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Queueing Problem with Dynamic and Heterogeneous Opportunity Costs

Sarvesh Bandhu
Assistant Professor of Economics
Indian Institute of Management Bangalore
sarvesh.bandhu@iimb.ac.in

Parikshit De
Assistant Professor of Economics
Department of Economics Sciences,
Indian Institute of Science Education and Research Bhopal
parikshitde@iiserb.ac.in

Devwrat Dube
Department of Industrial Engineering and Management
Ben Gurion, University of the Negev,
Beer Sheba, Israel
dubedevwrat@gmail.com

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Sarvesh Bandhu[1], Parikshit De[2], Devwrat Dube[3]

[1]Economics Department, IIM Bangalore, India.
[2]Department of Economics Sciences, IISER Bhopal, India.
[3]Department of Industrial Engineering and Management, Ben Gurion University of the Negev, Beer Sheba, Israel.

Contributing authors: sarvesh.bandhu@iimb.ac.in; parikshitde@iiserb.ac.in; dubedevwrat@gmail.com;

Abstract

We study queueing problems in which agents have heterogeneous per-period waiting costs (their private information), which can vary with queue position and are thus dynamic. Our goal is to implement a *Rawlsian* allocation rule that minimises the maximum of individual waiting costs among all agents. Under complete information, we introduce the *Just Algorithm*, a simple method that always selects a *Rawlsian* queue. However, in settings with incomplete information where agents possess *multidimensional private types* i.e., the vector of their per-period waiting costs for each period, we prove that no Dominant Strategy Incentive-Compatible (DSIC) mechanism can implement the Rawlsian queueing rule over an unrestricted domain of agent types. This result underscores the challenges of implementing allocational fairness in multidimensional environments even with quasi-linear utility structure. To address this impossibility, we explore the necessary domain restrictions that allow for the existence of deterministic DSIC mechanisms. We use the *Weak-Monotonicity* condition from Bikhchandani et al. (2006) to do this. This condition is both necessary and sufficient for deterministic DSIC mechanisms to exist in our convex domain setting. Further, we restrict the domain to *one-dimensional private information*, where agents' per-period waiting costs evolve according to publicly known agent-specific functions of their *privately* known first-period waiting costs. With this restriction, we construct a DSIC mechanism that implements the *Just Algorithm*, thereby ensuring that the allocational fairness objective is achieved. The results presented add to the growing literature on mechanism design in queueing problems by providing insights into the necessary and sufficient conditions for achieving fairness under strategic behaviour with heterogeneous waiting costs. This work highlights the complexities involved in mechanism design with multidimensional types and offers a viable solution within a significant and non-trivial restricted multidimensional domain with one-dimensional private information.

Keywords: Queueing, Dominant Strategy Implementation, Rawlsian

JEL Classification: D63 , D72 , D81

1 Introduction

Queueing theory, a fundamental area within operations research, examines the intricate dynamics of service systems where jobs are sequentially processed by servers. In the mechanism design approach to queueing problems, jobs are modelled as strategic agents possessing private information about their characteristics, particularly their waiting costs. Monetary transfers are allowed and agents have a quasilinear utility. Because agents incur disutility while waiting, they may strategically report their types to gain an advantage under the mechanism. This poses challenges for designing mechanisms that may aim to optimise aggregate welfare or achieve allocational fairness or any other desideratum.

Models of queueing have been studied extensively from various game-theoretic perspectives. In particular, a growing literature (see Subsection 1.1) on queueing problems with one-dimensional agents' types offers insight into mechanisms that are optimal, fair, or both. Under a similar setup to ours, Mitra (2001) investigate the existence of *dominant strategy incentive-compatible (DSIC)* mechanisms under which allocational efficiency is achievable along with *budget-balancedness*. A well established notion of allocational fairness or justice proposed by John Rawls (see in Rawls (1971)) is now called the *Rawlsian* allocation. De and Mitra (2017) provides a justification of Rawlsian allocation in sequencing problems with each agent having a constant private per-period opportunity cost. They introduce an algorithm that proposes an order consistent with Rawlsian fairness.

We consider queueing problems involving a finite set of agents characterised by agent-specific (hence *heterogeneous*) waiting cost vectors representing their opportunity costs. The waiting cost for each agent evolves over discrete periods or queue positions (hence, *dynamic*). Notice that an unrestricted cost vector, meaning specifying one scalar per agent would not allow for a computation of the vector for the agent, represents the *multidimensional type* of the agent.

In each period they wait, an agent incurs a (possibly different) cost, and the total waiting cost is the sum over all periods until service. The agents' utility is quasi-linear in total waiting time and monetary transfers. Our initial goal is to introduce an algorithm which ensures that the allocation is *Rawlsian*, minimising the maximum individual waiting cost among all agents.

Under complete information, we develop the *Just Algorithm*, a simple method that identifies a *Rawlsian* queue.

Under incomplete information, the problem essentially becomes one of *multidimensional* private information. This poses a significant challenge for dealing with unilateral manipulation. The strategy space of multidimensional-type agents is more sophisticated than the one-dimensional agents case, and hence achieving the objective is a difficult task. We demonstrate the impossibility of any Dominant Strategy Incentive-Compatible (DSIC) mechanism implementing our algorithm when agents' types are unrestricted. This result thus underscores the difficulty of achieving fairness in multi-dimensional settings, even within quasi-linear environments like ours.

To address this challenge, we restrict the domain to one-dimensional private information[1], where agents' per-period waiting costs evolve according to publicly known, agent-specific functions based on their initial private cost. This approach allows agents' opportunity costs to remain heterogeneous and dynamic while simplifying the strategic

[1] This is because even with two dimensions, the problem still persists. See Example 4.

complexity of the problem. Within this restricted domain, we propose a DSIC mechanism that successfully implements the *Just Algorithm*, thereby ensuring the realisation of the Rawlsian queue.

We present Example 1 to demonstrate the difference between an aggregate cost minimising i.e., *efficient* queue and a maximum individual cost minimising i.e., *Rawlsian* queue.

Example 1 *Consider a three-agent case,* $N = \{i, j, k\}$*. Let the reported waiting cost vectors be* $\theta_i = (2, 11, 1), \theta_j = (3, 4, 1),$ *and* $, \theta_k = (5, 9, 1)$*. The profile is given by:*

$$\theta = \begin{bmatrix} \theta_i \\ \theta_j \\ \theta_k \end{bmatrix} = \begin{bmatrix} 2 & 11 & 1 \\ 3 & 4 & 1 \\ 5 & 9 & 1 \end{bmatrix} \tag{1}$$

Table 1 summarises the problem.

Queue(s)→	**ijk**	**ikj**	**jik**	**jki**	**kij**	**kji**
$\sum_{h=1}^{\sigma_i} \theta_{ih}$	2	2	13	14	13	14
$\sum_{h=1}^{\sigma_j} \theta_{jh}$	7	8	3	3	8	7
$\sum_{h=1}^{\sigma_k} \theta_{kh}$	15	14	15	14	5	5
$\max_{l \in N} \sum_{h=1}^{\sigma_l} \theta_{lh}$	15	14	15	14	13	14
$\sum_{l \in N} \sum_{h=1}^{\sigma_l} \theta_{lh}$	24	24	31	31	26	26

Table 1 Individual costs, aggregate costs, and maximum individual costs in all possible queues for the given θ.

There are a total of six possible queues. Queue ***ijk*** *means that agent-i is served first, followed by agent-j in the second position and agent-k in the third position. Whenever agent-i is served in the first position (in queues* ***ijk*** *and* ***ikj****), the cost incurred is equal to the first column entry in row-i of profile* θ*, i.e., 2; whenever agent-i is served in the second position (in queues* ***jik*** *and* ***kij****), the cost incurred is equal to the sum of the first column and second column entry in row-i of profile* θ*, i.e., 2+11=13; and whenever agent-i is served in the third position (in queues* ***jki*** *and* ***kji****), the cost incurred is equal to the sum of the entries in the first three columns in row-i of profile* θ*, i.e., 2+11+1=14. The cost for other agents and queues is calculated* *similarly. Table 1 lists all possible queues in columns and the costs incurred by each of the agents in that queue. For each of the six queues, we calculate* $\max_{l \in N} \sum_{h=1}^{\sigma_l} \theta_{lh}$*, which is the maximum individual cost incurred by any agent in that queue in corresponding rows. For instance, in the queue* ***kji****, agent-i incurs a cost of 14, which is the maximum individual cost in that queue. There is only one Rawlsian queue:* ***kij*** $\in \arg\min_{\sigma \in \Sigma(N)} \max_{l \in N} \sum_{h=1}^{\sigma_l} \theta_{lh}$*. Note that there are two efficient queues:* ***ijk*** *and* ***kij*** $\in \arg\min_{\sigma \in \Sigma(N)} \sum_{l \in N} \sum_{h=1}^{\sigma_l} \theta_{lh}$*, but they are not Rawlsian.*

Example 1 demonstrates the distinction between efficient and *Rawlsian* queues.

In our set-up, an individual, by changing his reports can alter not only his but other agents allocations including the relative positions of two other agents. Therefore, we are dealing with a problem with *severe* externality imposable by agents on the society of agents. Further, not all queue positions may be accessible to an agent, as his type is varied over all possible types (where the agent does not get a positive utility from waiting), keeping the types of other agents fixed. In other words, the cut-off vector for an agent can have any number of points, and agents may differ in the dimensionality of their respective cut-off vectors. Example 5 illustrates such a case. Also, one agent can determine for another agent, several of the cut-off points of that other agent's type for distinct queue positions.

Investigating the existence of *first best* mechanisms under a similar set-up, Mitra (2001) highlights the importance of *Independence Property*: In the reduced problem obtained by making one agent absent from the problem, the relative position of any other two distinct agents cannot change under the same queueing rule. For queueing problems with heterogeneous and dynamic costs mechanisms employing the Rawlsian queueing rule violate the *Independence Property*. Example 2 demonstrates a three agent case where the mechanism employing an efficient queueing rule satisfies *Independence Property* while a mechanism employing the Rawlsian queue selected by the Just algorithm violates it. For the purpose of Example 2, we request the reader to accept our claims about which queue is selected by the *Just Algorithm*. The validity of claims is very obvious after the definition of the algorithm.

Example 2 *Consider the following three agent profile. Let the tie-breaking rule be* $\succ_{TB} := b \succ_{TB} a \succ_{TB} c$.

$$\theta = \begin{bmatrix} 2 & 6 & 10 \\ 3 & 5 & 7 \\ 8 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 8 & 18 \\ 3 & 8 & 15 \\ 8 & 12 & 14 \end{bmatrix} \tag{2}$$

The Just Algorithm selects the queue $\sigma^{Just}(\theta) := bac$. *If agent-a is absent from the problem, then* $\sigma^{Just}(\theta_{-a}) := cb$. *The presence or absence of agent-a can flip the relative positions of agent-b and agent-c in the Just queue. Therefore, the Independence Property is violated for the Rawlsian queue.*

Now consider the aggregate cost minimising queue $\sigma^*(\theta) := abc$. *If agent-a is absent from the problem, then* $\sigma^*(\theta_{-a}) := bc$. *Similarly,* $\sigma^*(\theta_{-b}) := ac$ *and* $\sigma^*(\theta_{-c}) := ab$. *Therefore, the profile* θ *is an example where the efficient queue satisfies the Independence Property but Rawlsian queue violates it.*

This completes the example.

