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Implementing Efficiency with Equality

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Abstract

We consider the implementation of efficiency with minimum inequality in a large population model of negative externalities. Formally, the model is one of tragedy of the commons with the aggregate strategy at the efficient state being lower than at the Nash equilibrium. A planner can restore efficiency by imposing an externality equivalent tax and then redistributing the tax revenue as transfers to lower inequality. We characterize the payment scheme that minimizes inequality, as measured by the Gini coefficient, at the efficient state subject to incentive compatibility and budget balance. We then construct a mechanism that implements efficiency with minimum inequality in dominant strategies. We also show that minimizing inequality at the efficient state maximizes the minimum payoff at efficiency. However, it is not equivalent to implementing the Rawlsian social choice function.

Keywords: Negative Externalities; Efficiency; Equality; VCG mechanism.

JEL classification: C72; D62; D63; D82.

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1 Introduction

The classical literature on welfare economics and mechanism design has considered the question of achieving social efficiency in great detail. Efficiency is generally interpreted as implementing the utilitarian social choice function that seeks to maximize aggregate payoff in society. Perhaps the most well-known mechanisms pursuing this objective is the class of Vickrey–Clarke–Groves (VCG) mechanisms (Vickrey [35], Clarke [6], Groves [17]), which take a dominant strategy implementation approach to this problem. As far as we know, however, this literature has not addressed one important question. That question is minimizing inequality at the socially efficient state subject to the usual incentive compatibility and budget balance constraints. One reason may be that in the usual setting of finite player mechanisms, dominant strategy implementation of efficiency with incentive compatibility is not possible with budget balance (Green and Laffont [16]).[1] Adding the objective of minimum inequality would only make the problem even more intractable. But equality is an important objective for any society. This is not just on grounds of fairness but also because higher inequality may prove detrimental to the utilitarian objective of economic growth by, for example, facilitating elite capture of institutions (Sokoloff and Engerman [32]). Therefore, reconciling efficiency with equality is a significant question.

The motivation behind this paper is to address this question. But since a finite player setting would prove intractable, we adopt a large population approach where there are a continuum of agents, each agent being of measure zero. Unlike in finite player mechanisms, we can accommodate goals of efficiency with incentive compatibility and budget balance in large population mechanisms (Lahkar and Mukherjee [22]). Therefore, it may also be possible to combine minimum inequality with efficiency in such an environment, which is what this paper seeks to establish. In particular, we seek to characterize and implement the efficient outcome with minimum inequality, as measured by the *Gini coefficient*, subject to incentive compatibility and budget balance.

There exists a literature on implementation in large population games. Such models, however, focus exclusively on efficiency. Thus, Sandholm [30, 31], who pioneered this literature, and Lahkar and Mukherjee [21, 23] consider the evolutionary implementation of efficiency in large population games by imposing an externality price that generates a potential game. Bandhu and Lahkar [2] show the evolutionary robustness of dominant strategy implementation that implements efficiency in a model of strategic complementarities. The present paper does not consider evolutionary implementation. Instead, its approach is closer to the more classical VCG-type mechanism of Lahkar and Mukherjee [22] adapted to the large population context. We consider the standard environment of incomplete information but in a large population instead of in a finite player model. The main

[1] Thus, VCG mechanism implements efficiency by leaving a budget surplus with the planner. Budget balance would require that there should be neither surplus nor deficit with the planner. An alternative to the VCG mechanism is the AGV mechanism (Arrow [1], d'Aspremont and Gérard–Varet [8]) that generates truthful revelation as a Bayesian Nash equilibrium. However, this mechanism requires a stronger assumption that the type distribution is common knowledge and does not satisfy ex-post individual rationality. More recently, Mukherjee et al. [24] established implementation of efficiency with budget balance in a finite player public goods model in "undominated strategies" rather than dominant strategies via a "social choice correspondence" rather than a social choice function.

objective of this paper is to implement not just efficiency but also minimum inequality in such an environment. We do so by implementing such an outcome in dominant strategies.[2] As far as we know, whether under the classical finite player approach or the large population approach, this is the first paper that seeks to implement not just efficiency but efficiency with minimum inequality.

We consider a large population model with strategic interlinkages and negative externalities. Total output is a function of aggregate strategy by agents, which creates interlinkages. Different types of agents have different effort cost functions, which is private information, and they receive a share of total output according to individual effort. Thus, payoffs are equivalent to that of a large population tragedy of the commons (Lahkar and Mukherjee [23]). The tragedy of the commons is, of course, a canonical model of negative externalities in economics. It is widely used to explain a range of socially inefficient phenomena like industrial pollution, overexploitation of natural resources and congestion in a traffic or computer network. Another application is to the financial sector that produces exotic new financial products (Chakrabarti and Lahkar [3]). As more and more institutions release exotic new financial products in, say, the housing market (a common resource), they also start holding each other's products thereby creating interlinkages. This can have detrimental consequences, as exemplified by the 2007-08 financial crisis.[3]

Our environment is one of incomplete information. Hence, agents do not know about the types of other agents. In fact, as is typical in any dominant strategy implementation mechanism, even the type distribution will not be known. Nevertheless, it is worth investigating what happens when agents have more information. We then observe that as in any tragedy of the commons, negative externality causes agents to play too high an aggregate strategy relative to the efficient level at the Nash equilibrium. Hence, merely having information doesn't help agents in coordinating at the efficient state. However, the mechanism we propose will achieve social efficiency with minimum inequality even under incomplete information.

We introduce a planner who taxes the externality causing activity. To this extent, our approach is similar to that of earlier models of implementation in large population games such as Sandholm [30, 31], Lahkar and Mukherjee [21, 22, 23] and Bandhu and Lahkar [2]. The key point of departure of this paper is that we also propose a transfer scheme that redistributes the tax revenue in a way that reduces inequality without sacrificing efficiency. Thus, a novel feature of this paper is to design a tax and transfer scheme that simultaneously resolves the problem of negative externalities and improves equality. From a policy perspective, this is an important finding. It illustrates that improving efficiency and enhancing equality are not contradictory goals. Instead, the resources the policymaker requires to promote equality can arise from the taxation of those very activities that harm efficiency.[4]

It is worth emphasizing that our implementation mechanism is one of simultaneous strategies.

[2] For theoretical foundations of dominant-strategy mechanism see Chung and Ely [5]; Chen and Li [4].

[3] See, for example, Elliot et al. [10] for an analysis of such financial interlinkages from a network theory perspective. Also see Chakrabarti and Lahkar [3] for a discussion of, besides finance, other industries like railways and information technology that may be modeled as large population tragedies of the commons.

[4] Thus, going back to our illustrative example, it may be possible to curb the speculative excesses of the financial sector through a financial transactions tax and use the proceeds for redistributive purposes.

