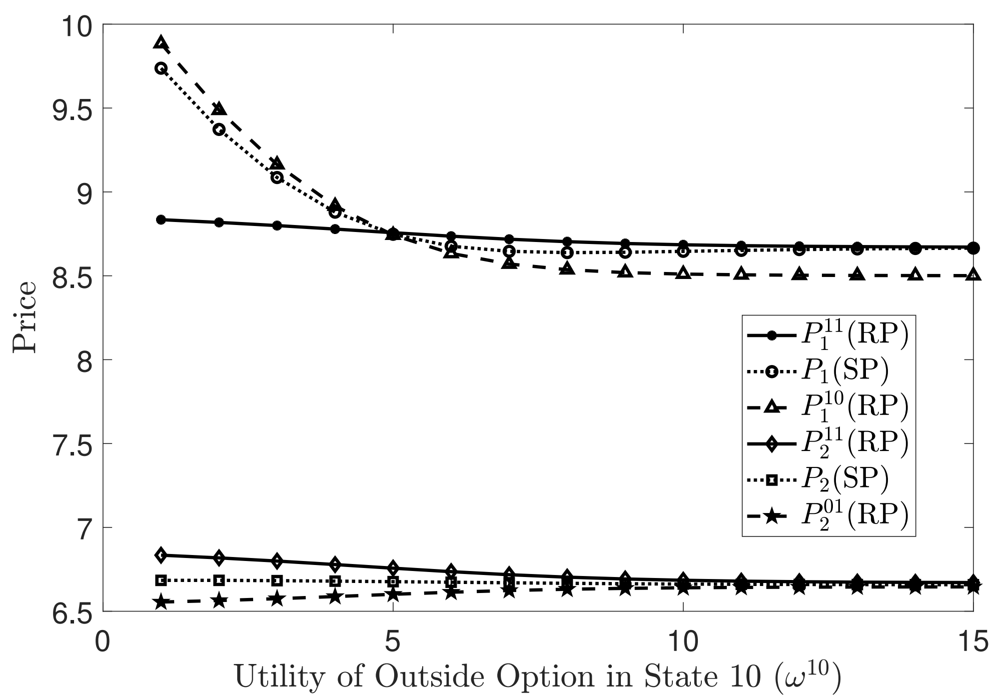


(a)



(b)

Figure 3: Optimal Prices in Static Pricing (SP) and Responsive Pricing (RP) Scenarios as a Function of Model Parameters

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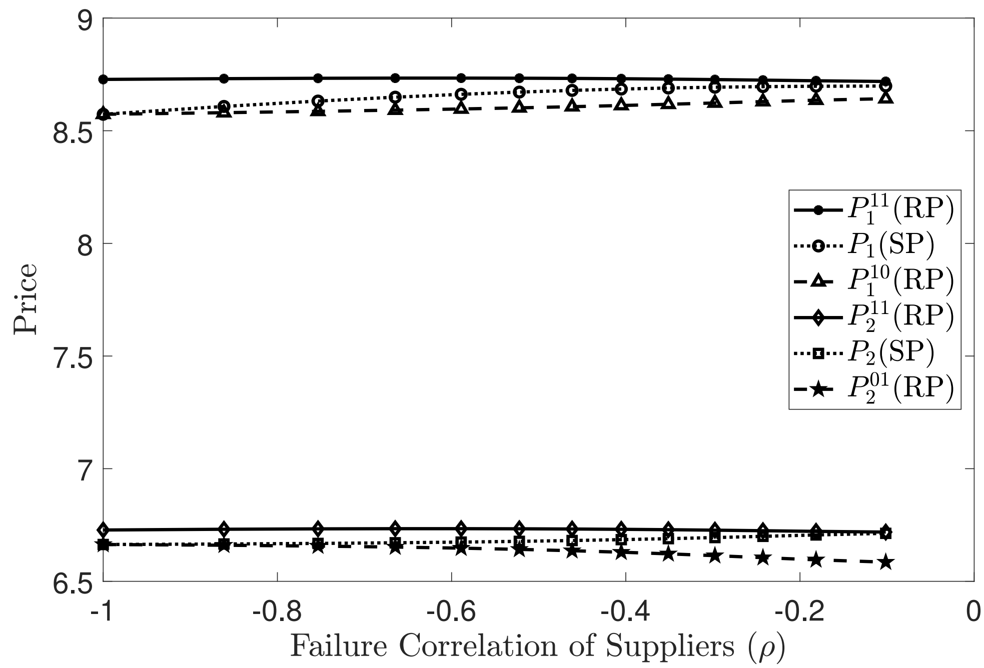
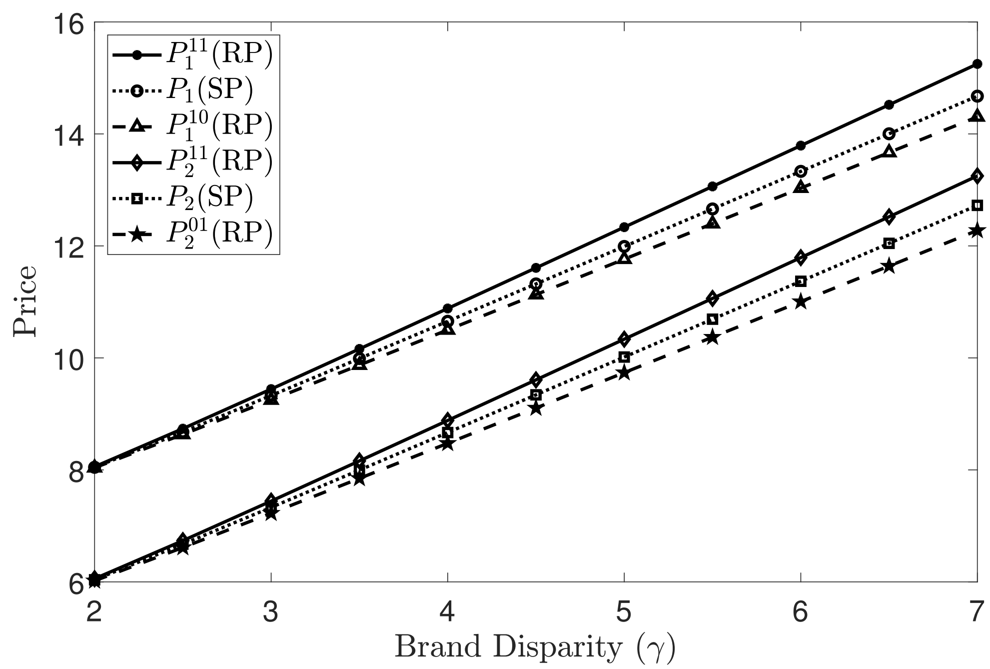


(a)



(b)

Figure 4: Optimal Prices in Static Pricing (SP) and Responsive Pricing (RP) Scenarios as a Function of Model Parameters

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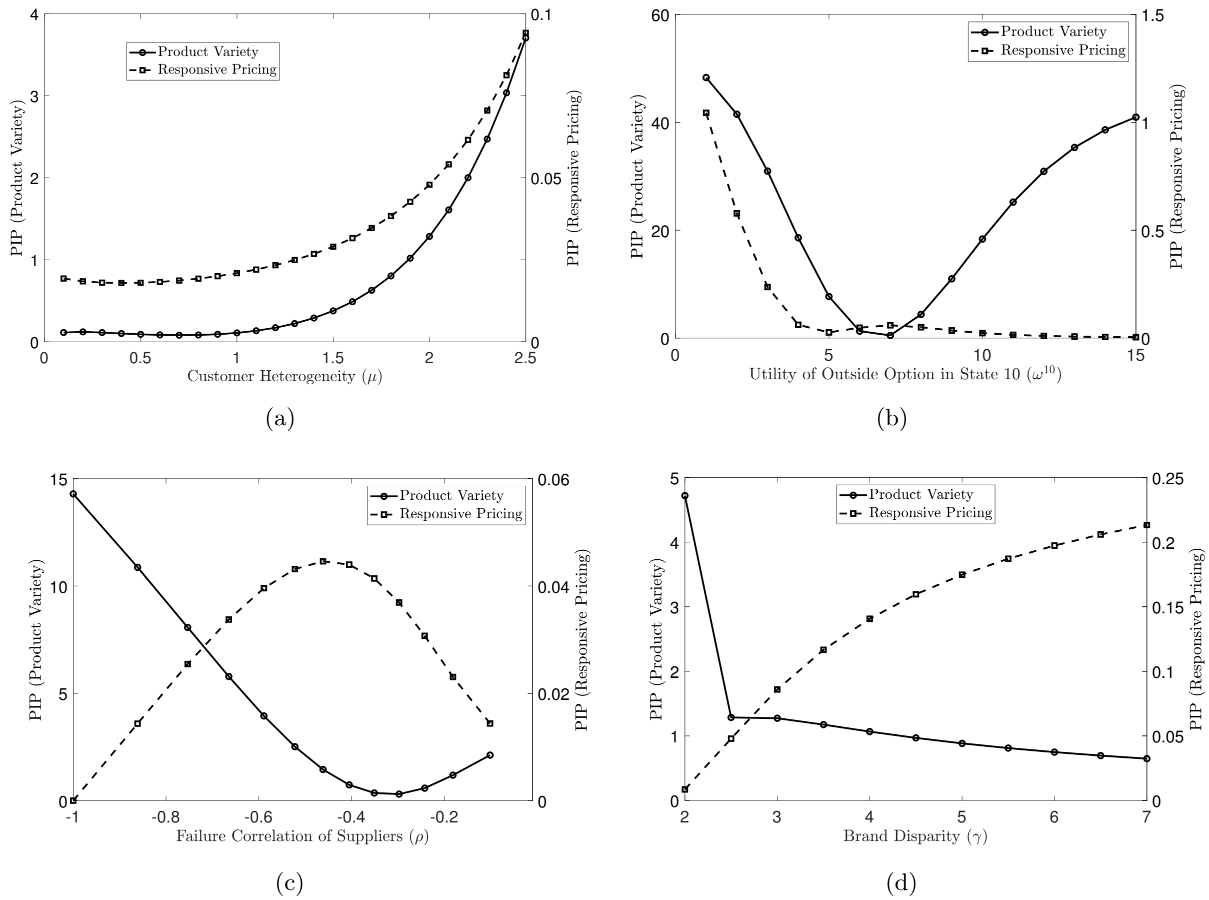

Figure 5: Profit Improvement due to Product Variety and Responsive Pricing Strategies as a Function of Model Parameters

This also means that the difference in the total number of product variants available in different states (for example, 11 and 10) increases. For example, the difference in the number of variants available in states 11 and 10 is n_2, and this difference increases with μ. This implies that it becomes more attractive to charge different prices for brand 1 in states 11 and 10 since price for a brand increases with the total number of variants offered (Proposition 5). Figure 3(a) confirms this rationale: the difference between optimal prices for brand 1 in states 11 and 10 is increasing in μ.