It would be interesting to identify the exact largest domain restriction under which the *Just Algorithm* works in such a way that the queueing problem with heterogeneous and dynamic opportunity costs satisfies the Independence property. But supposing that we retain the dynamic nature of per-period costs but *homogenise* the evolution of costs across all agents and then assume naturally that the evolution of costs is known except a one dimensional private information which can be without loss of generality the first period waiting cost, then the severity goes away, as well as each agent's cut-off vector will have one point less than the number of agents, meaning that all queue positions will be accessible to every agent when his type is varied keeping others fixed. A natural consequence is that if the evolution of costs has any weakly monotonic property along the private dimension then sorting agents in non-increasing order of their private type would yield a Rawlsian queue. Moreover, such a queue would also be an aggregate cost minimising i.e., efficient queue. Following Holmström (1979), such queues can be implemented if and only if the transfers are VCG transfers. Furthermore, following Mitra (2001), we also know the domain restrictions under which *first best* mechanisms are guaranteed. There is not much to inquire once the heterogeneity in agents' evolution of types is assumed away. If we assume away the dynamic nature of costs, we go back to the constant per period waiting cost queueing problems that began this literature. Heterogeneity and dynamic nature of costs are therefore collectively important.

Example 5 illustrates that, under our domain restriction, agents may be constrained in the queue positions they can obtain due to the interplay between their cost functions and those of other agents. It highlights that even with one-dimensional private information, the heterogeneity of agents' cost evolutions can prevent certain queue positions from being accessible. This underscores the importance of carefully designing mechanisms that account for these limitations while striving to implement the Rawlsian queue.

The findings presented here lay the groundwork for a comprehensive exploration of fair mechanisms in queueing problems with multidimensional private information. Our work contributes to the literature by highlighting the limitations of implementing fairness in complex settings and providing a viable solution within a restricted but significant domain. The rest of the paper is organised as follows. In subsection 1.1, we review the existing literature to place our work in context, highlighting how our contributions extend the current understanding of queueing problems and mechanism design. Section 2 explains the framework of queueing problems with heterogeneous waiting costs along with some necessary definitions. In Section 3, we develop the Just Algorithm. Subsection 3.1 contains our impossibility result for the unrestricted domain. Section 5 introduces the necessary and sufficient domain restriction characterised by the *Weak-Monotonicity condition* presented in Bikhchandani et al. (2006). We propose a transfer rule that implements a Rawlsian queueing rule in dominant strategies. Section 6 concludes.

1.1 Related Literature

In this subsection, we survey the existing literature on mechanism design in queueing problems, focusing on both strategic and fairness considerations. The mechanism design literature for optimal resource allocation rules (mechanisms) is rich. Myerson (1981) studies optimal mechanisms for single-item auctions and one-dimensional continuous type spaces of agents. In Hartline and Karlin (2007), the authors introduce optimal mechanism design with one-dimensional continuous types under Dominant Strategy Incentive Compatibility. The literature covers queueing problems involving strategic as well as fairness considerations.

Works such as Chun (2006b), Moulin (2007), Mishra and Rangarajan (2007), Maniquet (2003), Chun (2006a), and Chun (2011) study fairness aspects. These works address concepts like equitable sequencing, consistency in allocation, and the design of rules that satisfy various fairness criteria.

We apply the Rawlsian principle of minimising the maximum individual waiting cost. There is a large literature on social welfare rankings of society's income that applies this Rawlsian difference principle (see Barbarà and Jackson (1988), Moulin (1988), D'Aspremont and Gevers (1977), Hammond (1976), and Sen (1970)).

From a strategic standpoint, researchers have investigated mechanisms that encourage truthful reporting and efficient outcomes. Mitra (2001) examines efficient and budget-balanced mechanisms in queueing models, demonstrating that *first best* outcomes are attainable under certain conditions when agents have private information about their waiting costs. Similarly, Dolan (1978) and Suijs (1996) contribute to understanding incentive-compatible mechanisms in queueing systems, focusing on how to align individual incentives with social efficiency. Mitra (2002) explores the implementation of efficient allocation rules when agents have private waiting costs, emphasising the challenges of designing mechanisms that are both efficient and strategy-proof.

In the mechanism design literature with transfers, this compatibility of incentives and justice is indeed rare. Conclusions of Deb and Mishra (2014), and Lavi et al. (2003) show that the Rawlsian allocation is incompatible with implementability in dominant strategies. Velez (2011) studies the house allocation problems and shows a compatibility between incentives and justice. Velez (2011) show that the Generalised Money Rawlsian Fair solution implements the *no envy* solution as Nash and Strong Nash equilibria. Thus, in Velez (2011), incentive compatibility is achieved in the Nash sense and not in the dominant strategies sense like ours.

However, much of the existing literature tends to focus on agents with one-dimensional types, where each agent's private information is represented by a single parameter, typically their constant per-period waiting cost. This simplification facilitates the design of mechanisms but does not capture the complexity inherent in scenarios where agents have multidimensional private information. One departure from this is the work by Mitra (2001), which addresses efficient and budget-balanced mechanism design in a multidimensional queueing model. In their study, agents' waiting costs depend on their position in the queue, introducing a multidimensional aspect to their private information. However, even in Mitra (2001), an unrestricted domain does not admit *first best* mechanisms, and two conditions, *Independence Property* and *Combinatorial Property*, characterise the domain admitting *first best* mechanisms.

Duives et al. (2015) examines the problem in a setting where the optimal mechanism minimises the total expected transfers to all jobs while being Bayesian-Nash incentive-compatible. Recent progress in deriving optimal mechanisms for multidimensional settings often assumes that the type space is discrete. For example, Armstrong

(2000) investigates multi-object auction models where valuations are additive and drawn from a binary distribution (i.e., high or low), highlighting the challenges inherent in multidimensional, discrete type spaces. Similarly, Malakhov and Vohra (2009), Pai and Vohra (2014), Cai et al. (2012), and Hoeksma and Uetz (2013) make advances in optimal mechanism design under the assumption of discrete types, acknowledging the increased complexity compared to one-dimensional cases. Mishra and Roy (2013) consider deterministic dominant strategy implementation in multidimensional dichotomous domains with private values and quasi-linear utility, providing insights into mechanism design when agents have limited types.

The cut-off-based mechanisms are prevalent in varied mechanism design contexts (see Milgrom (2004), Börgers (2015), and Myerson (1985)). Such cut-off-based mechanisms were also derived for scheduling problems with multiple machines and varying speed by Mishra and Mitra (2010) and for multi-dimensional dichotomous domains by Mishra and Roy (2013).

The complexity of optimal mechanism design with multidimensional types is well-established, and the challenges are compounded when agents' private information is continuous, making strategic reporting a significant challenge. In such environments, designing mechanisms that are incentive-compatible and satisfy additional desiderata becomes significantly more difficult. It is not uncommon to find cut-off(s)-based mechanisms in settings with multidimensional types. Armstrong (2000) discusses how the seller can use personalised pricing schemes (akin to cut-off(s)) to maximise revenue. The mechanisms involve setting different prices or cut-off points for different bidders based on their multidimensional types. Armstrong and Rochet (1999) provides a comprehensive guide to multidimensional screening models, where a principal designs mechanisms to screen agents with private information along multiple dimensions. The authors discuss how cut-off strategies can be employed when agents have heterogeneous types and how these cut-off(s) can vary among agents. Thanassoulis (2004) examines bargaining and mechanism design when agents have private information about substitutable goods. The mechanisms involve setting individualised thresholds for agreement, which can be interpreted as agent-specific cut-off(s). Manelli and Vincent (2007) study revenue-maximising mechanisms in a multi-good monopoly setting. They show that optimal mechanisms may require offering menus of options (contracts) where different agents self-select based on their types, leading to differing cut-off(s). While Mussa and Rosen (1978) is a classic paper on quality differentiation, it introduces the concept of screening consumers through non-linear pricing, which effectively sets different cut-off(s) for consumers based on their willingness to pay. Other valuable works shedding light on personalised threshold mechanisms, which are essentially cut-off(s)-based mechanisms, include Wilson (1993), Jehiel et al. (1999), etc. These studies demonstrate that personalised mechanisms are a common feature in such settings. In complex mechanism design problems involving multidimensional types, it is common for agents to have different numbers of cut-off points due to heterogeneity in their private information and the design of optimal contracts. In Armstrong (1996), the optimal pricing scheme involves offering a menu of bundles with different prices, effectively creating different cut-off(s) for different consumers. The number of cut-off points (i.e., the number of bundles or pricing tiers) can vary depending on the heterogeneity of consumer types. In Rochet and Choné (1998), the optimal mechanism partitions the type space into different regions (akin to cut-off points). Due to the multidimensionality and heterogeneity of agents' types, the number and structure of these regions can differ among agents, implying that agents may face different numbers of cut-off(s). In our paper as well, although the private information is restricted to the first-period waiting cost, the evolution of costs remains heterogeneous across agents, and hence, the cut-off(s) of agent types to obtain queue positions is not the same. In fact, there may be agents who can obtain only a subset of the queue positions. Given the other agents' types, the functions determining the evolution of costs for an agent may exclude him from getting some of the queue positions, no matter what his type turns out to be. In the queueing and sequencing problems literature, this variation in cut-off(s) and the variation in the number of cut-off(s) for different agents is a novel feature. It follows from the heterogeneity of agents' waiting costs.

Our work contributes to this line of research by exploring fair mechanisms for queueing problems where agents have heterogeneous and position-dependent waiting costs, which is a setting where agents' types are multidimensional and continuous. Unlike previous studies that prioritise efficiency or budget balance, we aim to implement a Rawlsian allocation rule that minimises the maximum individual waiting cost among all agents. This focus on Rawlsian fairness distinguishes our work from that of Mitra (2001), who primarily seek to identify cost structures that enable *first best* implementability in terms of aggregate cost minimisation. We present an example to distinguish the two kinds of queuing rules in Example 1.

Consequently, implementing fairness notions like the Rawlsian criterion in multidimensional settings is difficult and less explored. Bikhchandani et al. (2006) show that a necessary condition for the existence of deterministic DSIC mechanisms is that the social choice rule satisfies *weak monotonicity* (W-Mon) on its domain. Furthermore, on convex domains, Saks and Yu (2005) establish that *W-Mon* is also sufficient for the existence of deterministic DSIC mechanisms implementing the rule. In the context of queueing problems with unrestricted multidimensional types, which form a convex set as noted in Mitra (2001), the Rawlsian allocation rule does not satisfy the W-Mon condition. This lack of compliance leads to the impossibility of designing DSIC mechanisms that implement the Rawlsian queueing rule in such settings.

To overcome this impossibility, we introduce a domain restriction to one-dimensional private information, allowing agents' per-period waiting costs to evolve according to publicly known, agent-specific functions based on their initial private cost. This restriction maintains the heterogeneity and dynamic nature of agents' waiting costs while simplifying the mechanism design problem. By doing so, we can design a DSIC mechanism that implements the Rawlsian queue, contributing to the broader understanding of mechanism design in complex, multidimensional environments.

Our study not only highlights the limitations of implementing fairness in multidimensional settings but also provides a viable solution within a significant and nontrivial restricted domain. This work opens avenues for further research into necessary and sufficient conditions for the existence of DSIC mechanisms in such contexts, potentially aligning with the weak monotonicity conditions identified by Bikhchandani et al. (2006) and others.

2 The Framework

Consider a finite set of agents $N = \{1, 2, \dots, n\}$ who need to get their jobs processed using a single server. The server can serve only one agent at a time, and a job, once started, cannot be stopped unless finished. Each agent's job takes one unit of time to get processed. Hence, the server needs to design a queue, which is an assignment of agents to queue positions [2].