However, it is more intuitive to think of the implementation in two parts. First, taxing agents induces them to choose the efficient outcome. Second, using the tax revenue to give transfers to disadvantaged groups to improve their welfare. Thus, this explanation decomposes the net payment to agents into two parts; a tax and a transfer. But in mechanism design terminology, it is a simultaneous move game. Agents announce their type and based on those announcements, they are assigned a (single) net payment. This net payment is the difference between transfer and tax and is assigned at a single instant of time rather than sequentially. The net payment approach is similar to the simultaneous move mechanism in Lahkar and Mukherjee [22] in a large population public goods game. Of course, the details of calculating the payments differ. That is because, in the earlier paper, the objective of the planner was only efficiency. In the present model, the planner also wants to minimize inequality at efficiency.[5]

We measure inequality using the Gini coefficient (Gini [13, 14]). This is arguably the most well-known measure of inequality. The planner's objective is to design a tax and transfer scheme or, equivalently, a net payment scheme, that minimizes the Gini coefficient of payoffs at the efficient state. In our mechanism, the planner assigns strategies and net payments to agents based on reported types. We characterize the net payments that make truthful revelation weakly dominant while minimizing the Gini coefficient at the efficient state. Due to considerations of incentive compatibility, the equality achieved is not perfect. Nevertheless, agents disadvantaged with a higher cost of effort still receive a higher transfer, due to which the inequality that remains is less than that achievable through any other payment scheme, such as an equal redistribution. Intuitively, equal redistribution, which suffices for efficiency, can be implemented with truthful revelation being strictly dominant. This leaves the planner enough scope to adjust incentive compatibility conditions to design a redistribution scheme that makes truthful revelation weakly dominant and, thereby, improve equality without compromising efficiency.

An important technical caveat to our results is that they hold for large population models or models where all agents are of measure zero. This is an important assumption because our analysis relies on the fact that changes in individual strategy do not affect aggregate variables. This adds considerably to the tractability of our problem. For example, the budget balance condition, which is crucial for us, is satisfied at efficiency in our large population context. But, as noted earlier, it is difficult to achieve in conventional finite player mechanisms. Of course, in real-world situations, no agent is ever of measure zero. But in most economic environments where public policy questions like redistribution assume importance, we would expect the number of people involved to be fairly large. Further, it is reasonable to assume that in such situations, agents would behave as if their individual actions cannot influence aggregate variables. In that case, as in models of competitive markets, we would expect our conclusions to be valid, at least approximately.[6]

Independent of efficiency, the classical implementation literature has considered equality from

[5]The present approach can also be extended to a model of positive externalities like the public goods game in Lahkar and Mukherjee [22]. The problem there would be to characterize a vector of inequality minimizing taxes that provides the revenue to subsidize a socially beneficial activity and restore efficiency.

[6]See the last paragraph of Section 6 for some remarks on how such approximations may hold.

the point of view of implementing the Rawlsian social choice function (Rawls [27]), which seeks to maximize the minimum welfare in society. It is known, for example, that this social choice function is not implementable. While this is not our main focus, our model will also provide some insights into combining utilitarian and Rawlsian objectives. In particular, we show that subject to the feasibility constraints, our transfer scheme not only minimizes inequality at efficiency but also maximizes the minimum payoff at the efficient state. Thus, in this sense, our transfer scheme is a Rawlsian one but restricted to the efficient state. It provides a partial reconciliation of the utilitarian objective of efficiency with the Rawlsian objective of maximizing the minimum payoff. The reconciliation, however, is not complete because by using a counterexample, we show that this is not equivalent to implementing the Rawlsian social choice function. If we are willing to sacrifice efficiency, then we can identify another state where the minimum payoff is higher but aggregate payoff is lower. This is the Rawlsian outcome subject to incentive compatibility and budget balance. Again, as far as we know, the existing literature on implementation theory has not addressed such differences between implementing the Rawlsian social choice function and implementing the efficient outcome with minimum inequality.

We emphasize that we are using the standard notion of efficiency as employed in, for example, VCG mechanisms. It is the allocation that maximizes the aggregate payoff for any given type profile. This is the concept of efficiency that has been used in other large population models of implementation (Sandholm [30, 31], Lahkar and Mukherjee [21, 22]). We apply standard dominant strategy incentive compatibility, which is stronger than Bayesian incentive compatibility, for implementing our desired allocation. Dominant strategies are prior free. Hence, our analysis does not incorporate any belief structure. Therefore, distinctions between belief-based efficiency concepts like ex-ante efficiency, interim efficiency and ex-post efficiency considered in, for example, Holmström and Myerson [18] are not strictly applicable to our model. Nevertheless, to the best of our understanding, our notion of efficiency is closest to ex-post efficiency. This is the efficient outcome the planner would have calculated had agents' types been known. Dominant strategy incentive compatibility allows the planner to implement that outcome without any knowledge of types or even the type distribution.

We can interpret our objective of minimizing inequality at the efficient state as seeking the most egalitarian outcome subject to efficiency. Thus, in a sense, efficiency is non-negotiable for us but we want the most egalitarian efficient outcome. There is a literature on other notions of egalitarianism. For example, Grant et al. [15] and Fleurbaey [11] consider the distinction between ex-ante and ex-post egalitarianism. This distinction arises due to the presence of uncertainty in their models.[7] However, there is no uncertainty in our paper. Hence, the difference does not arise. Another feature of our paper is the focus on implementing the egalitarian efficient outcome. For

[7] Ex-ante egalitarianism is relevant before the resolution of uncertainty while ex-post is after the resolution of uncertainty. There is a certain tension in these models between ex-ante efficiency and comparison of ex-post social allocations. In our model, we can clearly identify the most efficient egalitarian outcome. Hence, the tension is not present. The underlying reason is that distinctions between ex-ante and ex-post notions of efficiency and egalitarianism are not present in our model.

example, Kalai [19] considers the pure egalitarian outcome, which equalizes agents' utilities. Had implementability not been a constraint, we would have chosen this outcome as well. But as we discuss in Section 3, the pure egalitarian outcome is not implementable at efficiency.

There are also papers that address the issue of fairness at efficiency in other ways. A widely recognized axiom underlying fairness is envy freeness. No agent should desire the allocation of another agent. There are both positive and negative results on whether envy freeness is consistent with efficiency. For example, models by Tadenuma and Thomson [34] and Pápai [26] highlight conflict between envy freeness and other goals like efficiency, incentive compatibility and budget balance. On the other hand, Ohseto [25] and Sprumount [33] present models where efficiency and envy freeness are compatible. The details of these models are very different from ours. They are finite player object allocation models, while ours is a large population model of negative externalities. Hence, direct comparisons of results are difficult. Nevertheless, it is easy to see that our model does satisfy envy freeness at the efficient state. Transfers are designed in such a way that no agent prefers the allocation of any other type of agent.

The rest of the paper is as follows. Section 2 presents our model of the tragedy of the commons and characterizes its Nash equilibrium and efficient state. In Section 3, we identify the transfer vector that achieves the most equitable payoff at the efficient state subject to budget balance and incentive compatibility. Section 4 describes the dominant strategy mechanism that implements efficiency with minimum inequality. Section 5 presents the counterexample about the Rawlsian social choice function. Section 6 concludes. Some proofs are in the Appendix.