3. The PIP due to strategic selection of product variety has a unimodal shape with respect to ω^{10}, with a trough occurring at $\omega^{10} = 7$ (Figure 5(b)). As ω^{10} increases, the optimal number of variants for brand 1 decreases and that for brand 2 increases. For $\omega^{10} = 7$, these values are close to the corresponding values for the perfect supply model. (Recall that ω^{10} does not affect product variety in the perfect supply model.) This is why the PIP is close to zero at $\omega^{10} = 7$. For other values of ω^{10}, the optimal variety in the perfect supply and static pricing cases are differentiated, which results in greater PIP. On the other hand, the PIP for the responsive pricing strategy essentially decreases as ω^{10} increases. The relationship is monotonic but for a small peak when $\omega^{10} = 7$. The profit differential is high for low values of ω^{10} since it is optimal to carry a larger variety for such

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values of ω^{10} in the responsive pricing scenario. For larger values of ω^{10}, however, relative values of the optimal prices appear to explain the shape of the curve (see Figure 3(b)). In particular, the profit improvement is close to zero when $\omega^{10} = 5$ since P_1^{11} and P_1^{10} in the responsive pricing scenario become equal to the P_1 in the static pricing scenario. The prices diverge again as ω^{10} increases, which may explain the small peak corresponding to $\omega^{10} = 7$.

4. The impact of failure correlation of suppliers occurs in a roughly U-shaped fashion for the product variety strategy and an inverted U-shaped manner for the responsive pricing strategy (see Figure 5(c)). Given the modeling of correlation, the number of variants of brand 1 increases and that of brand 2 decreases as correlation increases, as discussed in item 3 in Subsection 5.2.3. These values are closest to the corresponding numbers for the perfect supply model when correlation is equal to -0.3. This is why the PIP for product variety has a trough when correlation is -0.3. The PIP takes greater value elsewhere since the product variety in the static pricing model is more differentiated from that in the perfect supply model. To understand the PIP due to responsive pricing, recall from item 3 in Subsection 5.2.3 that π^{11} increases and π^{01} decreases with correlation. In state 11, the profit margins for both brands are higher in the responsive pricing scenario compared to the static pricing scenario, though the gap reduces with correlation (Figure 4(a)). As correlation increases from -1, the increase in π^{11}, which appears as a scaling factor in (Eq. 3.3), appears to dominate the closure of the profit margin gap resulting in an increase in the profit improvement. The peak occurs at a correlation value of -0.5, implying that the reduction in profit margin gap dominates the increase in π^{11} for higher (less negative) correlation values. Moreover, greater profit margins in the static pricing scenario in states 10 and 01, which increase with correlation, also diminish the profit advantage of the responsive pricing scenario as correlation increases.

5. As brand disparity γ increases, the percent profit improvement decreases for the product variety strategy but increases for the responsive pricing strategy. For the product variety strategy, $Z_s^* - Z_s(n_1^{PS}, n_2^{PS})$ increases with γ since the difference in the number of variants (brand-wise as well as in aggregate) increases between the static pricing and perfect supply scenarios as γ increases. However, the denominator in Eq. (5.10), $Z_s(n_1^{PS}, n_2^{PS})$, increases at an even faster rate with γ, which leads to a decreasing curve for the PIP. On the other hand, the explanation for the responsive pricing strategy remains the same as that for μ; that is, the difference in product variety between states 11 and 10, n_2^*, and the difference between states 11 and 01, n_1^*, increases with γ (see Figure 2(d)), which makes it more attractive to charge differential prices in different states.

5.4 Implications for Inventory Requirements

In the nested logit model, the number of product variants and price together determine the probability that an incoming customer purchases some product variant. Since this probability in turn determines the distribution of the total customer demand over all the product variants, the variety and pricing decisions have implications for the inventory decisions as well as the safety stock, which is generally taken to be proportional to the standard deviation (SD) of demand. Accordingly, in this subsection,

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we compute the SD of total demand over all the product variants and examine how it changes due to responsive pricing. We also conduct a sensitivity analysis relative to several model parameters.

We begin by deriving the variance of total customer demand in a period in which both suppliers are available. Recall that if n_1 variants of brand 1 and n_2 variants of brand 2 are offered at respective prices of P_1^{11} and P_2^{11} in a period, then the total market share, which is equivalent to the probability that an incoming customer purchases a variant from either of the brands, is

$$\alpha_t^{11} = \frac{\left(n_1 e^{\frac{a_1 - P_1^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}} + \left(n_2 e^{\frac{a_2 - P_2^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + \left(n_1 e^{\frac{a_1 - P_1^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}} + \left(n_2 e^{\frac{a_2 - P_2^{11}}{\mu}}\right)^{\frac{\mu}{\gamma}}} = Q_1^{11} + Q_2^{11}.$$

If the number of customers in period t, N_t, is deterministic, then the number of customers who purchase the product is binomial distributed with parameters N_t and α_t^{11}. The variance of demand is thus equal to $N_t \alpha_t^{11}(1 - \alpha_t^{11})$.

The above expression can be easily modified when one of the suppliers is unavailable. Suppose that the probabilities that an incoming customer purchases a product variant in states 10 and 01 are denoted by α_t^{10} and α_t^{01}, respectively. The values of α_t^{10} and α_t^{01} are equal to

$$\alpha_t^{10} = \frac{\left(n_1 e^{\frac{a_1 - P_1^{10}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}} + \left(n_1 e^{\frac{a_1 - P_1^{10}}{\mu}}\right)^{\frac{\mu}{\gamma}}} = Q_1^{10}, \qquad \alpha_t^{01} = \frac{\left(n_2 e^{\frac{a_2 - P_2^{01}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{01}}{\gamma}} + \left(n_2 e^{\frac{a_2 - P_2^{01}}{\mu}}\right)^{\frac{\mu}{\gamma}}} = Q_2^{01},$$

and conditional on the state, the demand is again binomial distributed with parameters N_t and α_t^{ij}.

By conditioning on the Markov chain being in state ij, the variance of demand in a generic period t can be obtained as

$$Var(D_t) = \sum_{ij=11,10,01} \pi^{ij} \left(N_t \alpha_t^{ij}(1 - \alpha_t^{ij}) + N_t^2 (\alpha_t^{ij})^2\right) - \left(\sum_{ij=11,10,01} \pi^{ij} N_t \alpha_t^{ij}\right)^2.$$

This expression holds for both static pricing and responsive pricing models. Whereas in the static pricing model, the price used in the computation of α_t^{ij} remains the same for all states ij; in the responsive pricing model, the price varies with the state. The above approach, with some modifications, can also be utilized to compute the variance of demand for any other arrival distribution, for example, Poisson distribution.

In Table 3, we report the SD of total demand in a generic period t for the static pricing and responsive pricing models as a function of four different model parameters. We experimented with the number of arrivals being both deterministic and Poisson distributed. Since the results in both cases provide identical insights, we report only the deterministic case. The base parameter values for the table remain the same as in Table 2.

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Observe from the table that responsive pricing results in a lower SD than static pricing for all the parameters, which implies that the safety stock requirements decline if responsive pricing is utilized. The reason is as follows. When responsive pricing is used, as opposed to static pricing, the spread in probabilities of an arriving customer purchasing a variant across various states is smaller. More precisely, $\alpha_t^{11}, \alpha_t^{10}$, and α_t^{01} are closer to each other in the presence of responsive pricing than static pricing. Therefore, the likelihood of a customer purchasing a variant depends less significantly on the state of the Markov chain for responsive pricing compared to static pricing, which results in lower standard deviation of total customer demand.

μ	Responsive Pricing	Static Pricing
0.5	23.65	24.85
1.0	23.39	24.57
1.5	24.86	26.21
2.0	28.50	30.33
2.5	35.87	38.82
ω^{01}	Responsive Pricing	Static Pricing
1.0	196.22	207.14
4.0	56.31	57.67
7.0	34.53	35.86
10.0	42.49	42.78
13.0	43.38	43.47
ω^{10}	Responsive Pricing	Static Pricing
1.0	218.41	232.02
4.0	68.45	70.09
7.0	33.93	35.49
10.0	43.41	43.78
13.0	44.36	44.46
γ	Responsive Pricing	Static Pricing
2.5	28.50	30.33
3.5	46.52	52.89
4.5	54.39	64.27
5.5	58.53	70.88
6.5	61.02	75.17

Table 3: Standard Deviation of Total Demand in a Generic Period for Base Parameter Values in Table 2

6 Extension: Responsive Variety

In this section, we explore the scenario in which not only the price but also the number of variants for an available brand could be updated when the supplier for the other brand becomes unavailable. Similar to price, we include a superscript ij to n_k, provided the brand is available in state ij. All the other notations remain unchanged. Expressions for probability of selection for brand k, Q_k^{ij}, in different states also remain unchanged. Putting everything together, the optimal expected profit

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over the planning horizon, Z_v^*, is equal to

$$\begin{aligned} Z_v^* \quad = \quad & \max_{n_1^{11}, n_2^{11}, n_1^{10}, n_2^{01} \geq 0} \left\{ -F_1 \max(n_1^{11}, n_1^{10}) - F_2 \max(n_2^{01}, n_2^{11}) \right. \\ & + \sum_{t=1}^{T} \left\{ \pi^{11} \max_{P_1^{11}, P_2^{11} \geq 0} Z_t^{11}(n_1^{11}, n_2^{11}, P_1^{11}, P_2^{11}) \right. \\ & \left. \left. + \pi^{10} \max_{P_1^{10} \geq 0} Z_t^{10}(n_1^{10}, P_1^{10}) + \pi^{01} \max_{P_2^{10} \geq 0} Z_t^{01}(n_2^{01}, P_2^{10}) + \pi^{00} Z_t^{00} \right\} \right\}, \end{aligned}$$

where Z_t^{11}, Z_t^{10}, and Z_t^{01} are as defined in Subsection 3.3. Let $G_v(n_1^{11}, n_2^{11}, n_1^{10}, n_2^{01})$ be the maximand in the above formulation. We refer to this model as the *responsive variety model*.