Each agent incurs disutility while waiting for their job to be processed. The cost incurred by every agent in every period is variable and is the private information of the agents. A representative agent-i has per-period waiting cost θ_{i1} in the first period, θ_{i2} in the second period, and so on. $\theta_{ik} \in \mathbb{R}_{++}$ [3] indicates the k^{th} period unit waiting cost of agent-i. The vector $\theta_i = (\theta_{i1}, \theta_{i2}, \dots, \theta_{in}) \in \mathbb{R}^n_{++}$ is the waiting cost vector of agent-i. If agent-i is served in the k^{th} period (or position), his disutility is given by the sum of waiting cost incurred in each period until job completion, i.e., $\sum_{j=1}^{k} \theta_{ik}$.

The $n \times n$ positive matrix $\theta = [\theta_{ik}]_{1 \le i,k \le n}$ is called the waiting cost profile. Let $\Sigma(N)$ denote the set of all $n!$ possible orderings (queues) over N. We denote by $\sigma(\theta) \in \Sigma(N)$

[2] Throughout the paper we only consider assignments that are feasible and maximal. Every agent is assigned to a position. One and only one agent is assigned to each position. We will refer to these simply as queues.

[3] $\mathbb{R}_{++}$ denotes the positive orthant of real line $\mathbb{R}$.

a particular queue and write $\sigma_i(\theta) = k$ to mean that agent-i has position k in the queue $\sigma(\theta)$. A *queueing rule* is a function $\sigma : \mathbb{R}^{n\times n}_{++} \to \Sigma(N)$ that specifies, for each profile θ, a unique order $\sigma(\theta) = (\sigma_1(\theta), \ldots, \sigma_n(\theta)) \in \Sigma(N)$ [4]. A *transfer rule* is a function $\tau : \mathbb{R}^{n\times n}_{++} \to \mathbb{R}^n$ that specifies for each profile $\theta \in \mathbb{R}^{n\times n}_{++}$ a transfer vector $\tau(\theta) = (\tau_1(\theta), \ldots, \tau_n(\theta)) \in \mathbb{R}^n$, where $\tau_i(\theta) \in \mathbb{R}$ is the monetary transfer made to the agent. The term $\tau_i(\theta)$ is negative if the agent pays and positive if he receives monetary compensation.

A *mechanism* $\mu = (\sigma, \tau)$ constitutes a queueing rule σ and a transfer rule τ. The bundle of any agent-i under the mechanism μ at the reported profile θ is written as $\mu_i(\theta) = (\sigma_i(\theta), \tau_i(\theta))$.

The agents have quasi-linear utility functions of the form $u_i(\mu_i(\theta)) = -\sum_{k=1}^{\sigma_i(\theta)} \theta_{ik} + \tau_i(\theta)$. For any mechanism $\mu = (\sigma, \tau)$, if the reported profile is $(\hat{\theta}_i, \theta_{-i})$ [5] when the true waiting cost vector of agent-i is θ_i, then the utility of agent-i is $u_i(\mu_i(\hat{\theta}_i, \theta_{-i}); \theta_i) = -\sum_{k=1}^{\sigma_i(\hat{\theta}_i, \theta_{-i})} \theta_{ik} + \tau_i(\hat{\theta}_i, \theta_{-i})$.

$\mathcal{Q}^D$ denotes the class of queueing problems with heterogeneous waiting costs, and $\mathcal{Q}^D(N)$ denotes an instance of such a problem with a given set of agents (hence profile). If $\forall j, k \in N, \theta_{ik} = \theta_{ij}$, then agent-$i$ has a constant per-period waiting cost. If all agents have constant per-period waiting cost, we have the class of queueing problems $\mathcal{Q} \subset \mathcal{Q}^D$ with constant per-period waiting cost.

The heterogeneous waiting cost setting implies that each agent-$i \in N$ reports a vector $\theta_i = (\theta_{i1}, \theta_{i2}, \ldots, \theta_{in}) \in \mathbb{R}^n_{++}$. Hence, the agents are multidimensional, and $\mathcal{Q}^D$ are problems in *multidimensional mechanism design*. The profile $\theta \in \mathbb{R}^{n\times n}_{++}$ can be visualised as an $n \times n$ matrix where agents are labelled along the rows and periods along

[4]Since the queueing rule is a function and not a correspondence, tie-breaking may be required at some profiles.

[5]Here, $\theta_{-i} \in \mathbb{R}^{n\times(n-1)}_{++}$ is the set of waiting cost vector announcements by the other $(n-1)$ agents in $N \setminus \{i\}$.

the columns. Thus agent-i's report is row-i in the matrix.

$$[\theta] = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \dots & \theta_{1(n-1)} & \theta_{1n} \\ \theta_{21} & \theta_{22} & \dots & \theta_{2(n-1)} & \theta_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \theta_{n1} & \theta_{n2} & \dots & \theta_{n(n-1)} & \theta_{nn} \end{bmatrix} \tag{3}$$

We focus our attention on the queueing rule $\sigma^R \in \Sigma(N)$, which minimises the maximum waiting cost incurred by any agent out of all possible orders. We call such a queueing rule *Rawlsian* in keeping with Rawls' *Maxi-Min*[6] Principle (see Rawls (1971)).

Definition 1 **Rawlsian queueing rule σ^R:** A queueing rule σ^R is called a Rawlsian queueing rule if, for every profile $\theta \in \mathbb{R}^{n\times n}_{++}$, we have:

$$\sigma^R(\theta) \in \arg \min_{\sigma(\theta)\in\Sigma(N)} \max_{i\in N} \sum_{k=1}^{\sigma_i(\theta)} \theta_{ik}.$$

For an example of a queueing problem with heterogeneous waiting costs and identification of a Rawlsian queue, see Example 1.

We now turn our attention to defining mechanisms $\mu = (\sigma, \tau)$ that *implement* the queueing rule σ. As we are interested in truth-telling mechanisms, by the revelation principle we restrict attention to direct mechanisms. Implementation of a rule σ in Dominant Strategies via a mechanism (σ, τ) requires that the transfer rule τ be such that for any agent, truthful reporting (weakly) dominates false reporting *irrespective* of what others report. A mechanism $\mu = (\sigma, \tau)$ is called a Dominant Strategy Incentive-Compatible (DSIC) Mechanism if it implements the queueing rule σ in Dominant Strategies.

[6]The Maxi-Min Principle seeks to maximise the minimum utility obtained by any agent. In the case of disutility, it seeks to minimise the maximum disutility obtained by any agent.

Definition 2 **Dominant Strategies Implementation**: A mechanism $\mu = (\sigma, \tau)$ is Dominant Strategy Incentive-Compatible (DSIC) implementable if $\forall i \in N$, $\forall \theta_i, \hat{\theta}_i \in \mathbb{R}^n_{++}$, and $\forall \theta_{-i} \in \mathbb{R}^{n\times(n-1)}_{++}$:

$$u_i(\mu_i(\theta_i, \theta_{-i}); \theta_i) \geq u_i(\mu_i(\hat{\theta}_i, \theta_{-i}); \theta_i)$$

3 Unrestricted Domain

In order to implement the Rawlsian queueing rule, we need an algorithm to identify a Rawlsian queue at all profiles. In algorithm 1, we propose a method that always selects a unique queue $\sigma^{JA}(\theta)$ given any profile θ. This is followed by example 3 to demonstrate the working of the algorithm in a 4-agent case. It is easy to verify that the algorithm would select the queue **kij** when applied to example 1.

Algorithm 1 Just Algorithm

Tie-breaking rule

The tie-breaking order is given by $\succ_{TB} := 1 \succ_{TB} 2 \succ_{TB} \ldots \succ_{TB} \ n$. For any $m \in \{1, \ldots, n\}$, if

$$P^{n-m+1} \subseteq \arg \min_{j \in N^{n-m+1}(\theta)} \sum_{l=1}^{m} \theta_{jl},$$

then $\sigma_i^{JA}(\theta) = m$ whenever $i \in P^{n-m+1}$ and either $|P^{n-m+1}| = 1$, or $\forall j \in P^{n-m+1}$, such that $j \neq i$, $j \succ_{TB} i$.

First step

Let $N^1(\theta) = N$ be the set of agents and $\theta^1 = \theta$ be the reported profile for step-1. Let $i \in P^1 := \arg\min_{j \in N^1(\theta)} \sum_{l=1}^{n} \theta_{jl}$, such that either $|P^1| = 1$, or $\forall j \in P^1$, such that $j \neq i$, $j \succ_{TB} i$. Assign $\sigma_i^{JA}(\theta) = n$. Let $N^2(\theta) = N^1(\theta) \setminus \{i\}$. Update θ^1 to θ^2 by deleting the last column of θ^1 and the row corresponding to such agent-i.

kth step ($2 \leq k \leq n-1$)

$N^k(\theta) = N \setminus \bigcup_i \{i\} : \sigma_i^{JA}(\theta) \in \{n+2-k, n\}$. Let $i \in P^{n-k+1} := \arg\min_{j \in N^k(\theta)} \sum_{l=1}^{n-k+1} \theta_{jl}$, such that either $|P^{n-k+1}| = 1$, or $\forall j \in P^{n-k+1}$, such that $j \neq i$, $j \succ_{TB} i$. Assign $\sigma_i(\theta) = n-k+1$. Update θ^k to θ^{k+1} by deleting the last column of θ^k and the row corresponding to such agent-i.

nth step

$N^n(\theta) = N \setminus \bigcup_i \{i\} : \sigma_i^{JA}(\theta) \in \{2, n\}$. $||N^n(\theta)|| = 1$. For $i \in N^n(\theta)$, assign $\sigma_i^{JA}(\theta) = 1$.

Example 3 ***Working of Just Algorithm:*** *Consider a four-agent case.* $N^1(\theta) = N = \{i, j, k, l\}$. *Let the reported profile be* θ. *We use the tie-breaking rule* $i \succ_{TB} j \succ_{TB} k \succ_{TB} l$.

$$\theta = \theta^1 = \begin{bmatrix} \theta_i \\ \theta_j \\ \theta_k \\ \theta_l \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 & 8 & 12 \\ 1 & 3 & 6 & 10 \\ 1 & 3 & 6 & 7 \\ 1 & 3 & 6 & 10 \end{bmatrix} = \bar{\theta}^1 \tag{4}$$

We have transformed the matrix θ *into* $\bar{\theta}^1$ *as follows:* $\forall p \in N, \forall q \in \{1, 2, 3, 4\}$, $\bar{\theta}^1_{pq} = \sum_{m=1}^{q} \theta_{pm}$. *The cost incurred by agent-*$p \in N$ *when served in period* $q \in \{1, 2, 3, 4\}$ *can be read off directly as* $\bar{\theta}^1_{pq}$. *The algorithm works as follows.*

In the first step, we calculate each agent's cumulative waiting costs if served last. Agent-k has the lowest total cost of 7, so agent-k is assigned to the last position. Thus, $\sigma^{JA}_k(\theta) = 4$. $N^2(\theta) = N^1(\theta) \setminus \{k\} = \{i, j, l\}$. *We update* θ^1 *to* θ^2 *by removing the agent-k row and last column of* θ^1.

$$\theta^2 = \begin{bmatrix} \theta_i \\ \theta_j \\ \theta_l \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 & 8 \\ 1 & 3 & 6 \\ 1 & 3 & 6 \end{bmatrix} = \bar{\theta}^2 \tag{5}$$

In the second step, the algorithm calculates the cost of serving each remaining agent in the third period. The minimum cost that will be incurred by any agent getting served in the last period is 6 *if either agent-j or agent-l is served in period* 3. *The tie-breaking rule,* $i \succ j \succ k \succ l$, *favours agent-j, so he continues to be in the problem for an earlier period assignment, and agent-l losing the tie is awarded the third position,* $\sigma^{JA}_l(\theta) = 3$. *We update* θ^2 *to* θ^3 *by removing the agent-l row and last column of* θ^2.

$$\theta^3 = \begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix} = \bar{\theta}^3 \tag{6}$$

In the third step, agent-j is assigned to period two since $1 + 2 < 3 + 2$. *Thus,* $\sigma^{JA}_j(\theta) = 2$. *There is one remaining agent, and the agent is served in the first period,* $\sigma^{JA}_i(\theta) = 1$.