2 The Model

We consider a society consisting of a continuum of agents, each of measure zero. The society is divided into a finite set of populations, also called types, $\mathcal{P} = \{1, 2, \cdots, n\}$. The mass of type $p \in \mathcal{P}$ is $m_p \in (0, 1)$ with $\sum_{p\in\mathcal{P}} m_p = 1$. Thus, the total mass of the society is 1. We refer to the distribution $m = (m_1, m_2, \cdots, m_n)$ as the type distribution in the society. In certain parts of the paper, we require the type distribution to, for example, characterize the efficient state and the inequality minimizing transfers. However, when we present the most important result of the paper on implementing efficiency with minimum inequality in Section 4, we will assume incomplete information. The type distribution will not be known either to agents or the planner.

Every agent in the society has a common strategy set $\mathcal{S} = (0, \infty)$.[8] Throughout, we will interpret $x \in \mathcal{S}$ synonymously as the effort exerted by an agent. We denote the strategy distribution in a population by a finite positive measure μ_p such that $\mu_p(A) \in [0, m_p]$ is the mass of agents in population p who are playing strategies in $A \subseteq \mathcal{S}$. Hence, $\mu_p(\mathcal{S}) = m_p$. If every agent in population p plays the same strategy x, then we obtain a monomorphic population state which we denote it as $m_p\delta_x$. We interpret the vector of population states $\mu = (\mu_1, \mu_2, \cdots, \mu_n) \in \Delta$ as the state of the entire society or the social state. The aggregate strategy level in the society at the social state μ is

[8]For certain technical reasons explained later, we exclude the 0 strategy.

then

$$A(\mu) = \sum_{p \in \mathcal{P}} \int_{\mathcal{S}} x \mu_p(dx). \tag{1}$$

In our subsequent analysis, we will frequently refer to $A(\mu) \in (0, \infty)$ as α.

We consider an economy with interlinkages between agents. To capture such interlinkages, we assume that total output in the society depends upon the aggregate strategy level. Formally, we consider a smooth, strictly increasing and strictly concave production function $\pi : (0, \infty) \to \mathbf{R}_+$ such that $\pi(A(\mu))$ is the total output in the society when the aggregate strategy is $A(\mu)$. We assume that π satisfies the Inada conditions and that $\frac{\pi(\alpha)}{\alpha}$ is strictly declining for all $\alpha \in (0, \infty)$. An agent exerting effort x then receives a share $\frac{x}{A(\mu)}$ of the total output and incurs an effort cost $c_p(x)$. These cost functions are the source of type-specific distinctions in our model. They differ according to the type of agents but it is the same for all agents of a particular type. We assume that every such type specific cost function $c_p : \mathcal{S} \to \mathbf{R}_+$ is smooth, strictly increasing and strictly convex and satisfies $c_p(0) = c'_p(0) = 0$ if we extend the function to 0, where $c'_p(x)$ is type p's marginal cost. Thus, there are no fixed costs and the marginal cost also tends to zero as $x \to 0$. In addition, we make the following assumption about the cost functions.

Assumption 2.1 *For every $p, q \in \mathcal{P}$, $c_q(x) - c_p(x)$ is strictly increasing in x if $q > p$.*

This assumption can be equivalently written as $c'_q(x) > c'_p(x)$ for all $x \in \mathcal{S}$ if $q > p$, i.e. marginal cost at any level of effort is higher for higher types. It has an important implication. Recall that fixed cost is zero for all types. Hence, the area beneath the marginal cost is total cost. Assumption 2.1, therefore, generates the following observation.

Observation 2.2 *For every $x \in \mathcal{S}$, $c_p(x)$ is strictly increasing in $p \in \mathcal{P}$. Thus, for every $x \in \mathcal{S}$, $c_1(x) < c_2(x) < \cdots < c_n(x)$.*

This observation gives us an important labeling convention in our model. Higher-cost agents are classified as higher types. Hence, we may interpret agents labeled as being of a higher type as facing a greater disadvantage in exerting effort.

A population game is a weakly continuous mapping $F : \mathcal{S} \times \mathcal{P} \times \Delta \to \mathbf{R}$ such that $F_{x,p}(\mu)$ is the payoff of an agent from population p who plays strategy x at the social state μ. Given the production and cost functions, this payoff in our model takes the form

$$\begin{aligned} F_{x,p}(\mu) &= \frac{x}{A(\mu)} \pi(A(\mu)) - c_p(x) \\ &= x AP(A(\mu)) - c_p(x), \end{aligned} \tag{2}$$

where $AP(A(\mu)) = \frac{\pi(A(\mu))}{A(\mu)}$ is the average product of the production function when the aggregate effort is $A(\mu)$. Thus, our assumption that $\frac{\pi(\alpha)}{\alpha}$ is strictly declining is equivalent to the average product function being strictly declining. Formally, the payoff (2) is equivalent to a large population tragedy of the commons model with the aggregate output $\pi(A(\mu))$ being shared among agents in proportion to their individual effort x (Lahkar and Mukherjee [23]).[9] As noted in the Introduction, the large population structure is essential for our results to hold. These results will, however, hold for all type distributions m.

2.1 Nash Equilibrium and Efficient State

The population game F defined by (2) is an aggregative game as the payoff of an agent depends entirely upon his individual strategy and the aggregate strategy level $A(\mu)$ (Corchón [7]). We now use this aggregative structure of F to characterize its Nash equilibrium and efficient state.[10] For this purpose, we assume the agents know the type distribution m and can observe the aggregate strategy $A(\mu)$. This assumption will also help us characterize the efficient state of F. But when presenting our main result, Theorem 4.1, we will drop this assumption.

Let us denote the aggregate strategy level $A(\mu)$ as α and write (2) as $xAP(\alpha) - c_p(x)$. The strict convexity of $c_p(x)$ implies that for every given α, this function has a unique maximizer in $\mathcal{S}$. This maximizer, which we denote as $b_p(\alpha)$, is the unique best response of a type p agent to every social state μ such that $A(\mu) = \alpha$. The following proposition then characterizes the unique Nash equilibrium of our model. Further details of the proof are in Appendix A.1.

Proposition 2.3 *Consider the population game F defined by (2). Denote by α^N the unique solution to*

$$\sum_{p \in \mathcal{P}} m_p b_p(\alpha) = \alpha. \tag{3}$$

Then, F has a unique Nash equilibrium

$$\mu^N = \left(m_1 \delta_{\alpha_1^N}, m_2 \delta_{\alpha_2^N}, \cdots, m_n \delta_{\alpha_n^N} \right) \tag{4}$$

where $\alpha_p^N = b_p(\alpha^N)$ and $b_p(\alpha)$ is the unique best response function in F as characterized in (5). Thus, every agent of type $p \in \mathcal{P}$ plays strategy $\alpha_p^N = b_p(\alpha^N)$ at this Nash equilibrium. The aggregate strategy at μ^N is, therefore, $\alpha^N = \sum_{p \in \mathcal{P}} m_p \alpha_p^N$. The Nash equilibrium is characterized by

$$AP(\alpha^N) = c_p'(b_p(\alpha^N)). \tag{5}$$

Intuitively, (3) implies that a Nash equilibrium of an aggregative game is a social state such that when all agents play their best response to that state, the aggregate strategy level remains unchanged. The key to Proposition 2.3 is that $b_p(\alpha)$ is strictly declining due to our assumptions about $AP(\alpha)$ and the cost functions. Hence, (3) has a unique solution in our model, which characterizes the unique Nash equilibrium.