We find that all the analytical results presented in Section 4 except Proposition 3 extend to the above model with suitable modifications. Proposition 3 may break down since the number of variants of brand 1 (brand 2) in state 10 (state 01) may now exceed that in state 11, which may induce the corresponding price in state 10 (state 01) to exceed the price in state 11 even when $\omega^{10} \geq \omega^{11}$ ($\omega^{01} \geq \omega^{11}$). However, the result will hold for state 10 if $n_1^{10*} = n_1^{11*}$ and for state 01 if $n_2^{01*} = n_2^{11*}$. In contrast, Theorem 6 is an example of a result that extends directly, that is, G_v is jointly concave in $(n_1^{11}, n_2^{11}, n_1^{10}, n_2^{01})$.

We conduct computational experiments to evaluate the incremental profit improvement due to the responsive variety strategy. We refer to this metric as PIP (Responsive Variety) and compute it as the percentage profit improvement of the responsive variety model with respect to the responsive pricing model; that is,

$$\text{PIP (Responsive Variety)} = \frac{Z_v^* - Z_r^*}{Z_r^*} \times 100.$$

Similar to Section 5, we compute the PIP due to responsive variety by varying μ, ω^{10}, ρ (failure correlation), and γ. We find that the PIP ranges between 0 and 8.6% (see Figure 6 in the Appendix). The PIP is sensitive to changes in μ and ω^{10}, but is relatively insensitive to γ and ρ.

We also conduct experiments to understand how the responsiveness of product variety influences the relationship between optimal decision variables and model parameters. We find that the relationships remain broadly the same as in Figures 1, 3 and 4 (Figures 7 and 8 in the Appendix show a comparison). The only notable difference is that the number of variants for one of the brands may become significantly smaller in state 11 for some parameter combinations (e.g., for high values of μ, the number of variants of brand 1 reduces significantly in the responsive variety strategy compared to the responsive pricing strategy). Since the number of variants of brand 1 (brand 2) are no longer tied together in states 10 and 11 (states 01 and 11), the retailer's incentive to carry large number of variants of both brands is diminished in state 11. The optimal price for a brand may also reduce in such cases, though otherwise, prices exhibit similar pattern as in Figures 3 and 4.

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7 Conclusions

We consider a retailer that offers multiple product variants of two brands. Due to supply disruptions, the retailer may not be able to offer both the brands every period. In this context, we examine the relative role played by the retailer's product variety and price decisions in countering supply disruptions. Key insights from this study are as follows.

1. Retailers should pay special attention to product variety management as a strategy to safeguard against demand losses caused by the unavailability of products and brands. With effective product variety in place, the marginal improvement in profit due to price adjustments appears to be relatively small. This means that leaving prices unchanged in the event of supply breakdown is unlikely to cost the retailer much. Moreover, unchanged prices may also be good for public relations.

One explanation for greater impact of product variety management compared to responsive pricing is as follows. The effect of price is local in the sense that reduced price may induce some of the customers whose expected utility is marginally lower than that of the outside option to purchase an available product. Moreover, there is a limit to how much the price can be reduced since it affects the margin. In contrast, an additional variant may attract a whole new set of customers to shop from the retailer. Consequently, an adjustment in product variety has a greater effect on expected profit than responsive pricing.

2. Even though product variety may look substantially different when supply breakdowns are accounted for compared to when they are not, it may not be optimal to increase redundancy in the product variety in the presence of supply breakdowns. In fact, it is possible that the product variety may shrink. This may occur, for example, when either customers are relatively homogeneous or unavailability of a brand within the store makes it more attractive for the customers to switch to other stores.

3. All else being equal, the optimal price for variants of an available brand decreases when the other brand is not available. This also remains the case when a supply disruption makes the outside option more attractive. Interestingly, the converse of this insight is not true; it may not be optimal to increase price even if supply disruption makes the outside option less attractive. However, a less attractive outside option does increase the profit of the retailer in any period regardless of the state of the supplier availability.

4. Equal-margin pricing policy, which has been shown to be optimal in many settings, may become sub-optimal when supplier vulnerability to disruptions is considered.

5. Regardless of the degree of supplier unreliability and the quality of suppliers' brands, product variants of different brands are substitutable for each other. Specifically, this means that the marginal profit due to an additional variant of a brand decreases as the number of variants of the other brand increases.

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6. Although responsive pricing may not be useful in guarding against supply disruptions, it may reduce the variability of demand substantially. Since safety stock is typically proportional to the standard deviation of demand, the use of responsive pricing may result in a diminished safety stock leading to inventory carrying cost savings.

7. Whenever possible, the retailer should try to modulate the number of variants of each brand depending upon the supplier availability. Our experiments show that the responsiveness in variety could lead to a marginal profit improvement of up to 8.6%. This strategy appears to be particularly effective when customers are heterogeneous in their preference for product.

One future direction is to identify the extent to which our results extend to a competitive setting. In this model, two brands, which belong to two suppliers that are prone to disruption, compete (either on an online marketplace or through authorized exclusive retailers) and the product variety and price decisions for each brand will then be determined in equilibrium. A comparison of equilibrium decisions in this model with the optimal decisions in this study will permit analysis on how our results modify in a competitive environment.

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8 Appendix

8.1 Proof of Proposition 2

Since proofs for both parts are similar, we present it only for part 1.

$$\frac{\partial Z^{10}(n_1, Q_1^{10})}{\partial Q_1^{10}} = E(N_t)\left\{(P_1^{10} - c_1) + Q_1^{10}\frac{\partial}{\partial Q_1^{10}}(P_1^{10} - c_1)\right\} = E(N_t)\left\{(P_1^{10} - c_1) - \gamma\left(\frac{1}{1 - Q_1^{10}}\right)\right\}.$$

Setting $\frac{\partial Z^{10}(n_1, Q_1^{10})}{\partial Q_1^{10}}$ equal to 0, we get $P_1^{10*} = c_1 + \gamma\left(\frac{1}{1-Q_1^{10*}}\right)$. Let $\Pi^{10} = (P_1^{10} - c_1)Q_1^{10}$. Substituting for P_1^{10*}, $\Pi^{10*} = \gamma\left(\frac{Q_1^{10*}}{1-Q_1^{10*}}\right)$, which means that $P_1^{10*} = \Pi^{10*} + c_1 + \gamma$. Using this equation and $Q_1^{10} = \frac{\left(n_1 e^{\frac{a_1 - P_1^{10}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}} + \left(n_1 e^{\frac{a_1 - P_1^{10}}{\mu}}\right)^{\frac{\mu}{\gamma}}}$, $\frac{\Pi^{10*}}{\gamma} = \frac{\left(n_1 e^{\frac{a_1 - c_1 - \gamma - \Pi^{10*}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}} = e^{-\frac{\Pi^{10*}}{\gamma}} \frac{\left(n_1 e^{\frac{a_1 - c_1 - \gamma}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}}$ and so $\Pi^{10*} = \gamma \cdot W\left(\frac{\left(n_1 e^{\frac{a_1 - c_1 - \gamma}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}}\right)$. Substituting Π^{10*} in the above expression for P_1^{10*}, we get $P_1^{10*} = \gamma \cdot W\left(\frac{\left(n_1 e^{\frac{a_1 - c_1 - \gamma}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}}\right) + c_1 + \gamma$.

8.2 Proof of Proposition 3

1. We present the proof only for the case $\omega^{10} \geq \omega^{11}$; the proof for the case $\omega^{01} \geq \omega^{11}$ is similar and hence omitted. For any given n_1 and n_2, the optimal price for brand 1 in states 10 and 11 is equal to

$$P_1^{10*} = \gamma W\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}}\right) + \gamma + c_1, \text{ and} \tag{8.12}$$

$$P_1^{11*} = \gamma W\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - \gamma}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) + \gamma + c_1, \tag{8.13}$$

respectively. Since $\omega^{10} \geq \omega^{11}$, $e^{\frac{\omega^{10}}{\gamma}} \geq e^{\frac{\omega^{11}}{\gamma}}$. Further, since $n_1, \ n_2 \geq 0$,

$$\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}} \leq \frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}} \leq \frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - \gamma}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}$$

and so

$$W\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}}\right) \leq W\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - \gamma}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - \gamma}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right),$$