The maximum cost is incurred by agent-k in the queue $\sigma^{JA}(\theta)$ *and is equal to 7. Of the 24 possible queues, it is easily verified that there are six queues that serve agent-3 in period four, and the maximum cost in the other 18 queues will be either 12 or 10 depending upon* *which of the other agents, i, j, or l, is served last. All six queues serving agent-k in period 4 are Rawlsian queues, and the Just Algorithm for perceptive agents selected a queue that is Rawlsian. This example demonstrates that the Just Algorithm systematically assigns positions to minimise the maximum individual waiting cost, resulting in a Rawlsian queue.*

Example 3 demonstrates the step-by-step working of the *Just Algorithm*, including a tie-breaking situation for queue position 3 between agents j and l.

Proposition 1 *The Just Algorithm always selects a Rawlsian queue.*

Proof Consider the set of agents N, with any reported profile $\theta \in \mathbb{R}^{n\times n}_{++}$. Let $\sigma^{JA}(\theta)$ be the queue selected by the Just Algorithm. Let $p = \arg\max_{i\in N} \sum_{k=1}^{\sigma_i^{JA}(\theta)} \theta_{ik}$. Let agent-$p$, incurring the maximum cost in σ^{JA}, be served in position-q, i.e. $\sigma_p^{JA}(\theta) = q$.
For brevity of notation, we write $c_i(\sigma(\theta))$ to denote the cost incurred by agent-i in the queue $\sigma(\theta)$. Suppose that $\sigma^{JA}(\theta)$ is not a Rawlsian queue. Let $\sigma(\theta) \neq \sigma^{JA}(\theta)$ be one of the Rawlsian queues such that the maximum of individual cost borne by agents in $\sigma(\theta)$ is less than $c_p(\sigma^{JA}(\theta))$. Suppose $c_r(\sigma(\theta)) < c_p(\sigma^{JA}(\theta))$, where $r = \arg\max_{i\in N} c_i(\sigma(\theta)) = \arg\max_{i\in N} \sum_{k=1}^{\sigma_i(\theta)} \theta_{ik}$.
We have the following cases:
Case 1 Given $\sigma(\theta) \neq \sigma^{JA}(\theta)$, let $\sigma_p(\theta) \geq q$. Then, by definition, $c_r(\sigma(\theta)) = \max_{i\in N} c_i(\sigma(\theta))$, and hence, $c_r(\sigma(\theta)) \geq c_p(\sigma(\theta))$. But, $c_p(\sigma(\theta)) = \sum_{k=1}^{\sigma_p(\theta)} \theta_{pk} \geq \sum_{k=1}^{q} \theta_{pk} = \sum_{k=1}^{\sigma_p^{JA}(\theta)} \theta_{pk}$. This contradicts the claim that $c_r(\sigma(\theta)) < c_p(\sigma^{JA}(\theta))$, thus completing the proof.
Case 2 Let $\sigma_p(\theta) < q$. Then at least one of the predecessors of agent-p in the queue $\sigma^{JA}(\theta)$ is served in a position $s \geq q$. Let agent-$m(\neq p)$ be such an agent, i.e., $\sigma_m(\theta) = s \geq q$. Then, $c_r(\sigma(\theta)) = \max_{i\in N} c_i(\sigma(\theta)) \geq c_m(\sigma(\theta)) = \sum_{k=1}^{\sigma_m(\theta)} \theta_{mk} = \sum_{k=1}^{s} \theta_{mk} \geq \sum_{k=1}^{q} \theta_{mk} \geq \sum_{k=1}^{q} \theta_{pk}$. The last inequality follows from the algorithm. This contradicts the claim that $c_r(\sigma(\theta)) < c_p(\sigma^{JA}(\theta))$.
This completes the proof. □

3.1 Impossibility Results

Are there DSIC mechanisms that implement the queueing rule σ^{JA}? With the unrestricted type spaces, no such DSIC mechanism exists.

To demonstrate the impossibility of designing a Dominant Strategy Incentive-Compatible (DSIC) mechanism under unrestricted types, consider Example 4 with only two agents.

Example 4 *Consider a two-agent case, $N = \{1,2\}$, with reported waiting cost vectors $\theta_1 = (8,3)$ and $\theta_2 = (7,3)$. The Just Algorithm assigns $\sigma_1^{JA}(\theta) = 1$. The utility of agent-1 is $u_1(\sigma^{JA}(\theta), \tau(\theta)) = -(8) + \tau_1((8,3),(7,3))$. Suppose agent-1 were to misreport the waiting cost vector as $\tilde{\theta_1} = (5,4)$. $\sigma_1^{JA}(\tilde{\theta}_1, \theta_2) = 2$. Therefore, $u_1(\sigma^{JA}(\tilde{\theta}_1, \theta_2), \tau(\tilde{\theta}_1, \theta_2)) = -(8+3) + \tau_1(\tilde{\theta}_1, \theta_2)$. Implementation in Dominant Strategies demands: $u_1(\sigma^{JA}(\theta), \tau(\theta)) \geq u_1(\sigma^{JA}(\tilde{\theta}_1, \theta_2), \tau(\tilde{\theta}_1, \theta_2)) \equiv 3 \geq \tau_1(\tilde{\theta}_1, \theta_2) - \tau_1((8,3),(7,3))$.*

If the true waiting cost vector of agent-1 is $(5,4)$ and the misreport is $(8,3)$, then implementation in Dominant Strategies demands: $u_1(\sigma^{JA}(\tilde{\theta}_1, \theta_2), \tau(\tilde{\theta}_1, \theta_2)) \geq u_1(\sigma^{JA}(\theta), \tau(\theta)) \equiv \tau_1(\tilde{\theta}_1, \theta_2) - \tau_1((8,3),(7,3)) \geq 4$. One and only one of the two conditions can hold, and therefore, it is impossible to find any transfer rules $\tau(\theta), \tau(\tilde{\theta}_1, \theta_2))$ satisfying both conditions simultaneously.

Example 4 confirms that, even in a simple two-agent scenario, no transfer rule can satisfy the conditions required for DSIC implementation when agents have unrestricted types. It highlights the challenges of achieving fairness in multi-dimensional settings and motivates the need for domain restrictions.

Theorem 1 *Consider any problem $\mathcal{Q}^D(N)$, where N is the set of agents with reported profile $\theta \in \mathbb{R}^{n \times n}_{++}$. There is no DSIC mechanism $\mu = (\sigma^{JA}, \tau)$.*

Proof We prove this by construction of a generic counter-example.

Consider the set of agents $N = \{1, \ldots, n\}$. Arbitrarily choose any agent-i from N. Construct an admissible waiting cost vector $\theta_i \in \mathbb{R}^n_{++}$ such that $\theta_{i(k+1)} - \theta_{ik} > 0$ for some $k \in \{1, \ldots, n-1\}$. Because of the unrestricted domain, such construction is allowed. Let $\epsilon = \frac{\theta_{i(k+1)} - \theta_{ik}}{5} > 0$. We can write $\theta_i = (\theta_{i1}, \ldots, \theta_{ik}, \theta_{ik} + 5\epsilon, \theta_{i(k+2)}, \ldots, \theta_{in})$. Construct $\theta_m = (\theta_{i1}, \ldots, \theta_{i(k-1)}, \theta_{ik} + 4\epsilon, \theta_{ik} + 2\epsilon, \theta_{i(k+2)}, \theta_{in})$. Construct $\hat{\theta}_i = (\theta_{i1}, \ldots, \theta_{i(k-1)}, \theta_{ik} + 3.5\epsilon, \theta_{ik} + 3.5\epsilon, \theta_{i(k+2)}, \theta_{in}))$. The vectors θ_m, $\hat{\theta}_i$, and θ_i differ only in the k^{th} and $(k+1)^{th}$ coordinate. Let the report $\theta_{-i-m} \in \mathbb{R}^{n \times (n-2)}_{++}$ of agents other than agent-i and agent-m be such that 8, 7, and 9 hold.

Consider the profiles profile $\theta = (\theta_i, \theta_m, \theta_{-i-m})$ and another profile $\hat{\theta} = (\hat{\theta}_i, \theta_m, \theta_{-i-m})$.

$$i = \arg \min_{j \in N^{n-k}(\theta)} \sum_{l=1}^{k+1} \theta_{jl} \implies \sigma_i^{JA}(\theta) = k + 1 \tag{7}$$

$$m = \arg \min_{j \in N^{n-k+1}(\theta)} \sum_{l=1}^{k} \theta_{jl} \implies \sigma_m^{JA}(\theta) = k \tag{8}$$

$$m = \arg \min_{j \in N^{n-k}(\theta) \setminus \{i\}} \sum_{l=1}^{k+1} \theta_{jl} \tag{9}$$

Equation 7 means that, under the Just Algorithm, when queue position $(k + 1)$ is to be assigned to one of the agents in the set $N^{n-k}(\theta)$, agent-i has the least cost of getting served in period $(k + 1)$ amongst the agents in N^{n-k}. Equation 8 means that at the stage when queue position k is to be assigned to one of the agents in the set $N^{n-k+1}(\theta) = N^{n-k}(\theta) \setminus \{i\}$, agent-m has the least cost of getting served in period k amongst the agents in $N^{n-k+1}(\theta)$. Equation 9 states that if agent-i had not been present in the set $N^{n-k}(\theta)$, agent-m would have been the minimum cost agent to get served in period $(k + 1)$.