[9] See footnote 8. The reason for excluding the 0 strategy is to ensure that (2) is defined at all social states. Otherwise, if all agents play 0, the average product would be undefined.

[10] These results have also been established in Lahkar and Mukherjee [23]. Nevertheless, we present them here as well briefly in order to keep the present paper self–contained.

Condition (5) implies that this Nash equilibrium involves equating average product to marginal cost. That cannot be efficient. Instead, to characterize the efficient state, we consider the aggregate payoff. The aggregate payoff in a population game F at a social state μ, denoted $\bar{F}(\mu)$, is the total payoff earned by all agents at that state. Hence, given the payoff function (2), the aggregate payoff in our model is

$$
\begin{aligned}
\bar{F}(\mu) &= \sum_{p\in\mathcal{P}} \int_{\mathcal{S}} F_{x,p}(\mu)\mu_p(dx) \\
&= \sum_{p\in\mathcal{P}} \int_{\mathcal{S}} \left(\frac{x}{A(\mu)} \pi(A(\mu)) - c_p(x) \right) \mu_p(dx) \\
&= \frac{\pi(A(\mu))}{A(\mu)} \sum_{p\in\mathcal{P}} \int_{\mathcal{S}} x\mu_p(dx) - \sum_{p\in\mathcal{P}} \int_{\mathcal{S}} c_p(x)\mu_p(dx) \\
&= \pi(A(\mu)) - \sum_{p\in\mathcal{P}} \int_{\mathcal{S}} c_p(x)\mu_p(dx), \qquad (6)
\end{aligned}
$$

where the last equality follows from the definition of the aggregate strategy in (1). Thus, the aggregate payoff at a state μ is the total output generated by the society at that state minus the aggregate cost $\sum_{p\in\mathcal{P}} \int_{\mathcal{S}} c_p(x)\mu_p(dx)$ incurred by agents at that state. An *efficient state* of F is then a state μ^* *that maximizes* the aggregate payoff (6) in Δ.

The strategic interlinkages in our model imply there are externalities. Therefore, characterizing an efficient state would require us to account for such externalities. Let $e_{x,p}(\mu)$ denote the total externality imposed by an agent of type p who plays strategy x at the state μ on the entire society. Corollary 5.7 in Lahkar and Mukherjee [23] calculates this total externality in a tragedy of the commons model such as the present one to be

$$
e_{x,p}(\mu) = x\left(MP(A(\mu)) - AP(A(\mu))\right), \qquad (7)
$$

where $MP(\alpha) = \pi'(\alpha)$ is the marginal product of π at the aggregate strategy α. Our assumption that $AP(\alpha)$ is strictly declining implies $MP(\alpha) < AP(\alpha)$ at all $\alpha \in (0, \infty)$ so that $e_{x,p}(\mu) < 0$. Hence, externalities are negative in our model, which is another standard characteristic of the tragedy of the commons problems.[11]

It is known from Sandholm [29] that an efficient state of a population game F is also a Nash equilibrium of another game $\hat{F}$ we obtain by adding externalities in F to the original payoffs. We interpret the addition of this externality as the imposition of a tax that compels agents to internalize the externality they create. The payoff of a type p agent who plays strategy x in $\hat{F}$ is

$$
\begin{aligned}
\hat{F}_{x,p}(\mu) &= F_{x,p}(\mu) + e_{x,p}(\mu) \\
&= xAP(A(\mu)) - c_p(x) + x(MP(A(\mu)) - AP(A(\mu)))
\end{aligned}
$$

[11]See Appendix A.1.1 in Lahkar and Mukherjee [21] for the technical details of calculating externalities in large population games with a continuous strategy set. Also see Proposition 4.1 in Lahkar and Mukherjee [23] for a general derivation of externalities in aggregative games.

$$= xMP(A(\mu)) - c_p(x). \tag{8}$$

Like (2), (8) is also an aggregative game with the only difference being that the average product gets replaced by the marginal product. Hence, we can apply the same method as in Proposition 2.3 to obtain the Nash equilibrium of $\hat{F}$ or, equivalently, the efficient state of F. Thus, let $\hat{b}_p(\alpha)$ be the unique best response of a type p agent in $\hat{F}$ defined by (8) at a social state μ such that $A(\mu) = \alpha$. We then obtain the following result. Further details are in Appendix A.1.

Proposition 2.4 *Consider the population game $\hat{F}$ defined by (8) and the best response $\hat{b}_p(\alpha)$ characterized in (11). This game has a unique Nash equilibrium*

$$\mu^* = \left(m_1\delta_{\alpha_1^*}, m_2\delta_{\alpha_2^*}, \cdots, m_n\delta_{\alpha_n^*}\right), \tag{9}$$

where $\alpha_p^* = \hat{b}_p(\alpha^*)$ is the strategy of every agent of type p at μ^* and α^* is the unique solution to

$$\sum_p m_p\hat{b}_p(\alpha) = \alpha. \tag{10}$$

Hence, μ^* is also the efficient state of the original game F defined by (2). The aggregate strategy at μ^* is $\alpha^* = \sum_{p\in\mathcal{P}} m_p\alpha_p^*$. Further, for each p, $\alpha_p^* < \alpha_p^N$, the Nash equilibrium strategy level characterized in Proposition 2.3. Hence, $\alpha^* < \alpha^N$. Moreover, μ^* is characterized by

$$MP(\alpha^*) = c_p'\left(\hat{b}_p(\alpha^*)\right). \tag{11}$$

The intuition behind this result arises from negative externalities. Thus, as is any model of negative externalities, the efficient state involves a lower strategy level than at the Nash equilibrium. This is true for all types of agents and, therefore, at the aggregate level as well. Therefore, we arrive at the important conclusion in this section. The Nash equilibrium is the natural prediction of behavior when agents know the type distribution. Hence, even with such information, externalities imply agents' behavior will differ from the efficient state. However, once we introduce our mechanism in Section 4, we will show that efficiency can be achieved even with incomplete information. That will require a planner, whom we introduce in the next section.

At the efficient state, as implied by (11), every agent equates the marginal product of π to his type-specific marginal cost. This is, of course, the hallmark of efficiency. The following corollary provides a ranking of payoffs at the efficient state. The proof of the corollary is in Appendix A.1.

Corollary 2.5 *Consider the efficient state μ^* characterized in Proposition 2.4. Using (8) and Proposition 2.4, let us denote*

$$\hat{F}_{\alpha_p^*,p}(\mu^*) = \alpha_p^* MP(\alpha^*) - c_p(\alpha_p^*), \tag{12}$$

as the payoff of a type p agent at the efficient state of the tragedy of the commons F defined by (2). If $p < q$, then $\hat{F}_{\alpha_p^*,p}(\alpha^*) > \hat{F}_{\alpha_q^*,q}(\alpha^*)$. Moreover, $\alpha_p^* > \alpha_q^*$ i.e. $\alpha_1^* > \alpha_2^* > \cdots > \alpha_n^*$.

Quite intuitively, agents with lower levels of cost are better off. They also exert higher effort in at the efficient state. This result will be helpful for us in the next section in writing down post–redistribution payoffs once we introduce the planner.