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where the last inequality follows from the fact that $W(z)$ is an increasing function for $z \geq 0$. Thus, we can conclude that

$$P_1^{10*} \leq P_1^{11*}.$$

2. Consider the expected profit in period t in which the Markov chain is in state 11:

$$Z_t^{11}(n_1, n_2, Q_1^{11}, Q_2^{11}) = E(N_t)\left[(P_1^{11} - c_1)Q_1^{11} + (P_2^{11} - c_2)Q_2^{11}\right] - f(n_1, n_2),$$

where P_1^{11} and P_2^{11} are as given by Eq. (4.5). Differentiating the above function with respect to Q_1^{11}, we get

$$\begin{aligned}
\frac{\partial Z_t^{11}}{\partial Q_1^{11}} &= E(N_t)\left[(P_1^{11} - c_1) + Q_1^{11}\frac{\partial}{\partial Q_1^{11}}(P_1^{11} - c_1) + Q_2^{11}\frac{\partial}{\partial Q_1^{11}}(P_2^{11} - c_2)\right] \\
&= E(N_t)\left[(P_1^{11} - c_1) + \gamma Q_1^{11}\left(\frac{-1}{1 - Q_1^{11} - Q_2^{11}} + \frac{-1}{Q_1^{11}}\right) + \gamma Q_2^{11}\left(\frac{-1}{1 - Q_1^{11} - Q_2^{11}}\right)\right] \\
&= E(N_t)\left[(P_1^{11} - c_1) - \gamma - \gamma\left(\frac{Q_1^{11} + Q_2^{11}}{1 - Q_1^{11} - Q_2^{11}}\right)\right].
\end{aligned}$$

Setting $\frac{\partial Z_t^{11}}{\partial Q_1^{11}} = 0$ yields $P_1^{11} - c_1 = \gamma\left(\frac{1}{1-Q_1^{11}-Q_2^{11}}\right)$. Similarly, $\frac{\partial Z_t^{11}}{\partial Q_2^{11}} = 0$ yields $P_2^{11} - c_2 = \gamma\left(\frac{1}{1-Q_1^{11}-Q_2^{11}}\right)$. Recall that P_1^{11*} and P_2^{11*} are the optimal values of P_1^{11} and P_2^{11} given n_1 and n_2. From the above equations (and also as stated in Lemma 1), $P_1^{11*} - c_1 = P_2^{11*} - c_2$.

Using Lemma 1, $r^{11*} := \Pi^{11*} + \gamma$, and $Q_k^{11*} = \dfrac{\left(n_k e^{\frac{a_k - c_k - r^{11*}}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + \sum_{i=1}^{2}\left(n_i e^{\frac{a_i - c_i - r^{11*}}{\mu}}\right)^{\frac{\mu}{\gamma}}}, \qquad k = 1, 2.$

Using the relationship between P_k^{11*} and Q_k^{11*} and the above expression for Q_k^{11*},

$$r^{11*} = \gamma\left(\frac{1}{1 - Q_1^{11*} - Q_2^{11*}}\right) = \gamma\left(\frac{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right). \tag{8.14}$$

Similarly,

$$r^{10*} = \gamma\left(\frac{1}{1 - Q_1^{10*}}\right) = \gamma\left(\frac{e^{\frac{\omega^{10}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{10*}}{\gamma}}}{e^{\frac{\omega^{10}}{\gamma}}}\right), \text{ and} \tag{8.15}$$

$$r^{01*} = \gamma\left(\frac{1}{1 - Q_1^{01*}}\right) = \gamma\left(\frac{e^{\frac{\omega^{01}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{01*}}{\gamma}}}{e^{\frac{\omega^{01}}{\gamma}}}\right). \tag{8.16}$$

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Differentiating r^{11*} with respect to ω^{11},

$$\begin{aligned}
\frac{\partial r^{11*}}{\partial \omega^{11}} &= -\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial \omega^{11}} - \left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) \\
&= -(Q_1^{11*} + Q_2^{11*}) \qquad (8.17) \\
&\leq 0.
\end{aligned}$$

Thus, r^{11*} is decreasing in ω^{11}. Similarly, we can prove that $\frac{\partial r^{10*}}{\partial \omega^{10}} = -Q_1^{10*}$ and $\frac{\partial r^{01*}}{\partial \omega^{01}} = -Q_2^{01*}$. Thus, the optimal margins r^{10*} and r^{01*} are decreasing in ω^{10} and ω^{01}, respectively.

8.3 Proof of Proposition 5

Consider r^{11*}. Suppose we identify a value $\Pi^*(n_1, n_2)$ that satisfies Eq. (4.6). If n_1 increases by $\delta > 0$, then Lambert's W function will increase, which means that Π^* (and hence the margin) will also increase. Similarly, Π^* increases with n_2. Thus, the optimal margin increases as either n_1 or n_2 increases. Similarly, we can prove the result for r^{10*} (with respect to n_1) and r^{01*} (with respect to n_2).

Next, differentiating r^{11*} with respect to a_1,

$$\begin{aligned}
\frac{\partial r^{11*}}{\partial a_1} &= \left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) - \left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial a_1} \\
&= \left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}\right) \qquad (8.18) \\
&= Q_1^{11*} \\
&\geq 0.
\end{aligned}$$

Thus, r^{11*} is increasing in a_1. Similarly, we can prove that r^{10*} is increasing in a_1, and r^{11*} and r^{10*} are increasing in a_2. In the same manner, we can prove that r^{11*} and r^{10*} are decreasing in c_1, and r^{11*} and r^{01*} are decreasing in c_2.

Finally, differentiating r^{11*} with respect to μ,

$$\begin{aligned}
\frac{\partial r^{11*}}{\partial \mu} &= \left(\frac{\ln(n_1) n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + \ln(n_2) n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) - \left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial \mu} \\
&= \left(\frac{\ln(n_1) n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + \ln(n_2) n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}\right) \\
&= \ln(n_1) Q_1^{11*} + \ln(n_2) Q_2^{11*} \qquad (8.19) \\
&\geq 0.
\end{aligned}$$

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Since $n_1, n_2 \geq 1$, r^{11*} is increasing in μ. Similarly, we can prove that r^{10*} and r^{01*} are increasing in μ when $n_1 \geq 1$ and $n_2 \geq 1$, respectively.

8.4 Proof of Theorem 6

Let the optimal profit in period t in which the Markov chain is in state 11 be denoted by $Y_t^{11}(n_1, n_2)$. Thus,

$$Y_t^{11} = \max_{P_1^{11}, P_2^{11} \geq 0} Z_t^{11}(n_1, n_2, P_1^{11}, P_2^{11}).$$

By definition,

$$Y_t^{11}(n_1, n_2) = E(N_t) r^{11*} \{Q_1^{11*} + Q_2^{11*}\} - f(n_1 + n_2) \tag{8.20}$$

where $r^{11*} = \gamma\Big(\frac{1}{1 - Q_1^{11*} - Q_2^{11*}}\Big)$. Substituting for r^{11*} in the above equation using Eq. (8.14),

$$Y_t^{11}(n_1, n_2) = E(N_t)\gamma\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) - f(n_1 + n_2). \tag{8.21}$$

Differentiating with respect to n_1,

$$\begin{aligned}\frac{\partial Y_t^{11}}{\partial n_1} &= -f'(n_1 + n_2) + E(N_t)\mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) \\ &\quad - E(N_t)\frac{\partial r^{11*}}{\partial n_1}\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right).\end{aligned} \tag{8.22}$$

Differentiating r^{11*} (using Eq. (8.14)) with respect to n_1,

$$\frac{\partial r^{11*}}{\partial n_1} = \mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) - \frac{\partial r^{11*}}{\partial n_1}\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right),$$

which leads to

$$\frac{\partial r^{11*}}{\partial n_1} = \mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}\right) \geq 0. \tag{8.23}$$

Substituting for $\frac{\partial r^{11*}}{\partial n_1}$ in Eq. (8.22) using Eq. (8.23),

$$\frac{\partial Y_t^{11}}{\partial n_1} = -f'(n_1 + n_2) + E(N_t)\mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}\right). \tag{8.24}$$

Similarly,

$$\frac{\partial Y_t^{11}}{\partial n_2} = -f'(n_1 + n_2) + E(N_t)\mu\left(\frac{n_2^{\frac{\mu}{\gamma}-1} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^{11*}}{\gamma}}}\right). \tag{8.25}$$

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Differentiating Y_t^{11} again with respect to n_1,

$$\begin{aligned}\frac{\partial^2 Y_t^{11}}{\partial n_1^2} = -f''(n_1+n_2) + E(N_t)\mu\left(\frac{\mu}{\gamma}-1\right)\left(\frac{n_1^{\frac{\mu}{\gamma}-2}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}+n_1^{\frac{\mu}{\gamma}}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}+n_2^{\frac{\mu}{\gamma}}e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}\right)\\
+E(N_t)\mu\left(-\frac{1}{\gamma}\right)\left(\frac{n_1^{\frac{\mu}{\gamma}-1}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}+n_1^{\frac{\mu}{\gamma}}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}+n_2^{\frac{\mu}{\gamma}}e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial n_1}\\
+E(N_t)\mu\left(-\frac{\mu}{\gamma}\right)\left[\frac{n_1^{\frac{\mu}{\gamma}-1}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}n_1^{\frac{\mu}{\gamma}-1}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{\left(e^{\frac{\omega^{11}}{\gamma}}+n_1^{\frac{\mu}{\gamma}}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}+n_2^{\frac{\mu}{\gamma}}e^{\frac{a_2-c_2-r^{11*}}{\gamma}}\right)^2}\right]\\
+E(N_t)\mu\left(\frac{1}{\gamma}\right)\left[\frac{n_1^{\frac{\mu}{\gamma}-1}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}n_1^{\frac{\mu}{\gamma}}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{\left(e^{\frac{\omega^{11}}{\gamma}}+n_1^{\frac{\mu}{\gamma}}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}+n_2^{\frac{\mu}{\gamma}}e^{\frac{a_2-c_2-r^{11*}}{\gamma}}\right)^2}\right]\frac{\partial r^{11*}}{\partial n_1}\\
+E(N_t)\mu\left(\frac{1}{\gamma}\right)\left[\frac{n_1^{\frac{\mu}{\gamma}-1}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}n_2^{\frac{\mu}{\gamma}}e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{\left(e^{\frac{\omega^{11}}{\gamma}}+n_1^{\frac{\mu}{\gamma}}e^{\frac{a_1-c_1-r^{11*}}{\gamma}}+n_2^{\frac{\mu}{\gamma}}e^{\frac{a_2-c_2-r^{11*}}{\gamma}}\right)^2}\right]\frac{\partial r^{11*}}{\partial n_1}.\end{aligned} \tag{8.26}$$