Note $\theta_{ml} = \hat{\theta}_{il}$, for any $l \in \{1, 2, \ldots, k-1, k+2, \ldots, n\}$. Given equation 9 is true, 10 holds because $\sigma_i^{JA}(\hat{\theta}) > k + 1$ cannot be true and $\sum_{l=1}^{k+1} \hat{\theta}_{il} = \epsilon + \sum_{l=1}^{k+1} \theta_{ml} > \sum_{l=1}^{k+1} \theta_{ml}$. Also, given that 8 holds and $\sum_{l=1}^{k} \hat{\theta}_{il} = \sum_{l=1}^{k} \theta_{ml} - 0.5\epsilon$, 11 holds.

$$m = \arg \min_{j \in N^{n-k}(\hat{\theta})} \sum_{l=1}^{k+1} \theta_{jl} \implies \sigma_m^{JA}(\hat{\theta}) = k + 1 \tag{10}$$

$$i = \arg \min_{j \in N^{n-k+1}(\hat{\theta})} \sum_{l=1}^{k} \theta_{jl} \implies \sigma_i^{JA}(\hat{\theta}) = k \tag{11}$$

Implementation in Dominant Strategies requires 12 $\geq$ 13 and 14 $\geq$ 15. Both conditions together demand: $\theta_{ik} + 3.5\epsilon = \hat{\theta}_{i(k+1)} \geq \tau_i(\theta) - \tau_i(\hat{\theta}) \geq \theta_{i(k+1)} = \theta_{ik} + 5\epsilon$.

$$u_i(\mu_i(\theta); \theta_i) = -\sum_{l=1}^{k+1} \theta_{il} + \tau_i(\theta) \tag{12}$$

$$u_i(\mu_i(\hat{\theta}); \theta_i) = -\sum_{l=1}^{k} \theta_{il} + \tau_i(\hat{\theta}) \tag{13}$$

$$u_i(\mu_i(\hat{\theta}); \hat{\theta}_i) = -\sum_{l=1}^{k} \hat{\theta}_{il} + \tau_i(\hat{\theta}) \tag{14}$$

$$u_i(\mu_i(\theta); \hat{\theta}_i) = -\sum_{l=1}^{k+1} \hat{\theta}_{il} + \tau_i(\theta) \tag{15}$$

For any $\epsilon > 0$, it is *impossible* to find any functions $\tau_i(\theta), \tau_i(\hat{(\theta)})$ satisfying the implementation conditions. Hence, for the constructed profiles, allowed by an unrestricted domain, no DSIC mechanism can exist.

This completes the proof. □

In the following subsection, we identify domain restrictions on the agents' types that allow for the existence of DSIC mechanisms.

4 Domain Restrictions: Necessary

While we have achieved a negative result for the existence of DSIC mechanisms implementing Rawlsian queueing, it is well known that Rawlsian queueing can be implemented by DSIC mechanisms when the types of agents are restricted to have only constant per-period waiting costs (see, for example, De and Mitra (2017)). Exactly what domain restrictions are necessary for the existence of DSIC mechanisms?

Social choice rules that allow the existence of deterministic mechanisms must satisfy a necessary condition outlined in Bikhchandani et al. (2006) as the *Weak-Monotonicity (W-Mon)* condition. While Bikhchandani et al. (2006) establish the necessity of *W-Mon*, Saks and Yu (2005) establish the sufficiency of *W-Mon* over convex domains. Hence, for queueing problems with unrestricted multidimensional types (which are convex as noted in Mitra (2001)), *W-Mon* is a necessary and sufficient condition for the existence of deterministic DSIC mechanisms. The *W-Mon* requirement is the following: If changing one agent's type (while keeping the types of other agents fixed) changes the outcome under the social choice function, then the resulting difference in utilities of the new and original outcomes evaluated at the new type of this agent must be no less than this difference evaluated at the original type of this agent.

We present below the definition of *W-Mon* borrowed from Bikhchandani et al. (2006), in line with our notation. Then, we apply this definition to the utility structure of agents within the mechanism (σ^{JA}, τ) and obtain the necessary and sufficient condition for the domain of type of agents for which the rule σ^{JA} satisfies *W-Mon* because we know it does not satisfy *W-Mon* over an unrestricted domain.

Definition 3 **Weak-Monotonicity (W-Mon)**: A social choice function $\sigma(\cdot)$ is weakly monotone (W-Mon) if, for every $i \in N$, $\theta_i, \theta'_i \in \Theta_i$, and every $\theta_{-i} \in \prod_{j \in N \setminus \{i\}} \Theta_j$,

$$U_i(\sigma(\theta'_i, \theta_{-i}); \theta'_i) - U_i(\sigma(\theta_i, \theta_{-i}); \theta'_i) \geq U_i(\sigma(\theta'_i, \theta_{-i}); \theta_i) - U_i(\sigma(\theta_i, \theta_{-i}); \theta_i) \tag{16}$$

Bikhchandani et al. (2006) prove (Theorem 2 in their paper) that a social choice function on a completely ordered, bounded domain is truthful if and only if it is weakly monotone. The bounded restriction implies that θ_{ij} is finite $\forall i \in N$ and $\forall j \in \{1, \ldots, |N|\}$. The complete ordering restriction is already satisfied for our framework. All agents prefer a queue position earlier than later.[7] We let the rule be σ^{JA} and restrict $\theta_i, \theta'_i \in \Theta_i \subset (0, \infty)^n$, then condition 16 requires, for every $i \in N$, $\theta_i, \theta'_i \in \Theta_i$, and every $\theta_{-i} \in \prod_{j \in N \setminus \{i\}} \Theta_j$,

$$-\sum_{k=1}^{\sigma_i^{JA}(\theta'_i, \theta_{-i})} \theta'_{ik} - (-\sum_{k=1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} \theta'_{ik}) \geq -\sum_{k=1}^{\sigma_i^{JA}(\theta'_i, \theta_{-i})} \theta_{ik} - (-\sum_{k=1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} \theta_{ik}) \tag{17}$$

[7] If for some period some agent has unit waiting cost zero, this does not hold, but such indifference must hold for all types of agents to contradict complete ordering, which is not the case.

$$\sum_{k=\sigma_i^{JA}(\theta_i',\theta_{-i})+1}^{\sigma_i^{JA}(\theta_i,\theta_{-i})} \theta_{ik}' \geq \sum_{k=\sigma_i^{JA}(\theta_i',\theta_{-i})+1}^{\sigma_i^{JA}(\theta_i,\theta_{-i})} \theta_{ik} \tag{18}$$

Without loss of generality, let $\sigma_i^{JA}(\theta_i', \theta_{-i}) < \sigma_i^{JA}(\theta_i, \theta_{-i})$, then condition 18 must hold $\forall k \in \{\sigma_i^{JA}(\theta_i', \theta_{-i}) + 1,\ \sigma_i^{JA}(\theta_i, \theta_{-i})\}$. It is necessary that this be true for $k = \sigma_i^{JA}(\theta_i', \theta_{-i})+1 = \sigma_i^{JA}(\theta_i, \theta_{-i})$. That is $\theta_{ik}' \geq \theta_{ik}$. If it holds for all such $k \in \{2, \ldots, n\}$, then it is straightforward to show that condition 18 must be satisfied. Notice that the W-Mon condition does not include transfers, and especially in the case of quasi-linear utilities, all types of agents evaluate every equal difference in transfer exactly the same. The Bikhchandani et al. (2006) result tells us the restriction of types for which σ^{JA} is implementable but does not tell us anything about the transfer. Since the result must hold for all profiles θ_{-i}, we can always construct profiles for which $\sigma_i^{JA}(\theta_i, \theta_{-i})$ can take any value from $\{2, \ldots, n\}$. Whenever $\sigma_i^{JA}(\theta_i', \theta_{-i}) + 1 = \sigma_i^{JA}(\theta_i, \theta_{-i})$, we must have $\theta_{ik}' \geq \theta_{ik}, \forall k \in \{2, \ldots, n\}$. These restrictions do not apply to agents' reports for the first period. Hence, we let the agents be multidimensional but restrain the private information to first-period waiting costs only. In subsection 4.1, we propose a sufficient restriction on their admissible types, which allows for the existence of deterministic DSIC mechanisms.

4.1 One-Dimensional Private Information: Necessary and Sufficient Condition

Consider the set of agents $N = \{1, \ldots, n\}$. The agents can report their one-dimensional type $\theta_i \in \Theta_i \subseteq \mathbb{R}_{++} \setminus \{\infty\}$ and cost function $f_i(k, \theta_i)$, where k is the period for which cost is being reported. If $f_i(\cdot, \theta_i)$ is unrestricted, an agent can simply report $f_i(k, \theta_i) = \theta_{ik}$ as in the preceding discussion. We allow different agents to have different cost functions, but these are assumed to be public information and hence not a part of agents' strategic reports.

Proposition 2 *For a queueing problem $\mathcal{Q}^D$, with a set of agents N, each with a type $\theta_i \in \Theta_i \subseteq \mathbb{R}_{++} \setminus \{\infty\}$ and cost functions $f_i(k, \theta_i)$ where $k \in \{1, \ldots, n\}$, the queueing rule $\sigma^{JA}(\theta)$ is implementable in Dominant Strategies if and only if, $\forall i \in N, k \in \{1, \ldots, n\}, \theta_{-i} \in \prod_{j \neq i} \Theta_j$ and $\forall\ \theta_i, \theta_i' \in \Theta_i$, the functions $f_i(k, \theta_i) : \{1, \ldots, n\} \times \Theta_i \to \mathbb{R}_{++} \setminus \{\infty\}$ satisfy:*

$$\sigma_i^{JA}(\theta_i, \theta_{-i}) > \sigma_i^{JA}(\theta_i', \theta_{-i}) \implies \sum_{k=\sigma_i^{JA}(\theta_i', \theta_{-i})+1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i') \geq \sum_{k=\sigma_i^{JA}(\theta_i', \theta_{-i})+1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i) \quad (19)$$

Proof From the restriction $\theta_i \in \Theta_i \subseteq \mathbb{R}_{++} \setminus \{\infty\}$, the domain of types is bounded and complete, so the necessity and sufficiency of *W-Mon* follows from the (Theorem 2) result of Bikhchandani et al. (2006). The sufficiency of *W-Mon* also follows from the result of Saks and Yu (2005) since our domain is convex, as already noted in Mitra (2001) for the unrestricted domain. It only remains to prove that the queueing rule σ^{JA}, which is deterministic, satisfies *W-Mon* if condition 19 holds. Suppose the antecedent $\sigma_i^{JA}(\theta_i, \theta_{-i}) > \sigma_i^{JA}(\theta_i', \theta_{-i})$ is true, then $\sum_{k=1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i) \leq \sum_{k=1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i')$, in accordance with the algorithm. If condition 19 holds, then we have:

$$\sum_{k=\sigma_i^{JA}(\theta_i', \theta_{-i})+1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i') \geq \sum_{k=\sigma_i^{JA}(\theta_i', \theta_{-i})+1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i)$$

$$-\sum_{k=1}^{\sigma_i^{JA}(\theta_i', \theta_{-i})} f_i(k, \theta_i') + \sum_{k=1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i') \geq -\sum_{k=1}^{\sigma_i^{JA}(\theta_i', \theta_{-i})} f_i(k, \theta_i) + \sum_{k=1}^{\sigma_i^{JA}(\theta_i, \theta_{-i})} f_i(k, \theta_i)$$

$$U_i(\sigma_i^{JA}(\theta_i', \theta_{-i}); \theta_i') - U_i(\sigma_i^{JA}(\theta_i, \theta_{-i}); \theta_i') \geq U_i(\sigma_i^{JA}(\theta_i', \theta_{-i}); \theta_i) - U_i(\sigma_i^{JA}(\theta_i, \theta_{-i}); \theta_i)$$

In the last step of the calculation, we add the transfer terms $\tau_i(\theta_i', \theta_{-i}) - \tau_i(\theta_i, \theta_{-i})$ to both sides. Irrespective of the true type of agent-i, this transfer difference is evaluated as the same difference in utility by any agent type. If condition 19 holds, then the queueing rule σ^{JA} satisfies *W-Mon*.

This completes the proof. □

The necessary restrictions on the domain obtained by us are not easy to use in the search for mechanisms. More structure over the domain is needed to be able to identify mechanisms that are DSIC and implement the rule σ^{JA}. Section 5 furthers the discussion in this regard.