3 Efficiency and Equality

We now envisage a planner who wishes to implement the efficient state μ^* but with minimum possible inequality. In this section, we assume the planner knows the type distribution to be able to calculate μ^* as in Proposition 2.4. This assumption will also help us characterize the inequality minimizing transfer vector t^* at μ^* subject to budget balance and incentive compatibility. As noted earlier, we will drop this assumption about knowing the type distribution in Section 4 when we discuss the dominant strategy implementation of our desired outcome.

Consider the efficient state μ^* with every agent playing α_p^* as characterized in Proposition 2.4. The aggregate strategy level at μ^* is α^*. Each agent pays the tax $\alpha_p^*\left(AP(\alpha^*) - MP(\alpha^*)\right)$ equal (in absolute value) to the negative externality (7) they create at μ^*.[12] Hence, the total tax revenue the planner obtains at the efficient state is

$$T(\mu^*) = \sum_p m_p \alpha_p^* \left(AP(\alpha^*) - MP(\alpha^*)\right) = \alpha^* \left(AP(\alpha^*) - MP(\alpha^*)\right). \tag{13}$$

We now allow the planner to redistribute the entire tax revenue received among the agents as transfers. Notice from (8) that once the tax is paid, the payoff of every type p agent at the efficient state is only $\alpha_p^* AP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right) + \alpha_p^*\left(MP(\alpha^*) - AP(\alpha^*)\right) = \alpha_p^* MP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right)$, which is $\hat{F}_{\alpha_p^*,p}(\mu^*)$ as defined in (12). Due to (13), redistribution ensures that the entire aggregate payoff at the efficient state μ^*,

$$\begin{aligned}
&\sum_p m_p \left(\alpha_p^* MP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right)\right) + T(\mu^*) \\
=&\alpha^* MP(\alpha^*) - \sum_p m_p c_p\left(\alpha_p^*\right) + T(\mu^*) \\
=&\alpha^* AP(\alpha^*) - \sum_p m_p c_p\left(\alpha_p^*\right) \\
=&\pi(\alpha^*) - \sum_p m_p c_p\left(\alpha_p^*\right) \\
=&\pi(A(\mu^*)) - \sum_{p \in \mathcal{P}} \int_{\mathcal{S}} c_p(x) \mu_p^*(x),
\end{aligned} \tag{14}$$

accrues to the agents. Note from (6) that (14) is just $\bar{F}(\mu^*)$.

Throughout, we assume that during the redistribution exercise, the planner provides the same

[12] A slight clarification about notation. When we interpret the tax as a payment from the agent to the planner, we write it as the positive amount $\alpha_p^*(AP(\alpha^*) - MP(\alpha^*))$. When we interpret it as a payment from the planner to the agent as in (16) below, we write it as the negative amount $\alpha_p^*(MP(\alpha^*) - AP(\alpha^*))$.

transfer to every agent of a particular type, although the transfer may vary across types. Of course, the question arises how the planner will recognize the type of an agent. We address such questions in Section 4. For the moment, let $t = (t_1, t_2, \cdots, t_p)$ be a vector of such type-specific transfers. We also assume that any such transfer vector satisfies the *budget balance* condition

$$\sum_p m_p t_p = T(\mu^*), \tag{15}$$

where $T(\mu^*)$ is as defined in (13). Then, at μ^*, if a type p agent plays his type-specific efficient strategy α_p^*, we can use (12) to write his post redistribution payoff as

$$\hat{F}_{\alpha_p^*,p}(\mu^*) + t_p = \alpha_p^* MP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right) + t_p \tag{16}$$

Equivalently, using (8), we can write the post redistribution payoff as the sum of the original payoff $\alpha_p^* AP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right)$ and β_p, where

$$\beta_p = t_p - \alpha_p^*\left(AP(\alpha^*) - MP(\alpha^*)\right) \tag{17}$$

is the *net payment* made by the planner to a type p agent.[13] Recall that the planner wishes to enhance equality during redistribution through these transfers. The budget balance condition implies that whatever resources the planner needs to promote equality come from the taxation of the negative externality. Hence, taxation in our model will not only curb the negative externality but will also reduce inequality.

3.1 Gini Coefficient

We measure inequality at the efficient state of the post-redistribution payoffs (16) using the Gini coefficient (Gini [13, 14]). Every type p agent receives the payoff (16) at μ^* following redistribution. The budget balance condition (15) implies that the aggregate payoff following such redistribution is $\bar{F}(\mu^*)$. All type p agents receive $\hat{F}_{\alpha_p^*,p}(\mu^*) + t_p$ and all type q agents receive $\hat{F}_{\alpha_q^*,q}(\mu^*) + t_q$. Further, the total mass of agents is 1. Hence, the Gini coefficient at the payoffs (16) is

$$\mathcal{G}(\mu^*, t) = \frac{1}{2\bar{F}(\mu^*)} \sum_{p=1}^{n} \sum_{q=1}^{n} m_p m_q \left| \left(\hat{F}_{\alpha_p^*,p}(\mu^*) + t_p\right) - \left(\hat{F}_{\alpha_q^*,q}(\mu^*) + t_q\right) \right|. \tag{18}$$

The Gini coefficient is arguably the most widely accepted measure of inequality.[14] Its properties are well known. It equals 0 when there is perfect equality and approaches 1 as inequality becomes more extreme. Thus, if all agents receive the same payoff, then clearly (18) equals zero.[15] On the other hand, suppose only type 1 agents receive positive payoff and all other types receive zero

[13]Thus, the net payment is the tax minus the transfer. We discuss the net payment further at the end of this section.

[14]For a detailed discussion on various inequality measures, see Chapter 6 of Ray [27].

[15]This is because in that case, $\hat{F}_{\alpha_p^*,p}(\mu^*) + t_p = \hat{F}_{\alpha_q^*,q}(\mu^*) + t_q$.

payoff. Then, the aggregate payoff $\bar{F}(\mu^*) = m_1(\hat{F}_{\alpha_p^*,p}(\mu^*) + t_p)$. It is easily checked that in that case, (18) equals $1 - m_1 \to 1$ as $m_1 \to 0$. Thus, if the entire society's payoff gets concentrated in a small mass of agents, the Gini coefficient (18) is close to 1.

The planner's objective is to find a vector of redistributive transfers $t^* = (t_1^*, t_2^*, \cdots, t_n^*)$ that minimizes (18). In addition to the budget balance condition (15), t^* will have to satisfy incentive compatibility. Again, Section 4 will provide details of how the planner actually ensures incentive compatibility. Right now, given the transfer vector $(t_1, t_2, \cdots, t_n)$ and the aggregate strategy level α^* at the efficient state, we define by

$$\phi_p(q, \mu^*, t) = \alpha_q^* MP(\alpha^*) - c_p(\alpha_q^*) + t_q, \tag{19}$$

the payoff a type p agent who plays the type q efficient strategy level α_q^* and is assigned the transfer t_q meant for a type q agent.[16] Notice that using this notation, we can write (16) as $\phi_p(p, \mu^*, t)$. Incentive compatibility of the transfer vector $t = (t_1, t_2, \cdots, t_n)$ would then require

$$\phi_p(p, \mu^*, t) \geq \phi_p(q, \mu^*, t) \tag{20}$$

for every $p, q \in \mathcal{P}$. In words, every agent should find it at least weakly preferable to play his own type-specific strategy and receive his own type-specific transfer than the strategy and transfer of any other type when the society is at the efficient state.