Substituting for $\frac{\partial r^{11*}}{\partial n_1}$ in Eq. (8.26) using Eq. (8.23),

$$\begin{aligned}\frac{\partial^2 Y_t^{11}}{\partial n_1^2} &= -f''(n_1+n_2) - E(N_t)\left(\frac{\partial r^{11*}}{\partial n_1}\right)^2\left[\left(\frac{\gamma-\mu}{\mu\gamma Q_1^{11}}\right)+\left(\frac{2-Q_1^{11}-Q_2^{11}}{\gamma}\right)\right]\\
&= -\left\{f''(n_1+n_2)+E(N_t)\left(\frac{\partial r^{11*}}{\partial n_1}\right)^2\left[\beta_1^{11}+\zeta^{11}\right]\right\}\\
&\leq 0,\end{aligned} \tag{8.27}$$

where $\beta_1^{11}=\frac{\gamma-\mu}{\mu\gamma Q_1^{11}}\geq 0$ and $\zeta^{11}=\frac{\left(2-Q_1^{11}-Q_2^{11}\right)}{\gamma}\geq 0$. Similarly,

$$\begin{aligned}\frac{\partial^2 Y_t^{11}}{\partial n_2^2} &= -f''(n_1+n_2) - E(N_t)\left(\frac{\partial r^{11*}}{\partial n_2}\right)^2\left[\left(\frac{\gamma-\mu}{\mu\gamma Q_2^{11}}\right)+\left(\frac{2-Q_1^{11}-Q_2^{11}}{\gamma}\right)\right]\\
&= -\left\{f''(n_1+n_2)+E(N_t)\left(\frac{\partial r^{11*}}{\partial n_2}\right)^2\left[\beta_2^{11}+\zeta^{11}\right]\right\}\\
&\leq 0,\end{aligned} \tag{8.28}$$

where $\beta_2^{11}=\frac{\gamma-\mu}{\mu\gamma Q_2^{11}}\geq 0$.

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Differentiating $\frac{\partial Y_t^{11}}{\partial n_1}$ with respect to n_2,

$$\begin{aligned}
\frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2} &= -f''(n_1+n_2) + E(N_t)\mu\left(-\frac{1}{\gamma}\right)\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial n_2} \\
&\quad + E(N_t)\mu\left(\frac{1}{\gamma}\right)\left[\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{\left(e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}\right)^2}\right]\frac{\partial r^{11*}}{\partial n_2} \\
&\quad + E(N_t)\mu\left(-\frac{\mu}{\gamma}\right)\left[\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} n_2^{\frac{\mu}{\gamma}-1} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{\left(e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}\right)^2}\right] \\
&\quad + E(N_t)\mu\left(\frac{1}{\gamma}\right)\left[\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{\left(e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}\right)^2}\right]\frac{\partial r^{11*}}{\partial n_2}.
\end{aligned} \tag{8.29}$$

Similar to Eq. (8.23), the first derivative of r^{11*} with respect to n_2 is given as

$$\frac{\partial r^{11*}}{\partial n_2} = \mu\left(\frac{n_2^{\frac{\mu}{\gamma}-1} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}\right) \geq 0. \tag{8.30}$$

Substituting for $\frac{\partial r^{11*}}{\partial n_1}$ and $\frac{\partial r^{11*}}{\partial n_2}$ in Eq. (8.29) using Eq. (8.23) and Eq. (8.30), respectively,

$$\begin{aligned}
\frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2} &= -\left\{f''(n_1+n_2) + E(N_t)\left(\frac{\partial r^{11*}}{\partial n_1}\right)\left(\frac{\partial r^{11*}}{\partial n_2}\right)\left(\frac{2-Q_1^{11}-Q_2^{11}}{\gamma}\right)\right\} \\
&= -\left\{f''(n_1+n_2) + E(N_t)\left(\frac{\partial r^{11*}}{\partial n_1}\right)\left(\frac{\partial r^{11*}}{\partial n_2}\right)\zeta^{11}\right\} \\
&\leq 0.
\end{aligned} \tag{8.31}$$

Consider now the optimal profit in period t in which the Markov chain is in state 10. Let it be denoted by $Y_t^{10}(n_1)$. Thus,

$$Y_t^{10} = \max_{P_1^{10} \geq 0} Z_t^{10}(n_1, P_1^{10}).$$

By definition,

$$Y_t^{10}(n_1) = Y_t^{11}(n_1, n_2)\bigg|_{n_2=0} \tag{8.32}$$

which implies that

$$\frac{\partial^2 Y_t^{10}}{\partial n_1^2} = \frac{\partial^2 Y_t^{11}}{\partial n_1^2}\bigg|_{n_2=0} \leq 0$$

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using Eq. (8.27). Similarly,

$$\frac{\partial^2 Y_t^{01}}{\partial n_2^2} = \left.\frac{\partial^2 Y_t^{11}}{\partial n_2^2}\right|_{n_1=0} \leq 0.$$

Next, we define the expected profit function over the planning horizon as

$$\begin{aligned} G(n_1, n_2) &= -F_1 n_1 - F_2 n_2 + \sum_{t=1}^{N} \left[\pi^{11} Y_t^{11}(n_1, n_2) + \pi^{10} Y_t^{10}(n_1) + \pi^{01} Y_t^{01}(n_2)\right] \\ &= -F_1 n_1 - F_2 n_2 + \sum_{t=1}^{N} G_t(n_1, n_2). \end{aligned} \tag{8.33}$$

Observe that it is enough to prove the concavity of $G_t(n_1, n_2)$. The Hessian matrix for $G_t(n_1, n_2)$ is equal to

$$\begin{aligned} H(G_t(n_1, n_2)) &= \begin{bmatrix} \frac{\partial^2 G_t}{\partial n_1^2} & \frac{\partial^2 G_t}{\partial n_1 \partial n_2} \\ \frac{\partial^2 G_t}{\partial n_1 \partial n_2} & \frac{\partial^2 G_t}{\partial n_1^2} \end{bmatrix} \\ &= \begin{bmatrix} \pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1^2} + \pi^{10} \frac{\partial^2 Y_t^{10}}{\partial n_1^2} & \pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2} \\ \pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2} & \pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_2^2} + \pi^{01} \frac{\partial^2 Y_t^{01}}{\partial n_2^2} \end{bmatrix}. \end{aligned} \tag{8.34}$$

The first principal minor of the matrix $H(G_t(n_1, n_2))$ is equal to

$$\begin{aligned} Minor\left[H(G_t(n_1, n_2))\right]_1 &= \pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1^2} + \pi^{10} \frac{\partial^2 Y_t^{10}}{\partial n_1^2} \\ &\leq 0. \end{aligned} \tag{8.35}$$

On the other hand, the second principal minor of the matrix $H(G_t(n_1, n_2))$ is equal to

$$\begin{aligned} Minor\left[H(G_t(n_1, n_2))\right]_2 &= \left(\pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1^2} + \pi^{10} \frac{\partial^2 Y_t^{10}}{\partial n_1^2}\right) \left(\pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_2^2} + \pi^{01} \frac{\partial^2 Y_t^{01}}{\partial n_2^2}\right) - \left(\pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2}\right)^2 \\ &= (\pi^{11})^2 \frac{\partial^2 Y_t^{11}}{\partial n_1^2} \frac{\partial^2 Y_t^{11}}{\partial n_2^2} + \pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1^2} \pi^{01} \frac{\partial^2 Y_t^{01}}{\partial n_2^2} \\ &+ \pi^{10} \frac{\partial^2 Y_t^{10}}{\partial n_1^2} \left(\pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_2^2} + \pi^{01} \frac{\partial^2 Y_t^{01}}{\partial n_2^2}\right) - \left(\pi^{11} \frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2}\right)^2 \end{aligned} \tag{8.36}$$

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On the right hand side, second and third terms are positive. Consider the sum of the first and the last terms (omitting $(\pi^{11})^2$).

$$
\begin{aligned}
&\frac{\partial^2 Y_t^{11}}{\partial n_1^2}\frac{\partial^2 Y_t^{11}}{\partial n_2^2} - \left(\frac{\partial^2 Y_t^{11}}{\partial n_1 \partial n_2}\right)^2 \\
&= E(N_t)^2 \left(\frac{\partial r^{11*}}{\partial n_1}\right)^2 \left(\frac{\partial r^{11*}}{\partial n_2}\right)^2 \left[\beta_1^{11}\beta_2^{11} + \zeta^{11}(\beta_1^{11} + \beta_2^{11}) + (\zeta^{11})^2\right] \\
&+ f''(n_1+n_2)E(N_t)\left[\beta_1^{11}\left(\frac{\partial r^{11*}}{\partial n_1}\right)^2 + \beta_1^{11}\left(\frac{\partial r^{11*}}{\partial n_2}\right)^2\right] \\
&+ f''(n_1+n_2)E(N_t)\zeta^{11}\left[\left(\frac{\partial r^{11*}}{\partial n_1}\right)^2 + \left(\frac{\partial r^{11*}}{\partial n_2}\right)^2\right] \\
&+ f''(n_1+n_2)^2 - f''(n_1+n_2)^2 - E(N_t)^2\left(\frac{\partial r^{11*}}{\partial n_1}\right)^2\left(\frac{\partial r^{11*}}{\partial n_2}\right)^2 (\zeta^{11})^2 \\
&- 2f''(n_1+n_2)E(N_t)\left(\frac{\partial r^{11*}}{\partial n_1}\right)\left(\frac{\partial r^{11*}}{\partial n_2}\right)(\zeta^{11}) \\
&= E(N_t)^2 \left(\frac{\partial r^{11*}}{\partial n_1}\right)^2 \left(\frac{\partial r^{11*}}{\partial n_2}\right)^2 \left[\beta_1^{11}\beta_2^{11} + \zeta^{11}(\beta_1^{11} + \beta_2^{11})\right] \\
&+ f''(n_1+n_2)E(N_t)\zeta^{11}\left[\left(\frac{\partial r^{11*}}{\partial n_1}\right) - \left(\frac{\partial r^{11*}}{\partial n_2}\right)\right]^2 \\
&+ f''(n_1+n_2)E(N_t)\left[\beta_1^{11}\left(\frac{\partial r^{11*}}{\partial n_1}\right)^2 + \beta_1^{11}\left(\frac{\partial r^{11*}}{\partial n_2}\right)^2\right] \\
&\geq 0.
\end{aligned} \tag{8.37}
$$

Since the second differential terms in Eq. (8.36) are all negative, using Eq. (8.37), we get

$$
Minor\left[H(G_t(n_1,n_2))\right]_2 \geq 0. \tag{8.38}
$$

Since the first and the second principal minors are negative and positive, respectively, the Hessian matrix $H(G_t(n_1,n_2))$ is negative semi-definite. This implies that $G_t(n_1,n_2)$ is jointly concave in (n_1,n_2), which, in turn, means that $G(n_1,n_2)$ is jointly concave in (n_1,n_2).