5 Domain Restriction: One-Dimensional Private Information

When per-period waiting costs are constants, i.e., for all agents $i \in \mathcal{N}$, $\theta_{ik} = \theta_i \in \mathbb{R}_+$ for all $k \in \{1, 2, \dots, n\}$, then the Rawlsian queueing rule (which coincides with the aggregate cost-minimising queueing rule) can be implemented by DSIC mechanisms (see Mitra (2001), Chun (2006a), Hashimoto and Saitoh (2012), etc.).

5.1 Domain Restriction

We restrict the domain to a one-dimensional private-information setting but not constant per-period costs. We use the notation $f_i(k, \theta_i) > 0$ to denote the k^{th}-period waiting cost of agent-i of one-dimensional private-type $\theta_i \in \mathbb{R}_+$. $F_i^k(\theta_i) = \sum_{l=1}^{k} f_i(l, \theta_i)$ denotes the total waiting cost of agent-i when he waits for $k \in \{1, \dots, n\}$ periods. We put the following restrictions:

1. **(public-information)** The functions $f_i(k, \cdot)$ are public information for all periods $k \in \{1, \dots, n\}$ and all agents $i \in \mathcal{N}$. In general, $f_i(k, \theta_i) \neq f_j(k, \theta_i)$ for two distinct agents i and j [8].
2. **(private-information)** The only private information is the first-period waiting cost for all agents, i.e., $f_i(1, \theta_i) = \theta_i$ for all agents-$i \in \mathcal{N}$.

[8] Since $f_i(k, \cdot) \neq f_j(k, \cdot)$ in general, the functions also specify a type for each agent, but this is public information.

3. **(per-period costs)** The functions $f_i(k, \cdot)$ are continuous and non-decreasing in their second argument, satisfying $\lim_{\theta_i \to 0} f_i(k, \theta_i) \to 0$ for all agents $i \in \mathcal{N}$ and for all periods $k \in \{2, \ldots, n\}$.

The literature on evolving opportunity costs can benefit by restricting attention to simpler functions such as linear or quadratic. However, we do not put any further restrictions on the cost structures for two reasons: *one*, that the heterogeneity or dynamism of cost structure leaves almost nothing to explore for a *Rawlsian* queue in light of the discussion in Section 1, and *second*, in healthcare, triage scheduling, dynamic pricing for ride sharing apps etc. there are cost structures that evolve with not much discipline. Further, as in Mitra (2001), who investigates large classes of cost structures, we see no motivation to limit our inquiry, except perhaps some plausible particular application where cost structures are known to be of a certain kind.

5.2 Domain Restriction: Implications

The class of queueing problems with the restricted domain is $\mathcal{Q}^D =< \mathcal{N}, \{f_i(k, \cdot)\}_{i \in \mathcal{N}} >$. Given our domain restriction, the Just Algorithm works as follows: $\sigma_i^{JA}(\theta_i, \theta_{-i}) = n$ if $i = \arg\min_{j \in \mathcal{N}} F_j^n(\theta_j)$, where tie(s) are assumed to be resolved. Then, $\sigma_k^{JA}(\theta_i, \theta_{-i}) = n - 1$ if $k = \arg\min_{j \in \mathcal{N} \setminus \{i\}} F_j^{n-1}(\theta_j)$, where tie(s) are assumed to be resolved. By looking at the allocation of positions, we cannot decide the order between $F_i^{n-1}(\theta_i)$ and $F_k^{n-1}(\theta_k)$. Suppose agent-i reports a very high type $\bar{\theta}_i$ such that for some fixed θ_{-i}, $\sigma_i(\bar{\theta}_i, \theta_{-i}) = 1$. Such $\bar{\theta}_i$ exists because $F_i^k(\theta_i)$ are increasing functions of θ_i for all periods $k \in \{1, \ldots, n\}$ and θ_{-i} is fixed. Similarly, since $\forall i \in \mathcal{N}$, $\lim_{\theta_i \to 0} F_i^n(\theta_i) \to 0$ has full range in $\mathbb{R}_+$, for some arbitrarily small $\underline{\theta}_i$, $\sigma_i^{JA}(\underline{\theta}_i, \theta_{-i}) = n$. However, unlike a queueing problem with constant per-period costs, such domain restriction does not guarantee, for a fixed θ_{-i}, that agent-i can obtain every queue position by reporting some waiting cost. Example 5 illustrates a case where agent-3 can never get queue position-2.

Example 5 ***An Illustration of Limited Accessibility to Queue Positions*** *Consider a set of three agents, $\mathcal{N} = \{1, 2, 3\}$. Let the tie-breaking rule be $1 \succ_{TB} 2 \succ_{TB} 3$. Fix $\theta_1 = 5$ and $\theta_2 = 7$. The cost functions are given by:*

- *Agent 1: $f_1(2, \theta_1) = \theta_1,\ f_1(3, \theta_1) = 18\theta_1$.*
- *Agent 2: $f_2(2, \theta_2) = 2\theta_2,\ f_2(3, \theta_2) = 11\theta_2$.*
- *Agent 3: $f_3(2, \theta_3) = 3\theta_3,\ f_3(3, \theta_3) = 3\theta_3$.*

We examine how agent 3's reported type θ_3 affects their position in the queue.

$$\theta = \begin{bmatrix} 5 \\ 7 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 90 \\ 7 & 14 & 77 \\ \theta_3 & 3\theta_3 & 3\theta_3 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 10 & 100 \\ 7 & 21 & 98 \\ \theta_3 & 4\theta_3 & 7\theta_3 \end{bmatrix}$$

If agent-3 reports his cost of waiting for three periods more than 98, only then will he not be served in the third position. If he is not served third, then agent-2 will be served third. Agent-3 cannot be served in the second position if he reports a waiting cost for two periods totalling more than 10. But as agent-3 changes his reports from zero to any arbitrarily large number, he crosses the threshold waiting cost of 10 for position-2 before he can cross the threshold waiting cost of 98 for position-3. An agent can be served in an earlier position only if he reports his total waiting cost for all later positions more than the respective threshold waiting costs.
This example demonstrates that agent 3 cannot access position 2 regardless of their reported type. The structure of the cost functions and the agents' reported types result in agent 3 being assigned either to position 3 (when $\theta_3 \leq 14$) or position 1 (when $\theta_3 > 14$), but never to position 2. This concludes the example.

Consider the profile $(\bar{\theta}_i, \theta_{-i})$ with $\sigma_i^{JA}(\bar{\theta}_i, \theta_{-i}) \neq n$. For every agent $k \neq i$, let $\sigma_k^{JA}(\bar{\theta}_i, \theta_{-i}) = \hat{k}$. The cost cut-off(s) of agent-i for all positions $\hat{k} \in \{2, \ldots, n\}$ are defined as the costs of the agent getting served in position-$\hat{k}$ ($=F_k^{\hat{k}}(\theta_k)$). Since θ_{-i} is fixed, we suppress the dependence of cut-off on θ_{-i} for ease of notation. We now define a cost cut-off for agent i for queue positions $\hat{k}$.

Definition 4 (Cost Cut-off of agent-i for position $\hat{k}$) For a given θ_{-i} and per-period cost functions $f_j(\hat{k}, \theta_k)$ for all agents $j \in \mathcal{N}$ and all positions $\hat{k} \in \{2, \ldots, n\}$, the cost cut-off of agent-i for position-$\hat{k}$ is $(F_k^{\hat{k}}(\theta_k))$.

For every position $\hat{k} \in \{2, \ldots, n\}$, we can calculate agent-i's type cut-off as the highest type that agent-i should have been so that he could obtain position $\hat{k}$ in the sequence or the lowest type that he should have been to obtain a position earlier than $\hat{k}$. This type is found by equating agent-i's cost of waiting for $\hat{k}$ periods to the cost cut-off for that position.

Definition 5 (Type Cut-off of agent-i for position $\hat{k}$) For a given θ_{-i} and per-period cost functions $f_j(\hat{k}, \theta_k)$ for all agents $j \in \mathcal{N}$ and all positions $\hat{k} \in \{2, \ldots, n\}$, the type cut-off of agent-i for position-$\hat{k}$ is $\theta_i^{\hat{k}} = (F_i^{\hat{k}})^{-1}(F_k^{\hat{k}}(\theta_k))$.

Agent-i can obtain a position earlier than $\hat{k}$ only if his reported type $\theta_i \geq \theta_i^{\hat{k}}$ [9]. However, this is not sufficient. Because of the way that the Just Algorithm works, an agent cannot get a position $\hat{k} - 1$ before passing the cost cut-off(s) for all positions $\hat{k}, \ldots, n$. If agent-i reporting θ_i obtains a position $\hat{k} - 1$, then it must be the case that $\theta_i \geq \theta_i^p$ for all $p \in \{\hat{k}, \ldots, n\}$. Given θ_{-i}, the set of type cut-off of agent-i for all positions is the set $= \{\theta_i^n, \ldots, \theta_i^2\}$. But there is no position-based ordering of the cut-off(s). For any report, $\theta_i \in [0, \theta_i^{\hat{k}})$, agent-$i$ cannot get a position earlier than position $\hat{k}$, which means that if $\theta_i^{\hat{k}} \leq \theta_i^n$, then as agent-$i$'s report increases from zero to θ_i^n, his position continues to be position-n, and if his report increases any further, he has already crossed the cost cut-off for period $\hat{k}$. Therefore, he never obtains position $\hat{k}$ for any of his possible reports. Agent-i may be the agent with the lowest cost in position $\hat{k}$ for his report $\theta_i \in [0, \theta_i^{\hat{k}}]$, but the agent only obtains the last position for all of his reports $\theta_i \in [0, \theta_i^n]$. If $\theta_i^{\hat{k}} \leq \theta_i^n$, then $[0, \theta_i^{\hat{k}}] \subseteq [0, \theta_i^n]$. Every type cut-offs of agent-i

[9] In case of a tie, the tie-breaking rule decides the position of the agent. But the type cut-off(s) can be calculated without considering explicitly how ties are resolved.

for all positions $\hat{k}$ satisfying $\theta_i^{\hat{k}} \leq \theta_i^n$ are irrelevant. Since $F_i^k(\theta_i) = \sum_{l=1}^k f_i(l, \theta_i) = \theta_i + \sum_{l=1}^k f_i(l, \theta_i)$ is increasing in θ_i, every agent-i, for any θ_{-i}, can also obtain the first position in the sequence selected by Just Algorithm. Consider the set of type cut-off(s) of agent-i for all positions $:= \{\theta_i^n, \ldots, \theta_i^2\}$. We order this set in decreasing order of sequence positions to obtain the vector $(\theta_i^n, \ldots, \theta_i^2)$. From this vector, we delete all irrelevant type cut-off(s) $\theta_i^{\hat{k}} \leq \theta_i^n$ to obtain the reduced vector $(\theta_i^{m_0} = \theta_i^n, \ldots, \theta_i^s)$ for some $s \in \{2, \ldots, n-1\}$ where the elements are ordered in decreasing order of sequence positions. Let $\theta_i^{m_1}$ be the second element in the reduced vector $(\theta_i^{m_0} = \theta_i^n, \theta_i^{m_1}, \ldots, \theta_i^s)$. From this reduced vector, we preserve $\theta_i^{m_0} = \theta_i^n$ and delete all irrelevant type cut-off(s) $\theta_i^{\hat{k}} \leq \theta_i^{m_1}$ to obtain the reduced vector $(\theta_i^n, \ldots, \theta_i^s)$ for some $s \in \{2, \ldots, n-1\}$ where the elements are ordered in decreasing order of sequence positions. We continue such reduction iteratively until we get a vector $(\theta_i^n = \theta_i^{m_0}, \theta_i^{m_1}, \ldots, \theta_i^{m_{M(i)}})$ for some $M(i) \in \{0, \ldots, n-2\}$ where the elements are ordered in decreasing order of sequence positions and $\theta_i^{m_l} < \theta_i^{m_{l+1}}$ for all $l \in \{0, \ldots, M(i)-1\}$. This is the type cut-off vector for agent-i. Next, we transform these cost cut-offs into type cut-offs by inverting the function $F_i^{\hat{k}}$.