One feature of (19) that deserves comment is that it seems when an agent reports some other type q, he only plays the efficient strategy α_q^* of that type. Can't the agent play some other strategy? It suffices to consider only α_q^* because, as we will show in the dominant strategy mechanism of Section 4, the planner always allocates the optimal strategy of the reported type based on the reported type distribution. Further, transfers will be such that agents will report true type. Hence, eventually, due to truthful revelation, t^* will have to satisfy (20). For that reason, provisionally in this section, it suffices to characterize t^* by defining the payoff in (19) as if the agent reporting type q will only play α_q^*.

The planner, therefore, seeks a transfer vector that minimizes the Gini coefficient (18) while satisfying the budget balance condition (15) and the incentive compatibility condition (20). Before presenting the solution to this problem, let us consider two other alternatives which, as we will argue, cannot be that transfer scheme. First is the *equal redistribution transfer scheme*. Under this scheme, the planner redistributes an equal amount to every agent, irrespective of type. Budget balance then implies that every agent of every type p receives $t_p = T(\mu^*)$ as defined in (13). The resulting post redistribution payoffs (16) will then be strategically equivalent to $\hat{F}_{\alpha_p^*,p}(\mu^*)$ as defined in (12). But μ^* is the unique Nash equilibrium of $\hat{F}$ and α_p^* is the unique best response of every type p to μ^* (Proposition 2.4). Therefore, the equal redistribution rule will satisfy all IC constraints (20) and will do so strictly. Hence, it may be possible to further improve equality by making truthful

[16] Notice in (19) that even if the type p agent plays α_q^* instead of α_p^*, that will not change the aggregate strategy level α^*. That is due to the measure zero characteristic of agents.

revelation weakly dominant and this is what we will discuss in Sections 3.2 and 4.

The second possibility is a transfer scheme that ensures *perfect equality*. This outcome would make the post-redistribution payoff (16) of all agents perfectly equal. In the terminology of Kalai [19], this would be the *pure egalitarian* outcome at the efficient state. The planner would like to choose a transfer scheme $\tilde{t}$ such that at μ^*,

$$\alpha_p^* MP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right) + \tilde{t}_p = \alpha_q^* MP\left(\alpha^*\right) - c_q\left(\alpha_q^*\right) + \tilde{t}_q \tag{21}$$

for all $p, q \in \mathcal{P}$. For a planner concerned with equality at the efficient state, this is the *first best solution*. It is, however, easy to see that such a transfer scheme cannot satisfy incentive compatibility. Suppose $p < q$ so that, by Observation 2.2, $c_p(x) < c_q(x)$ for all $x \in \mathcal{S}$. Hence, $c_p(\alpha_q^*) < c_q(\alpha_q^*)$ which means

$$\begin{aligned} \alpha_q^* MP\left(\alpha^*\right) - c_p\left(\alpha_q^*\right) + \tilde{t}_q &> \alpha_q^* MP\left(\alpha^*\right) - c_q\left(\alpha_q^*\right) + \tilde{t}_q \\ &= \alpha_p^* MP\left(\alpha^*\right) - c_p\left(\alpha_p^*\right) + \tilde{t}_p, \end{aligned} \tag{22}$$

where the equality follows from (21). Thus, we have $\phi_p(q, \mu^*, \tilde{t}) > \phi_p(p, \mu^*, \tilde{t})$ so that type p's IC constraint (19) is violated. Hence, the first best solution cannot be achieved by the planner. We now discuss the *second best solution*.

3.2 Incentive Compatible Minimum Inequality at the Efficient State

Recall the Gini coefficient (18), the budget balance condition (15), and the IC constraints (20). Formally, the planner's objective is to choose a transfer vector $t = (t_1, t_2, \cdots, t_n)$ so as to

$$\underset{t}{\text{Minimize}}\ \mathcal{G}(\mu^*, t) \text{ such that } \phi_p(p, \mu^*, t) \geq \phi_p(q, \mu^*, t) \text{ and } \sum_p m_p t_p = T(\mu^*), \tag{23}$$

for all $p, q \in \mathcal{P}$. We characterize the solution $t^* = (t_1^*, t_2^*, \cdots, t_n^*)$ to this problem through the following lemmas leading up to Proposition 3.4. All proofs are in Appendix A.2.[17]

Lemma 3.1 *Recall the IC conditions (20). Consider a type $p \in \{1, 2, \cdots, n-1\}$ and an arbitrary transfer scheme $t = (t_1, t_2, \ldots, t_n)$ such that*

$$\phi_p(p, \mu^*, t) = \phi_p(p+1, \mu^*, t). \tag{24}$$

Then,

$$\phi_p(p, \mu^*, t) > \phi_p(q, \mu^*, t), \tag{25}$$

for all $q \in \{p+2, p+3, \cdots, n\}$.

[17] Notice that the type distribution m enters the budget balance condition in (23). This is another way we use the assumption the planner knows m in this section to characterize t^*.

Lemma 3.1 implies that to ensure that agents do not claim to be of a higher type at μ^*, it suffices to equate their true payoff to the payoff they would obtain by claiming to be the next higher type. It is worth noting that this lemma doesn't consider the other two case, $\phi_p(p, \mu^*, t) < \phi_p(p+1, \mu^*, t)$ and $\phi_p(p, \mu^*, t) > \phi_p(p+1, \mu^*, t)$. The former is a violation of incentive compatibility. The latter leaves further room to adjust transfers and, therefore, wouldn't characterize inequality minimizing transfers. Suppose now that we have a transfer scheme $t = (t_1, \cdots, t_n)$ that satisfies Lemma 3.1. The following lemma then establishes certain characteristics of the payoffs resulting from those transfers as well as the transfers themselves.

Lemma 3.2 *Recall the payoff (19). Suppose the transfer scheme $t = (t_1, t_2, \cdots, t_n)$ satisfies (24) in Lemma 3.1. Then, the following hold.*

1. *$\phi_p(p, \mu^*, t) > \phi_{p+1}(p+1, \mu^*, t)$ for all $p = 1, 2, \ldots, n-1$.*
2. *$t_1 < t_2 < \ldots < t_n$.*

Lemma 3.2, therefore, establishes that under a transfer scheme that satisfies Lemma 3.1, types with a lower cost function obtain a higher payoff than types with a higher cost function. This is despite the fact, as part 2 of the lemma shows, high-cost types obtain a higher transfer. Part 2 of this lemma also leads to Lemma 3.3 that shows that agents will not have any incentive to claim to be of a lower type. Hence, Lemmas 3.1 and 3.3 suffice to rule out incentives for false representation.