The submodularity of $G(n_1,n_2)$ follows from the submodularity of $Y_t^{11}(n_1,n_2)$ as $\frac{\partial^2 Y_t^{11}(n_1,n_2)}{\partial n_1 \partial n_2} \leq 0$.

8.5 Proof of Theorem 7

The first derivatives of $G(n_1,n_2)$ are equal to

$$
\frac{\partial G(n_1,n_2)}{\partial n_1} = \sum_{t=1}^{T}\left[\pi^{11}\frac{\partial Y_t^{11}}{\partial n_1} + \pi^{10}\frac{\partial Y_t^{10}}{\partial n_1}\right] - F_1, \tag{8.39}
$$

$$
\frac{\partial G(n_1,n_2)}{\partial n_2} = \sum_{t=1}^{T}\left[\pi^{11}\frac{\partial Y_t^{11}}{\partial n_2} + \pi^{01}\frac{\partial Y_t^{01}}{\partial n_2}\right] - F_2, \tag{8.40}
$$

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where Y_t^{ij} for $ij = \{11, 10, 01\}$ are as defined in the proof of Theorem 6. When $n_1^{RP}, n_2^{RP} > 0$, which is what we assume for simplicity, the above two equations should be equal to zero at $(n_1, n_2) = (n_1^{RP}, n_2^{RP})$.

Suppose that we reduce n_1 down to zero. Then, both of the first derivatives of $G(n_1, n_2)$ with respect to n_1 and n_2 will increase and become positive since both the second derivatives and the cross-partial of G are negative. Consequently, we need to increase n_2 to make $\frac{\partial G(0,n_2)}{\partial n_2} = 0$ again. $G(0, n_2)$ is concave in n_2 and so it has a unique maximizer. Thus,

$$n_2^{RP} \leq n_2^{\#}.$$

Similarly, we can show that

$$n_1^{RP} \leq n_1^{\#}.$$

8.6 Proof of Proposition 8

Recall the definition of Y_t^{11} from the proof of Theorem 6. Differentiating Y_t^{11} with respect to a_1,

$$\frac{\partial Y_t^{11}}{\partial a_1} = E(N_t)\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) - E(N_t)\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial a_1}$$

Substituting $\frac{\partial r^{11*}}{\partial a_1} = Q_1^{11*}$ (using Eq. (8.18)), we get

$$\begin{aligned}
\frac{\partial Y_t^{11}}{\partial a_1} &= E(N_t)\left(\frac{Q_1^{11*}}{1 - Q_1^{11*} - Q_2^{11*}}\right) - E(N_t)\left(\frac{Q_1^{11*} + Q_2^{11*}}{1 - Q_1^{11*} - Q_2^{11*}}\right)Q_1^{11*} \\
&= E(N_t)Q_1^{11*} \\
&\geq 0.
\end{aligned}$$

Thus, $Y_t^{11}(n_1, n_2)$ is increasing in a_1. Similarly, we can show that $Y_t^{10}(n_1)$ and $Y_t^{01}(n_2)$, which are defined in the proof of Theorem 6, are increasing in a_1. In turn, the optimal expected profit Z_r^* is also increasing in a_1. Using the same approach, we can show that the optimal expected profit is increasing in a_2 and decreasing in both c_1 and c_2.

Next, differentiating Y_t^{11} with respect to μ,

$$\begin{aligned}
\frac{\partial Y_t^{11}}{\partial \mu} = E(N_t)&\left(\frac{\ln(n_1) n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + \ln(n_2) n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right) \\
&- E(N_t)\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial \mu}.
\end{aligned}$$

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Substituting $\frac{\partial r^{11*}}{\partial \mu} = \ln(n_1)Q_1^{11*} + \ln(n_2)Q_2^{11*}$ (using Eq. (8.19)), we get

$$\begin{aligned}\frac{\partial Y_t^{11}}{\partial \mu} &= E(N_t)\left(\frac{\ln(n_1)Q_1^{11*} + \ln(n_2)Q_2^{11*}}{1 - Q_1^{11*} - Q_2^{11*}}\right) - E(N_t)\left(\frac{Q_1^{11*} + Q_2^{11*}}{1 - Q_1^{11*} - Q_2^{11*}}\right)\left(\ln(n_1)Q_1^{11*} + \ln(n_2)Q_2^{11*}\right) \\ &= E(N_t)\left(\ln(n_1)Q_1^{11*} + \ln(n_2)Q_2^{11*}\right) \\ &\geq 0\end{aligned}$$

when $n_1, n_2 \geq 1$. Thus, $Y_t^{11}(n_1, n_2)$ is increasing in μ when $n_1, n_2 \geq 1$. Similarly, we can show that $Y_t^{10}(n_1)$ and $Y_t^{01}(n_2)$ are increasing in μ, provided $n_1 \geq 1$ and $n_2 \geq 1$, respectively. This means that the optimal expected profit Z_r^* is also increasing in μ, provided $n_1^*, n_2^* \geq 1$.

Finally, differentiating $Y_t^{11}(n_1, n_2)$ with respect to ω^{11},

$$\begin{aligned}\frac{\partial Y_t^{11}}{\partial \omega^{11}} = E(N_t)\gamma\left(-\frac{1}{\gamma}\right)\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial \omega^{11}} + E(N_t)\gamma\left(-\frac{1}{\gamma}\right)\left(\frac{n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)\frac{\partial r^{11*}}{\partial \omega^{11}} \\ + E(N_t)\gamma(-1)\left(\frac{1}{\gamma}\right)\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right).\end{aligned}$$

Substituting $\frac{\partial r^{11*}}{\partial \omega^{11}} = -(Q_1^{11*} + Q_2^{11*})$ (using Eq. (8.17)), we get

$$\begin{aligned}\frac{\partial Y_t^{11}}{\partial \omega^{11}} &= E(N_t)\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1-c_1-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)(Q_1^{11*} - 1) + E(N_t)\left(\frac{n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2-c_2-r^{11*}}{\gamma}}}{e^{\frac{\omega^{11}}{\gamma}}}\right)(Q_2^{11*} - 1) \\ &\leq 0.\end{aligned}$$

Thus, $Y_t^{11}(n_1, n_2)$ is decreasing in ω^{11}. This means that the optimal expected profit Z_r^* is also decreasing in ω^{11}. Similarly, we can prove the result for ω^{10} and ω^{01}.

8.7 Proof of Proposition 9

Since both suppliers are assumed to be perfectly reliable, we drop the superscript ij in this proof. Consider the following function: $\Pi(Q_1, Q_2) = (P_1 - c_1)Q_1 + (P_2 - c_2)Q_2$, where P_1 and P_2 are as given by Eq. (4.5). Differentiating the above function with respect to Q_1, we get

$$\begin{aligned}\frac{\partial}{\partial Q_1}\Pi(Q_1, Q_2) &= (P_1 - c_1) + Q_1\frac{\partial}{\partial Q_1}(P_1 - c_1) + Q_2\frac{\partial}{\partial Q_1}(P_2 - c_2) \\ &= (P_1 - c_1) + \gamma Q_1\left(\frac{-1}{1 - Q_1 - Q_2} + \frac{-1}{Q_1}\right) + \gamma Q_2\left(\frac{-1}{1 - Q_1 - Q_2}\right) \\ &= (P_1 - c_1) - \gamma - \gamma\left(\frac{Q_1 + Q_2}{1 - Q_1 - Q_2}\right).\end{aligned}$$

Setting $\frac{\partial}{\partial Q_1}\Pi(Q_1, Q_2) = 0$ yields $P_1 - c_1 = \gamma\left(\frac{1}{1-Q_1-Q_2}\right)$. Similarly, $\frac{\partial}{\partial Q_2}\Pi(Q_1, Q_2) = 0$ yields $P_2 - c_2 = \gamma\left(\frac{1}{1-Q_1-Q_2}\right)$. Recall that P_1^* and P_2^* are the optimal values of P_1 and P_2 given n_1 and

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n_2. From the above equations, $P_1^* - c_1 = P_2^* - c_2$ (and also as stated in Lemma 1), and let it be denoted by r^*. Using Lemma 1, $r^* = \Pi^* + \gamma$, and $Q_k^* = \frac{\left(n_k e^{\frac{a_k - c_k - r^*}{\mu}}\right)^{\frac{\mu}{\gamma}}}{e^{\frac{\omega 11}{\gamma}} + \sum_{i=1}^{2}\left(n_i e^{\frac{a_i - c_i - r^*}{\mu}}\right)^{\frac{\mu}{\gamma}}}, \qquad k = 1, 2.$

Using the relationship between P_k^* and Q_k^* and the above expression for Q_k^*,

$$r^* = \gamma\left(\frac{1}{1 - Q_1^* - Q_2^*}\right) = \gamma\left(\frac{e^{\frac{\omega 11}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}}}\right).$$