Definition 6 (Type Cut-off vector of agent-i) For all agents $j \in \mathcal{N}$, a given θ_{-i}, per-period cost functions $f_j(\hat{k}, \theta_k)$, and all positions $\hat{k} \in \{2, \ldots, n\}$, agent-$i$'s type cut-off vector is defined as $\theta_{\mathbf{i}}^{\mathbf{cfs}} := (\theta_i^n = \theta_i^{m_0}, \theta_i^{m_1}, \ldots, \theta_i^{m_{M(i)}})$ where every $\theta_i^{m_l}$ is a type cut-off of agent-i for some position $\hat{m}_l \in \{2, \ldots, n\}$ satisfying $\hat{m}_l > \hat{m}_{l+1}$ and $\theta_i^{m_l} < \theta_i^{m_{l+1}}$ for all $l \in \{0, \ldots, M(i)-1\}$.

Every agent can obtain the first and the last position for some report. The number of positions that agent-i can obtain by varying his reports is equal to $M(i)+2$. If agent-i's report $\theta_i \in [0, \theta_i^n]$ [10], he is served last. If $\theta_i \in (\theta_i^{m_l}, \theta_i^{m_{l+1}})$, then $\sigma_i^{JA}(\theta_i, \theta_{-i}) = \hat{m}_{l+1}$ because agent-i has more than the minimum cost in all positions after $\hat{m}_{l+1}$, and he has the minimum cost for that position. The case presented in Example 5 is one

[10] If his reported cost is tied with any cut-off, the tie-breaking rule $\succ_{TB}$ allocates the position to agent-i.

where agent-3 has only one type Cut-off which is 14. As a result only two positions are obtainable by agent-3. Whenever his type is below 14 he is served last, else first. We define the transfer rule τ^{JA} below.

Definition 7 The transfer rule $\tau^{JA}(\theta_i, \theta_{-i})$ for any profile $(\theta_i, \theta_{-i}) \in \mathbb{R}^n_+$, every agent-$i \in \mathcal{N}$ with cut-off(s) vector $\theta_{\mathbf{i}}^{\mathbf{cfs}} := (\theta_i^n = \theta_i^{m_0}, \ldots, \theta_i^{m_{M(i)}})$, and arbitrary $h_i(\theta_{-i}) : \mathbb{R}^{n-1}_+ \to \mathbb{R}$ is defined as:

$$\tau_i^{JA}(\theta) = \begin{cases} h_i(\theta_{-i}) & \text{if } \sigma_i^{JA}(\theta) = n \\ h_i(\theta_{-i}) \; - \sum_{r=1}^{l} \sum_{j=\hat{m}_r+1}^{\hat{m}_{r-1}} \; f_i(j, \theta_i^{m_{r-1}}) & \text{if } \sigma_i^{JA}(\theta) = \hat{m}_l \end{cases} \tag{20}$$

The transfer of agent-i according to the rule τ^{JA} is the following:

- If the agent is served last, he gets an arbitrary amount $h(\theta_{-i})$.
- If his position (say position-$k = \hat{m}_r$) is not the last position, then for each position $k+1, k+2, \ldots, \hat{m}_{r-1}$ where $\theta_i^{m_{r-1}}$ is the lowest type for which agent-i could get position $\hat{m}_r$, he pays the cost $\sum_{j=k+1}^{\hat{m}_r} f_i(j, \theta_i^{m_{r-1}})$, for all positions $\hat{m}_{r-1}+1, \hat{m}_{r-1}+2, \ldots, \hat{m}_{r-2}$, the lowest type he should have been to be served in position $\hat{m}_{r-1}$ is the cut-off $\theta_i^{m_{r-2}}$, so he pays the cost $\sum_{j=\hat{m}_{r-1}+1}^{\hat{m}_{r-2}} f_i(j, \theta_i^{m_{r-2}})$, and so on.

We state our main result as Theorem 2.

Theorem 2 *For any $\mathcal{Q}^D =< \mathcal{N}, \{f_i(k, \cdot)\}_{i \in \mathcal{N}} >$ and any profile $\theta \in \mathbb{R}^n_+$, the mechanism $\mu^{JA} = (\sigma^{JA}, \tau)$ is DSIC if and only if the transfer rule is τ^{JA}.*

Proof For any arbitrary agent i, fix any $\theta_{-i} \in \mathbb{R}^{n-1}_+$. Consider any mechanism $\mu = (\sigma^{JA}, \tau)$. Let $\mu_i(\theta) = (\sigma_i^{JA}(\theta), \tau_i(\theta))$ denote agent-i's bundle under the mechanism μ when profile θ is reported. Let $u_i(\mu_i(\theta_i', \theta_{-i}); \theta_i)$ denote the utility of agent-i from the bundle $\mu_i(\theta_i', \theta_{-i})$ when his true type is θ_i and he reports θ_i'.
For any $k \in \{1, \ldots, n\}$, let $\theta_i^k, \hat{\theta}_i^k \in \mathbb{R}_+$ be any two reports of agent-i such that $\sigma_i^{JA}(\theta_i^k, \theta_{-i})$

$=\sigma_i^{JA}(\hat{\theta}_i^k, \theta_{-i}) = k$ [11].

$$u_i(\mu_i(\theta_i^k, \theta_{-i}); \theta_i^k) = -F_i^k(\theta_i^k) + \tau_i(\theta_i^k, \theta_{-i}) \tag{21}$$

$$u_i(\mu_i(\hat{\theta}_i^k, \theta_{-i}); \theta_i^k) = -F_i^k(\theta_i^k) + \tau_i(\hat{\theta}_i^k, \theta_{-i}) \tag{22}$$

$$u_i(\mu_i(\hat{\theta}_i^k, \theta_{-i}); \hat{\theta}_i^k) = -F_i^k(\hat{\theta}_i^k) + \tau_i(\hat{\theta}_i^k, \theta_{-i}) \tag{23}$$

$$u_i(\mu_i(\theta_i^k, \theta_{-i}); \hat{\theta}_i^k) = -F_i^k(\hat{\theta}_i^k) + \tau_i(\theta_i^k, \theta_{-i}) \tag{24}$$

$$\tau_i(\theta_i^k, \theta_{-i}) = \tau_i(\hat{\theta}_i^k, \theta_{-i}) \tag{25}$$

If the mechanism μ implements the queueing rule σ^{JA} in Dominant Strategies, then 21 $\geq$ 22 and 23 $\geq$ 24. The transfer of any agent must be independent of his own report if his position in the queue does not change, i.e., condition 25 is necessary.

We now consider an agent's reports when the reports lead to different queue positions. Since θ_{-i} is fixed, there is some agent j satisfying: $j = \arg\min_{l \in \mathcal{N} \setminus \{i\}} F_l^n(\theta_l)$ [12]. Let θ_i^n be the highest report. $\theta_i \in \mathbb{R}_+$ such that $i \in \arg\min_{l \in \mathcal{N}} F_l^n(\theta_l)$ [13]. Notice that θ_i^n is the lowest report for which agent-i can obtain a better position than the last position if the tie-breaking rule favours him. Therefore $F_i^n(\theta_i^n) = F_j^n(\theta_j) = \min_{l \in \mathcal{N}} F_l^n(\theta_l)$, and hence $\theta_i^n = F_i^{n^{-1}}(F_j^n(\theta_j)$ - the cut-off for agent-i for position-n. Implementation in Dominant Strategies would demand that the utility of agent-i be the same no matter how the tie is resolved, i.e., the mechanism be essentially single-valued. If this is not true, then agent-i can misreport to be in a tie (or not in a tie) to get the advantage (or avoid the disadvantage) of the tie-breaking rule. Hence we can calculate his utility i at position-n and the position he would get if the tie is resolved differently. The tie-breaking rule is the same, but with an arbitrary choice of agent-i and arbitrary θ_{-i}, all cases need consideration. Let $\hat{m}_1$ denote the position of agent-i if the tie is resolved in his favour. This demands that the utilities in equation 26 and in 27 be equal. Let $\theta_i^{m_1}$ denote the highest report for which $\sigma^{JA}(\theta_i, \theta_{-i})$ may be $\hat{m}_1$.

$$u_i(\mu_i(\theta_i^n, \theta_{-i}); \theta_i^n\ ) = -F_i^n(\theta_i^n) + \tau_i(\theta_i^n, \theta_{-i}) \tag{26}$$

$$u_i(\mu_i(\theta_i^n, \theta_{-i}); \theta_i^n\ ) = -F_i^{\hat{m}_1}(\theta_i^n) + \tilde{\tau}_i(\theta_i^n, \theta_{-i}) \tag{27}$$

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[11] It can be verified that $\sigma^{JA}(\theta_i^k, \theta_{-i}) = \sigma^{JA}(\hat{\theta}_i^k, \theta_{-i})$

[12] If there are more than one such agents, consider any such agent arbitrarily.

[13] The functions $F_i^k(\theta_i)$ are increasing functions of θ_i. Hence, we can find a unique θ_i corresponding to any value of $F_i^k(\theta_i)$ for all periods, all agents, and all reports θ_i.

Thus, another necessary condition for the transfer rule is condition 28.

$$\tau_i(\theta_i^n, \theta_{-i}) - \tilde{\tau}_i(\theta_i^n, \theta_{-i}) = F_i^n(\theta_i^n) - F_i^{\hat{m}_1}(\theta_i^n) = \sum_{l=\hat{m}_1+1}^{n} f_i(l, \theta_i^n) \tag{28}$$

Suppose $\theta_i^{m_{l-1}}$ is the lowest type of agent-i so that he may obtain position $\hat{m}_l \in \{1, \ldots, n-1\}$ and the highest type so that he can obtain position $\hat{m}_{l-1} \in \{2, \ldots, n\}$. Clearly, $\hat{m}_l < \hat{m}_{l-1}$ and $\theta_i^{m_{l-1}}$ is the type cut-off of agent-i for position $\hat{m}_{l-1}$. Since DSIC demands essentially single-valuedness, we need the utilities in equations 29 and 30 to be equal.

$$u_i(\mu_i(\theta_i^{m_{l-1}}, \theta_{-i}); \theta_i^{m_{l-1}}) = -F_i^{\hat{m}_{l-1}}(\theta_i^{m_{l-1}}) + \tau_i(\theta_i^{m_{l-1}}, \theta_{-i}) \tag{29}$$

$$u_i(\mu_i(\theta_i^{m_{l-1}}, \theta_{-i}); \theta_i^{m_{l-1}}) = -F_i^{\hat{m}_l}(\theta_i^{m_{l-1}}) + \tilde{\tau}_i(\theta_i^{m_{l-1}}, \theta_{-i}) \tag{30}$$

Thus, another necessary condition for the transfer rule is condition 31 for all positions $\hat{m}_l$ and $\hat{m}_{l-1}$ obtainable by agent-i.

$$\tau_i(\theta_i^{m_{l-1}}, \theta_{-i}) - \tilde{\tau}_i(\theta_i^{m_{l-1}}, \theta_{-i}) = F_i^{\hat{m}_{l-1}}(\theta_i^{m_{l-1}}) - F_i^{\hat{m}_l}(\theta_i^{m_{l-1}}) = \sum_{l=\hat{m}_l+1}^{\hat{m}_{l-1}} f_i(l, \theta_i^{m_{l-1}}) \tag{31}$$

From conditions 25 and 31, if $\theta_i \in \mathbb{R}_+$[14] is any report such that $\sigma^{JA}(\theta_i, \theta_{-i}) = \hat{m}_l$, then for all obtainable positions $\hat{m}_l \in \{1, \ldots, n-1\}$, equation 32 is necessary.

$$\tau_i(\theta_i, \theta_{-i}) = \tilde{\tau}_i(\theta_i^{m_{l-1}}, \theta_{-i}) = \tau_i(\theta_i^{m_{l-1}}, \theta_{-i}) - \sum_{l=\hat{m}_l+1}^{\hat{m}_{l-1}} f_i(l, \theta_i^{m_{l-1}}) \tag{32}$$

From equation 32, we have $\tilde{\tau}_i(\theta_i^{m_{l-1}}, \theta_{-i}) = \tau_i(\theta_i^{m_l}, \theta_{-i})$.