Lemma 3.3 *Recall the IC conditions (20) and suppose a transfer scheme $t = (t_1, t_2, \cdots, t_n)$ satisfies part 2 of Lemma 3.2. Suppose $p > q$. Then, $\phi_p(p, \mu^*, t) > \phi_p(q, \mu^*, t)$. Therefore, if t satisfies (24), then all IC constraints (20) are satisfied.*

The key condition in Lemmas 3.1–3.3 is (24). The condition ensures there is no misrepresentation as a higher type in Lemma 3.1. It also gives rise to the ordering between the transfers in Lemma 3.2(2), which then leads to Lemma 3.3 that rules out misrepresentation as a lower type. Notice that the three preceding lemmas are independent of the budget balance condition. But once we combine the IC constraints with the budget balance condition, we obtain the unique solution to the planner's problem (23). The following proposition formalizes that solution. The proposition also shows that the solution satisfies *individual rationality*, which means the post-redistribution payoffs will be positive for all types of agents. This is important because it means no agent has to be coerced to participate in the mechanism.

Proposition 3.4 *Consider the system of n linear equations consisting of the $n-1$ equations $\phi_p(p, \mu^*, t) = \phi_p(p+1, \mu^*, t)$ as specified in (24) for types $p \in \{1, 2, \cdots, n-1\}$ and the budget balance equation $\sum_{p \in \mathcal{P}} m_p t_p = T(\mu^*)$, where $T(\mu^*)$ is as defined in (13). Denote the solution to these n equations as $t^* = (t_1^*, t_2^*, \cdots, t_n^*)$. Then, t^* is the solution to the planner's problem (23).*

Thus, t^* satisfies $t_1^* < t_2^* < \cdots < t_n^*$. With this transfer vector, the payoff of every type p agent at μ^* is

$$\phi_p(p, \mu^*, t^*) = \alpha_p^* MP(\alpha^*) - c_p(\alpha_p^*) + t_p^* \tag{26}$$

Moreover, among all transfer vectors t at the efficient state that satisfy incentive compatibility and budget balance, t^* maximizes the post redistribution payoff of type n agents, $\phi_n(n, \mu^*, t)$. Hence, at t^*,

$$\phi_1(1, \mu^*, t^*) > \phi_2(2, \mu^*, t^*) > \cdots > \phi_n(n, \mu^*, t^*) > 0, \tag{27}$$

which means t^* also ensures individual rationality.

Proposition 3.4 is the most important technical result of our paper. It characterizes the optimal transfer vector $t^* = (t_1^*, t_2^*, \cdots, t_n^*)$ as the solution to a set of linear equations. Even though t^* does not ensure perfect equality, it does minimize inequality at the efficient state subject to incentive compatibility and budget balance.

To understand the intuition behind Proposition 3.4, we note that any other transfer vector that satisfies these two conditions must generate a higher Gini coefficient (18) in the post-redistribution payoffs. One such transfer scheme is the equal redistribution scheme $T(\mu^*)$ as defined in (13). Compared to equal redistribution, t^* increases the payoff of higher cost types and reduces the payoff of lower cost types thereby reducing inequality while still satisfying incentive compatibility. Consistent with Lemma 3.2(2), the Gini coefficient minimizing transfer t_p^* favors high-cost agents over low-cost ones. Hence, $t_1^* < t_2^* < \cdots < t_n^*$.

The order among the payoffs in (27) arises from part 1 of Lemma 3.2. All these payoffs are strictly positive because the equal redistribution transfer $T(\mu^*)$ itself ensures that the payoff of the highest cost type n is strictly positive.[18] But the proof of Proposition 2.4 not only shows that t^* minimizes the Gini coefficient but also maximizes the payoff of type n agents among all incentive compatible and budget balanced transfer vectors.[19] Hence, the payoff of type n agents must be even higher than under equal redistribution. Therefore, not only do we minimize the Gini coefficient at the efficient state but also maximize the minimum payoff. In this sense, implementing (μ^*, t^*) would be one way to reconcile the utilitarian objective of achieving efficiency with the Rawlsian objective of maximizing the minimum payoff. However, as we discuss in more detail in Section 5, this is not equivalent to implementing the Rawlsian social choice function. Additionally, the payoffs (27) also satisfy envy–freeness. This follows from our large population structure and incentive compatibility. No agent of type p would desire the allocation (α_q^*, t_q^*) of any other type $q \neq p$.

So far, we have presented our results as the planner first imposing the externality equivalent tax $\alpha_p^*(AP(\alpha^*) - MP(\alpha^*))$ on agents of type p and then providing them the transfer t_p^*. This interpretation has a clear economic intuition. This tax is equal to the negative externality an agent is generating at the efficient state. Hence, by imposing this particular tax, it is as if the planner is using the tax to achieve efficiency and then using the transfers as a redistributive measure while retaining efficiency.

Nevertheless, it is worth ending this section with an alternative but equivalent presentation of our solution. This is in terms of a (single) net payment vector and not in terms of tax and transfer.

[18] Even without $T(\mu^*)$, the fact that there are no fixed costs in our model ensures that the pre–redistribution payoff $\alpha_n^* MP(\alpha^*) - c_n(\alpha_n^*)$ in (16) is strictly positive.

[19] See Claim 5 of that proof.

Recall from (17) the net payment β_p received by a type p agent. Then the fact that the transfer t_p^* and the tax $\alpha_p^*(AP(\alpha^*) - MP(\alpha^*))$ minimizes inequality at efficiency is equivalent to the fact that the net payment vector $\beta^* = (\beta_1^*, \beta_2^*, \cdots, \beta_n^*)$ such that

$$\beta_p^* = t_p^* - \alpha_p^*(AP(\alpha^*) - MP(\alpha^*)) \tag{28}$$

would also achieve the same objective. It is immediate from (2) that once the net payoff (28) is applied, a type p agent's payoff at μ^* is

$$\begin{aligned} F_{\alpha_p^*,p}(\mu^*) + \beta_p^* &= \alpha_p^* AP(\alpha^*) - c_p(\alpha_p^*) + \beta_p^* \\ &= \alpha_p^* MP(\alpha^*) - c_p(\alpha_p^*) + t_p^* \\ &= \phi_p(p, \mu^*, t^*) \end{aligned}$$

as defined in (26).

In (28), we do not have to think of the planner as first taxing an agent and then providing a transfer. Instead, the planner goes through the calculations leading to Proposition 3.4 without actually imposing any tax. Once the planner solves for μ^* and t^*, there is a single transaction with every agent. If $\beta_p^* > 0$, then the agent receives the payment (28) from the planner. If $\beta_p^* < 0$, then the agent makes an equal transfer to the planner. Thus, the planner's transactions are more parsimonious and happen at a single instant of time. This interpretation, framed as a single net payment, will be crucial in the subsequent section, where we introduce our simultaneous strategy mechanism.

4 Dominant Strategy Implementation

We now consider the main question of this paper. How does the planner implement the efficient state μ^* and the transfer vector t^* that minimizes inequality at the efficient state μ^* subject to the two constraints (15) and (20). We seek to apply a dominant strategy implementation mechanism to solve this problem. As is standard in such a mechanism, agents or the planner should not know the type distribution m. Accordingly, *we drop the assumption that m is known* that we had made in Sections 2 and 3 to characterize μ^* and t^*.