Differentiating r^* with respect to n_1, $\frac{\partial r^*}{\partial n_1} = \mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}}}\right) - \frac{\partial r^*}{\partial n_1}\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}}}\right)$, which leads to $\frac{\partial r^*}{\partial n_1} = \mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}\right)$. Define $G(n_1, n_2)$ as the maximand for n_1 and n_2. For simplicity, let $E(N_t)$ be time-invariant and so we omit the subscript t. Thus,

$$\begin{aligned}
G(n_1, n_2) &= T\left(E(N) r^* \{Q_1^* + Q_2^*\} - f(n_1 + n_2)\right) - F_1 n_1 - F_2 n_2 \\
&= T\left(E(N)\gamma\left\{\frac{Q_1^* + Q_2^*}{1 - Q_1^* - Q_2^*}\right\} - f(n_1 + n_2)\right) - F_1 n_1 - F_2 n_2 \\
&= T\left(E(N)\gamma\left\{\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}}}\right\} - f(n_1 + n_2)\right) - F_1 n_1 - F_2 n_2.
\end{aligned}$$

Suppose there exists an optimal solution n_1^* and n_2^*. Then, n_1^* satisfies the following first order condition:

$$\begin{aligned}
\frac{\partial G(n_1, n_2)}{\partial n_1} &= 0 = T\left(-f'(n_1 + n_2) + E(N)\mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}}}\right)\right. \\
&\qquad \left. - E(N)\frac{\partial r^*}{\partial n_1}\left(\frac{n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}}}\right)\right) - F_1.
\end{aligned}$$

Substituting for $\frac{\partial r^*}{\partial n_1}$, we get

$$\frac{\partial G(n_1, n_2)}{\partial n_1} = 0 = T\left(-f'(n_1 + n_2) + E(N)\mu\left(\frac{n_1^{\frac{\mu}{\gamma}-1} e^{\frac{a_1 - c_1 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}\right)\right) - F_1.$$

Similarly,

$$\frac{\partial G(n_1, n_2)}{\partial n_2} = 0 = T\left(-f'(n_1 + n_2) + E(N)\mu\left(\frac{n_2^{\frac{\mu}{\gamma}-1} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}{e^{\frac{\omega 11}{\gamma}} + n_1^{\frac{\mu}{\gamma}} e^{\frac{a_1 - c_1 - r^*}{\gamma}} + n_2^{\frac{\mu}{\gamma}} e^{\frac{a_2 - c_2 - r^*}{\gamma}}}\right)\right) - F_2.$$

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When $a_1 - c_1 > a_2 - c_2$, $e^{\frac{a_1-c_1-r^*}{\gamma}} > e^{\frac{a_2-c_2-r^*}{\gamma}}$. Moreover, $F_1 = F_2$. Together, these conditions imply that the first order conditions can only be satisfied if $n_1^* > n_2^*$. Additionally, observe that $\frac{\partial G(n_1,n_2)}{\partial n_2} > 0$ for sufficiently small n_2 as $\gamma > \mu$. This along with the concavity of G implies that $n_2^* > 0$.

On the other hand, when $a_1 - c_1 = a_2 - c_2$ and $F_1 = F_2$, $\frac{\partial G(n_1,n_2)}{\partial n_1} = \frac{\partial G(n_1,n_2)}{\partial n_2}$ for any (n_1, n_2). Moreover, both first derivatives have identical coefficients for n_1 and n_2. As a consequence, $n_1 = n_2$ is optimal.

8.8 Discussion on Modeling of Suppliers' Availability

That the supplier availability is governed by a Markov chain implies that the retailer does not receive any advance information about supplier failures. The Markov chain assumption also means that the probability of disruption in a period is independent of how long the supplier has been in state 1. Although some disruptions may come with an advance warning (e.g., labor unrest), most come without any warning or at most a short notice (e.g., natural catastrophes, terrorist events). Therefore, the onset of disruptions is characterized well by a Markov chain model.

However, the Markov chain assumption also implies that the supply resumes without any advance information. This is a strong assumption since it is fair to believe that in many instances the supplier has some idea about the time it will take to resume production capacity and it communicates it to the retailer. This information may not become available immediately after the disruption. However, once the damage is assessed, the supplier may know an approximate time table in which it can resume delivery.

A more sophisticated stochastic process that can address this issue is a multi-state semi-Markov process. A semi-Markov process may spend a random amount of time in a state, whose distribution depends on that state, before shifting to another state. The state space of a semi-Markov process that would characterize suppliers' availability may include the state of normal operations and different degrees of possible damage to the production facility, for instance, low damage, high damage, and complete destruction. Subsequently, a distribution may be defined for each state of damage that would correspond to the time it takes before the production resumes normally. While such a model would overcome some of the apparent weaknesses in our model, we believe the results would not fundamentally change due to the average-cost criterion that we employ. In this criterion, the sample path process of supplier availability over time is not important given our modeling approach. What matters, instead, is the fraction of time a supplier is available. Therefore, to keep the analysis simple, we have used the discrete-time Markov chain model.

Another feature of our model of supplier availability is that it considers only extreme possibilities such that either a supplier is fully available or is completely unavailable. When disruptions are massive, it is very likely for a supplier to lose all of its production capacity, and it will not be able to deliver the product at all. However, the recovery of the production capacity is sometimes gradual. While recovering, the supplier may first be able to deliver the best-selling product variants before

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resuming supply of all the variants. To keep the analysis simple, we ignore this possibility and assume that a supplier resumes supply only when its capacity is fully restored.

8.9 Modeling of Supplier Failure Correlation

In this subsection, we present an approach to define the *correlation* of suppliers' availability. Our approach is similar to Babich et al. (2007). Define $\pi^{1*} := \pi^{11} + \pi^{10}$ and $\pi^{*1} := \pi^{11} + \pi^{01}$ as the total fraction of time suppliers 1 and 2, respectively, are available. Since $\pi^{11} + \pi^{01} + \pi^{10} + \pi^{00} = 1$, fixing the values of π^{11}, π^{*1} and π^{1*} gives us the values of π^{01}, π^{10}, and π^{00}.

Recall that we use X_1^t and X_2^t to denote the availabilities of supplier 1 and 2, respectively, in a generic period t. We drop the superscript t to keep the notation simple. Both X_1 and X_2 are Bernoulli random variables that take two values, 0 and 1. X_1 is equal to 1 with probability π^{1*}. Similarly, X_2 assumes a value of 1 with probability π^{*1}. Also, $P(X_1 = 1, X_2 = 1) = \pi^{11}$ and $P(X_1 = 0, X_2 = 0) = \pi^{00}$. The correlation of X_1 and X_2, $\rho_{X_1X_2}$, is equal to

$$\rho_{X_1X_2} = \frac{E(X_1X_2) - E(X_1)E(X_2)}{\sqrt{(E(X_1^2) - E(X_1)^2)(E(X_2^2) - E(X_2)^2)}} = \frac{\pi^{11} - \pi^{1*}\pi^{*1}}{\sqrt{\pi^{1*}\pi^{*1}(1-\pi^{1*})(1-\pi^{*1})}}. \tag{8.41}$$

The supplier availability correlation can be varied by changing any one of the three parameters, π^{11}, π^{1*} or π^{*1}.

Observe that if we set $\pi^{11} = \pi^{*1} = \pi^{1*}$, then $\pi^{01} = \pi^{10} = 0$, and the two suppliers are perfectly positively correlated. When the two suppliers have a correlation of 1, in any period, either both suppliers are available or both suppliers are unavailable. On the other hand, if $\pi^{11} = \pi^{00} = 0$, then either supplier 1 or supplier 2 is available (but not both) in any period. Thus, the two suppliers are perfectly negatively correlated. The condition $\pi^{11} = \pi^{00} = 0$ is equivalent to $\pi^{*1} + \pi^{1*} = 1$.

Therefore, our approach can potentially allow us to examine a full range of correlation values between -1 and +1. As we vary correlation, suppose we change the value of π^{00}, the fraction of the planning horizon when none of the brands is available. However, as π^{00} increases, the cumulative profit earned through the sale of the product over all the variants decreases. Since the design and launch costs remain unaffected, the total variety must decrease as π^{00} increases. Therefore, changing the value of π^{00} produces a confounding effect in the experiment. To exclude this effect, we set it to zero. As a consequence, we are able to consider only negative values of the failure correlation.

For the plots in Figures 2(c), 4(a) and 5(c), we set $\pi^{10} = 0.25$ and vary the sum of π^{11} and π^{01} between 0 and 0.75 to conduct the analysis.

9 Comparison of Optimal Number of Variants for the Responsive and Static Pricing Models

In Table 4, we present the optimal number of variants for each brand in the responsive and static pricing models as a function of four parameters: μ, ω^{10}, ρ, and γ.

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μ	Responsive pricing		Static pricing	
	n_1^*	n_2^*	n_1^*	n_2^*
0.1	0.1101	0.0952	0.1100	0.0952
0.7	0.4757	0.4155	0.4756	0.4153
1.3	0.7569	0.6477	0.7566	0.6474
1.9	1.0953	0.8966	1.0948	0.8958
2.5	1.6262	1.2100	1.6253	1.2074
ω^{10}				
1	3.3323	0.3710	3.2758	0.3983
4	2.0431	0.6750	2.0398	0.6769
7	0.8354	1.0563	0.8343	1.0557
10	0.3270	1.2426	0.3263	1.2427
13	0.1912	1.2957	0.1909	1.2957
ρ				
-1.0000	0.5318	1.5250	0.5318	1.5250
-0.6647	0.7125	1.3860	0.7114	1.3857
-0.4616	0.8705	1.1940	0.8695	1.1936
-0.2977	1.0503	0.9475	1.0499	0.9469
-0.1015	1.2684	0.6494	1.2686	0.6486
γ				
2	0.6448	0.3124	0.6448	0.3123
3	1.4690	1.2653	1.4677	1.2633
4	1.7999	1.6075	1.7971	1.6036
5	1.9761	1.7878	1.9724	1.7826
6	2.0855	1.8992	2.0811	1.893

Table 4: Optimal Number of Variants of Each Brand for Responsive and Static Pricing Models

Since we assume product variety to be continuous for analytical convenience, we keep that assumption in our numerical experiments as well. The optimal product variety values that fall between 0 and 1 are still qualitatively indicative of how model parameters influence the optimal variety decision.