Suppose $\bar{\theta}_i$ is such that $\sigma_i^{JA}(\bar{\theta}_i, \theta_{-i}) = \hat{m}_r$ for some $r \in \{0, \ldots, M(i)\}$. From 32, $\tau_i(\bar{\theta}_i, \theta_{-i}) = \tilde{\tau}_i(\theta_i^{m_{r-1}}, \theta_{-i}) = \tau_i(\theta_i^{m_r}, \theta_{-i})$. Let $\hat{m}_r > \hat{m}_l$, without loss of generality.

$$u_i(\mu_i(\theta_i, \theta_{-i}); \theta_i) = -F_i^{\hat{m}_l}(\theta_i) + \tau_i(\theta_i, \theta_{-i}) = -F_i^{\hat{m}_l}(\theta_i) + \tau_i(\theta_i^{m_l}, \theta_{-i}) \tag{33}$$

$$u_i(\mu_i(\bar{\theta}_i, \theta_{-i}); \bar{\theta}_i) = -F_i^{\hat{m}_r}(\bar{\theta}_i) + \tau_i(\bar{\theta}_i, \theta_{-i}) = -F_i^{\hat{m}_r}(\bar{\theta}_i) + \tau_i(\theta_i^{m_r}, \theta_{-i}) \tag{34}$$

$$u_i(\mu_i(\bar{\theta}_i, \theta_{-i}); \theta_i) = -F_i^{\hat{m}_r}(\theta_i) + \tau_i(\bar{\theta}_i, \theta_{-i}) = -F_i^{\hat{m}_r}(\theta_i) + \tau_i(\theta_i^{m_r}, \theta_{-i}) \tag{35}$$

$$u_i(\mu_i(\theta_i, \theta_{-i}); \bar{\theta}_i) = -F_i^{\hat{m}_l}(\bar{\theta}_i) + \tau_i(\theta_i, \theta_{-i}) = -F_i^{\hat{m}_l}(\bar{\theta}_i) + \tau_i(\theta_i^{m_l}, \theta_{-i}) \tag{36}$$

[14] We know from the way the Just Algorithm works that such $\theta_i \in [\theta_i^{m_{l-1}}, \theta_i^{m_l}]$

The DSIC condition requires that 26 $\geq$ 35 and 27 $\geq$ 36, which together demand condition 37.

$$\sum_{l=\hat{m}_l+1}^{\hat{m}_r} f_i(l,\theta_i) \geq \tau_i(\theta_i^{m_r},\theta_{-i}) - \tau_i(\theta_i^{m_r},\theta_{-i}) \geq \sum_{l=\hat{m}_l+1}^{\hat{m}_r} f_i(l,\bar{\theta}_i) \tag{37}$$

Let $\{1,\ldots,\hat{m}_l,\ldots,\hat{m}_r,\ldots,n\}$ be obtainable positions for agent-i.

Also, let $(\theta_i^n,\ldots,\theta_i^{m_r},\ldots,\theta_i^{m_l},\ldots,\theta_i^{m_{M(i)}})$ be the type cut-off(s) vector. Let $l = r+t$ for some $1 \leq t \leq M(i)-1$. Then $\theta_i^{m_r} \leq \theta_i^{m_{r+t}} \leq \theta_i^{m_l}$ for all t.

The cost functions $f_i(k,\theta_i)$ are non-decreasing for all $k \in \{2,\ldots,n\}$. Hence, the inequality 37 is always valid. Moreover, if the necessary conditions 25, 31, and 32 hold, then condition 37 always holds and is thus not a binding condition. This means that if the mechanism is DSIC for reports that change obtainable positions locally, then the mechanism is also DSIC for reports that change the agent's position globally. Adding condition 28 to other necessary conditions, we get $\tau = \tau^{JA}$. This completes the only-if part of the proof. It is easy to verify that the transfer rule τ^{JA} satisfies conditions 25, 28, 31, and 32. The verification is left to the reader.

This completes the proof. □

Remark 1 *In the transfer rule τ^{JA}, notice that if for every agent-$i \in N$ and $\forall \theta_{-i} \in \mathbb{R}_{++}$, we let $h_i(\theta_{-i}) = 0$, then the sum of transfers is negative. Therefore, the identified class of mechanism includes feasible mechanisms.*

We end this section with a demonstration of the proposed mechanism. We take the same values as in Example 5 and demonstrate that agent-2 cannot gainfully misreport.

Example 6 *Consider three agents $\mathcal{N} = \{1,2,3\}$. Let the tie-breaking rule be $1 \succ_{TB} 2 \succ_{TB} 3$. Let $\theta_1 = 5$, $\theta_2 = 7$, and $\theta_3 = 15$. The cost matrix is given below.*

$$\theta = \begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix} = \begin{bmatrix}\theta_1 & \theta_1 & 18\theta_1\\ \theta_2 & 2\theta_2 & 11\theta_2\\ \theta_3 & 3\theta_3 & 3\theta_3\end{bmatrix} \to \begin{bmatrix}\theta_1 & 2\theta_1 & 20\theta_1\\ \theta_2 & 3\theta_2 & 14\theta_2\\ \theta_3 & 4\theta_3 & 7\theta_3\end{bmatrix} = \begin{bmatrix}5 & 10 & 100\\ \theta_2 & 3\theta_2 & 14\theta_2\\ 15 & 60 & 105\end{bmatrix}$$

If agent-2 reports truthfully, then he is served in position 3 and obtains a transfer of $h_2(\theta_{-2})$*. His total utility is* $u_2(\sigma_2^{JA}(\theta), \tau_2^{JA}(\theta)) = h_2(\theta_{-2}) - 98$*.*

If agent-2 reports his type $\theta_2' \in (0, \frac{100}{14}]$*, then* $\sigma_2^{JA}(\theta_2', \theta_{-2}) = 3$*, and* $\tau_2^{JA}(\theta_2', \theta_{-2}) = h_2(\theta_{-2})$*. Therefore,* $u_2(\sigma_2^{JA}(\theta_2', \theta_{-2}), \tau_2^{JA}(\theta_2', \theta_{-2}); \theta_2) = -98 + h_2(\theta_{-2}) = u_2(\sigma_2^{JA}(\theta), \tau_2^{JA}(\theta))$*.*

If he reports his type $\hat{\theta}_2 \in (\frac{100}{14}, 20]$*, then* $\sigma_2^{JA}(\hat{\theta}_2, \theta_{-2}) = 2$ *and* $\tau_2^{JA}(\hat{\theta}_2, \theta_{-2}) = h_2(\theta_{-2}) - 11(\frac{100}{14})$*. Therefore,* $u_2(\sigma_2^{JA}(\hat{\theta}_2, \theta_{-2}), \tau_2^{JA}(\hat{\theta}_2, \theta_{-2}); \theta_2) = -21 + h_2(\theta_{-2}) - 11(\frac{100}{14}) \approx -99.57 + h_2(\theta_{-2}) < u_2(\sigma_2^{JA}(\theta), \tau_2^{JA}(\theta))$*.*

Similarly, for any report $\bar{\theta}_2 \in (20, \infty)$*,* $\sigma_2^{JA}(\bar{\theta}_2, \theta_{-2}) = 1$ *and* $\tau_2^{JA}(\bar{\theta}_2, \theta_{-2}) = h_2(\theta_{-2}) - 11(\frac{100}{14}) - 2(20)$*. Therefore,* $u_2(\sigma_2^{JA}(\bar{\theta}_2, \theta_{-2}), \tau_2^{JA}(\bar{\theta}_2, \theta_{-2}); \theta_2) = -7 + h_2(\theta_{-2}) - 11(\frac{100}{14}) - 2(20) \approx -125.57 + h_2(\theta_{-2}) < u_2(\sigma_2^{JA}(\theta), \tau_2^{JA}(\theta))$*.*

Hence, with the transfer rule τ^{JA}*, agent-2 can never gainfully manipulate. The same may be verified for other agents and considering other true types of agent-2. This completes the example.*

Example 6 demonstrates the working of the proposed mechanism in a simple three-agent case.

6 Conclusion

In this paper, we examined the challenge of implementing a Rawlsian queueing rule in queueing problems where agents have heterogeneous per-period waiting costs. We introduced the *Just Algorithm*, a straightforward method that consistently selects a Rawlsian queue under complete information by minimising the maximum individual waiting cost among all agents. Our primary objective was to design mechanisms that implement the Rawlsian queue selected by the *Just Algorithm* in Dominant Strategies, thereby ensuring allocational fairness even when agents act strategically.

We found that within the unrestricted domain of agents' types, where agents possess multidimensional private information, no Dominant Strategy Incentive-Compatible (DSIC) mechanism exists that can implement the Rawlsian queue selected by the *Just*

Algorithm. This negative result underscores the inherent challenges of designing fair mechanisms in multidimensional environments, even under quasilinear preferences.

To address this impossibility, we introduced a domain restriction to *one-dimensional private information*. Specifically, while agents differ in how their per-period waiting costs evolve over periods, this aspect is public information. Their private information is confined to their first-period waiting cost. This restriction is nontrivial because it does not allow us to identify a Rawlsian queue solely by ordering agents' private information, contrasting with the achievements in seeking *first best* mechanisms as discussed in Mitra (2001). If agents differ only in the private type and the publicly known aspect is identical for all agents, then the aggregate cost-minimising queue is also a Rawlsian queue, and results from Mitra (2001) would apply. But we did not impose any such restriction.

Within the restricted domain, we identified a class of DSIC mechanisms that implement the Rawlsian queueing rule in dominant strategies. An interesting observation is that while the cut-off(s) approach is well studied in the mechanism design literature, the same approach applied to our frameworks yields different numbers of cut-off(s) for different agents. Further, one agent may be pivotal to determining cut-off(s) for multiple queue positions for another agent, and not all queue positions might be accessible for an agent given the other's types. The origin of this novel feature lies in the functions determining how agents' costs evolve with queue positions. The DSIC mechanism we present is robust in the sense that none of this lies beyond the scope of our mechanism.

These findings contribute to the broader investigation of implementing fair social choice or public decision rules in quasi-linear environments. It highlights the complexities involved in mechanism design when dealing with *multidimensional private information* and the pursuit of fairness.

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Declarations

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- The authors declare no conflict of interest/Competing interests.
- The author ordering is alphabetical last name ordering.

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