10 Additional Figures

In this section, we present three figures corresponding to the responsive variety strategy. The figures describe the percent profit improvement (PIP) due to the responsive variety strategy, the optimal number of variants for each brand in different states, and the optimal price for each brand in different states as a function of customer heterogeneity (μ), utility of outside option in state 10 (ω^{10}), correlation (ρ), and brand disparity (γ). All the model parameters remain same as in Table 2 except a_2, which now is equal to 6. For this value of a_2, the difference in optimal decisions for the responsive pricing and responsive variety strategies is clearer, which allows us to develop additional insights.

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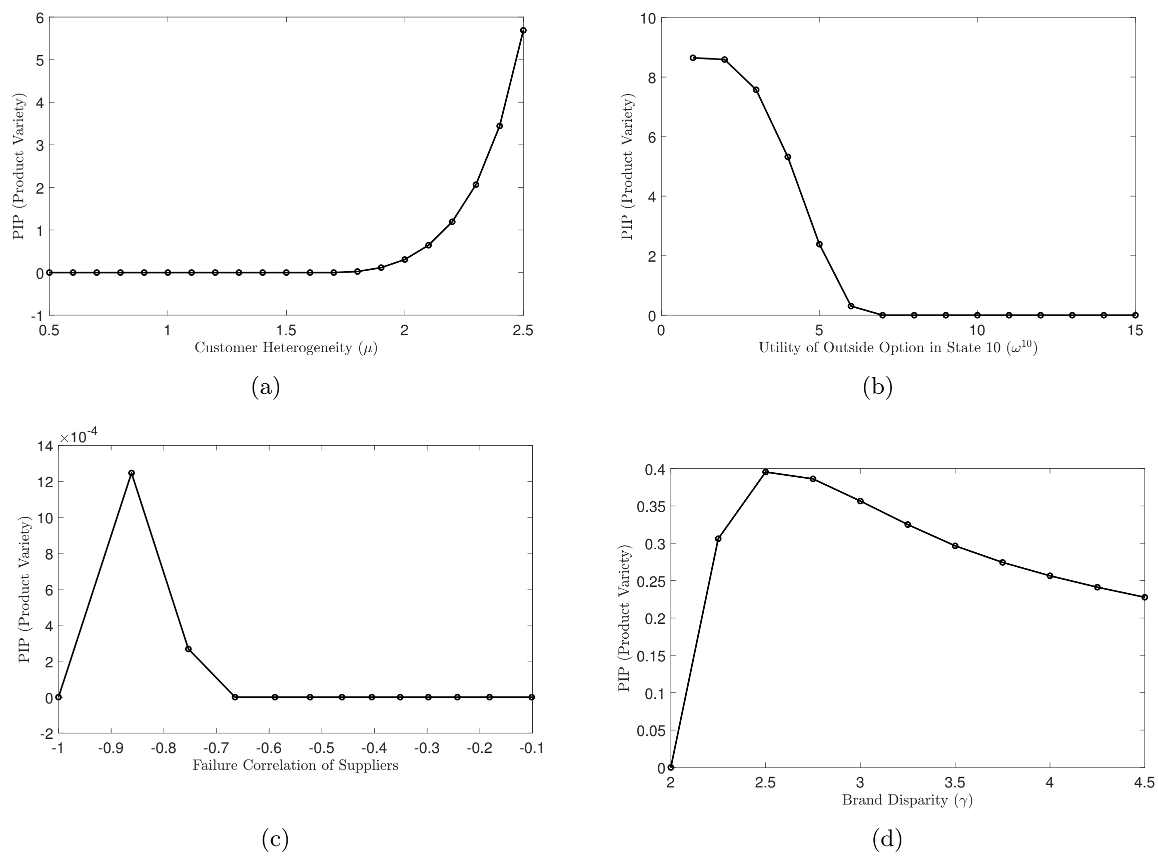

Figure 6: Profit Improvement due to Responsive Variety as a Function of Model Parameters

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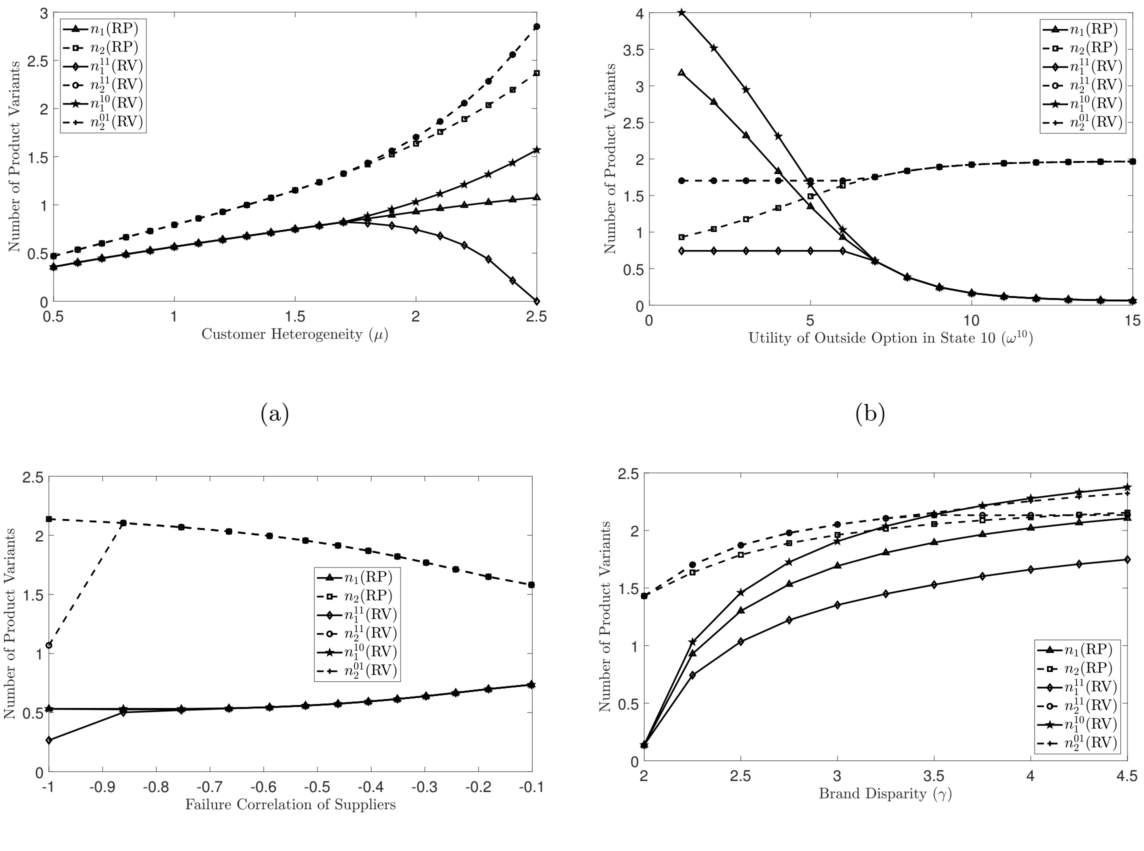

Figure 7: Number of Variants for Each Brand in the Responsive Pricing (RP) and Responsive Variety (RV) Models as a Function of Model Parameters

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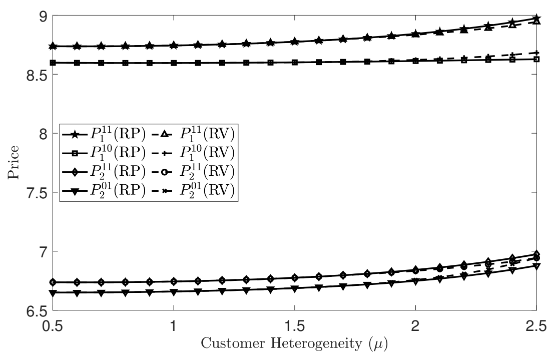
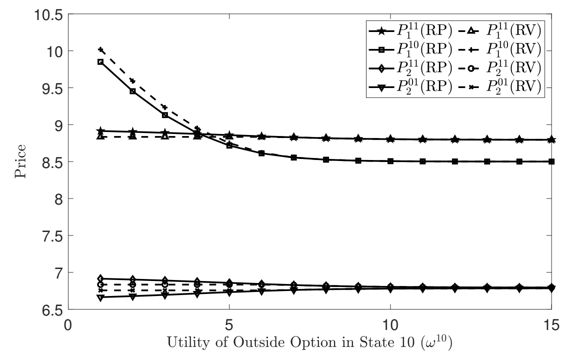
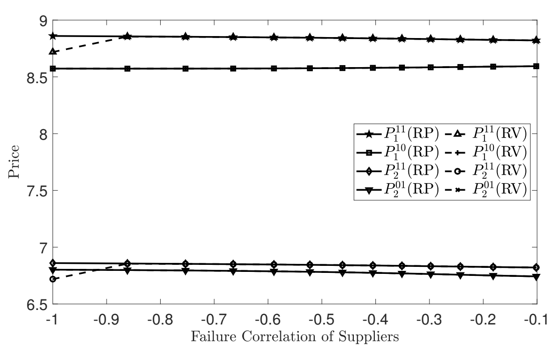


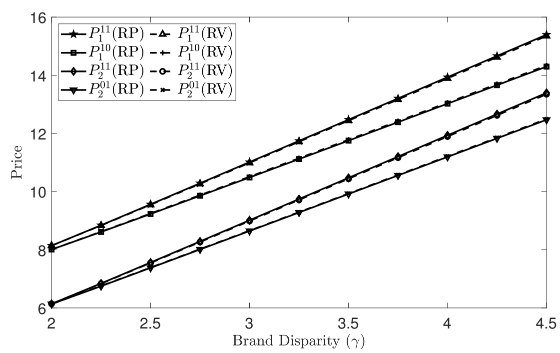
(a)



(b)



(c)



(d)

Figure 8: Optimal Prices in Responsive Pricing (RP) and Responsive Variety (RV) Models as a Function of Model Parameters

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