Recall the net payment β_p^* from (28). We denote by $\beta^* = (\beta_1^*, \beta_2^*, \cdots, \beta_n^*)$ the vector of net payments for all types. We formally state the *planner's objective* to be to implement the social choice function $m \mapsto (\mu^*, \beta^*)$. Thus, for any type distribution m, the planner wishes to implement the associated efficient state μ^* given by (9) and the net payment vector β^* given by (28). If this social choice function is implemented, every type p agent will play the efficient strategy level α_p^* and receive the net payment β_p^*. Consequently, as noted following (28), such an agent will receive the payoff $\phi_p(p, \mu^*, t^*)$ as defined in (26). We state the social choice function in terms of the single net payment β^* instead of externality tax and the transfer vector t^* because that will allow us to represent our mechanism as a simultaneous move mechanism.

By the revelation principle, it suffices to consider direct mechanisms. Hence, the planner designs a direct mechanism, which we denote as Φ, as follows. The planner asks each agent to report his type. Suppose $\tilde{m} = (\tilde{m}_1, \tilde{m}_2, \cdots, \tilde{m}_n)$ is the reported type distribution. Thus, $\tilde{m}_p$ is the proportion of agents who report their type to be p. As agents can report type falsely, it is possible that $\tilde{m}_p \neq m_p$. Using the reported type distribution $\tilde{m}$, the planner calculates the efficient state corresponding to $\tilde{m}$. This can be done by proceeding as in Proposition 2.4 once m is replaced with $\tilde{m}$ in (10). Let the efficient state corresponding to the distribution $\tilde{m}$ be $\tilde{\mu}^*$ and the strategy level of a type p agent at that state be $\tilde{\alpha}^*_p$.

Thus, $\tilde{\mu}^* = \left(\tilde{m}_1\delta_{\tilde{\alpha}^*_1}, \tilde{m}_2\delta_{\tilde{\alpha}^*_2}, \cdots, \tilde{m}_n\delta_{\tilde{\alpha}^*_n}\right)$. Denote the corresponding aggregate strategy level $A(\tilde{\mu}^*) = \sum_{p\in\mathcal{P}} \tilde{m}_p\tilde{\alpha}^*_p = \tilde{\alpha}^*$. Further, analogous to (13) and (19), we define

$$T(\tilde{\mu}^*) = \tilde{\alpha}^* \left(AP(\tilde{\alpha}^*) - MP(\tilde{\alpha}^*)\right). \tag{29}$$

and

$$\phi_p(q, \tilde{\mu}^*, \tilde{t}) = \tilde{\alpha}^*_q MP(\tilde{\alpha}^*) - c_p(\tilde{\alpha}^*_q) + \tilde{t}_q. \tag{30}$$

for some arbitrary transfer vector $\tilde{t} = (\tilde{t}_1, \tilde{t}_2, \cdots, \tilde{t}_n)$. Following Proposition 3.4, we denote as $\tilde{t}^* = (\tilde{t}^*_1, \tilde{t}^*_2, \cdots, \tilde{t}^*_n)$ the solution to the system of equations

$$\begin{aligned} &\phi_p(p, \tilde{\mu}^*, \tilde{t}) = \phi_p(p+1, \tilde{\mu}^*, \tilde{t}), \text{ for all } p \in \{1, 2, \cdots, n-1\} \\ &\sum_{p\in\mathcal{P}} \tilde{m}_p\tilde{t}_p = T(\tilde{\mu}^*), \end{aligned} \tag{31}$$

where $T(\tilde{\mu}^*)$ is as defined in (29). The first set of equalities are the minimal IC constraints that need to satisfied as in Lemma 3.1 but with respect to $\tilde{m}$. The second equality is the budget balance condition similar to (15) but with respect to $\tilde{m}$. Finally, analogous to (28), we define the net payment

$$\tilde{\beta}^*_q = \tilde{t}^*_p - \tilde{\alpha}^*_p(AP(\tilde{\alpha}^*) - MP(\tilde{\alpha}^*)) \tag{32}$$

where $\tilde{t}^*_q$ arises from the solution to (31).

The planner then assigns the type specific strategy $\tilde{\alpha}^*_q$ and the net payment $\tilde{\beta}^*_q$ to any agent who announces type to be q. Thus, in the conventional terminology of mechanism design, the planner designs the direct mechanism

$$\Phi : (q, \tilde{m}) \mapsto (\tilde{\alpha}^*_q, \tilde{\beta}^*_q), \tag{33}$$

where $\tilde{\beta}^*_q$ is as defined in (32). The mechanism Φ takes the reported type q of an agent and the reported type distribution $\tilde{m}$ as inputs and generates the type specific strategy and net payment $(\tilde{\alpha}^*_q, \tilde{\beta}^*_q)$ as output as described above. The resulting payoff of a type p agent who reports type to be type q is then $\phi_p(q, \tilde{\mu}^*, \tilde{t}^*)$ as defined in (30).

We emphasize that (33) is a simultaneous move mechanism. Recall our discussion following (28). When we presented our solution in terms of the net payment β^*_p, we envisaged the planner engaging in a single transaction with an agent. Similarly, in the mechanism (33), the planner makes a single net payment to an agent instead of going through the sequence of first taxing the agent and then providing a transfer. Intuitively, the planner goes through the calculations leading up to (31) to derive $\tilde{\mu}^*$ and $\tilde{t}^*$. The planner doesn't actually collect the tax $T(\tilde{\mu}^*)$ or make the transfers $\tilde{t}^*$. Instead, the planner uses these values to derive (32) and makes the net payment to an agent as one transaction at a single instant of time.

We arrive at the main result of this paper. The result is stated in the following theorem. The proof is in Appendix A.3

Theorem 4.1 *The direct mechanism Φ defined by (33) implements (μ^*, β^*) in weakly dominant strategies, where μ^* is the efficient state characterized in Proposition 2.4 and β^* is the net transfer vector defined in (28). The mechanism also satisfies budget balance and individual rationality.*

Once (μ^*, β^*) is implemented, each type p agent receives the payoff $\phi_p(p, \mu^*, t^*)$ defined in (26). The resulting Gini coefficient in payoffs is

$$\mathcal{G}(\mu^*, t^*) = \frac{1}{2\bar{F}(\mu^*)} \sum_{p=1}^{n} \sum_{q=1}^{n} m_p m_q \Big| \phi_p(p, \mu^*, t^*) - \phi_q(q, \mu^*, t^*) \Big|, \tag{34}$$

which is the lowest possible Gini coefficient at the efficient state subject to incentive compatibility and budget balance.

The logic behind Theorem 4.1 is the same as that of Proposition 3.4. That result implies that at the actual type distribution m, if the planner assigns strategies α_p^* and net payment β_p^*, then agents will reveal their type truthfully. But mathematically, there is nothing special about m. Hence, if the planner announces that he will calculate $(\tilde{\mu}^*, \tilde{\beta}^*)$ and assign strategies and net payments based on the reported type, it becomes a weakly dominant strategy for every agent to report their type truthfully. The type distribution that gets revealed is the true one m and, therefore, the outcome that is implemented is (μ^*, β^*).

The mechanism Φ defined in 33 has similarities to a classical finite player VCG mechanism. It extends the large population VCG type mechanism of Lahkar and Mukherjee [22] to the present context of implementing not just efficiency but also minimum inequality. As in a VCG mechanism, Φ assigns the allocation $(\tilde{\alpha}_p^*, \tilde{\beta}_p^*)$ to an agent who reports being of type p, so as to make truthful revelation weakly dominant. The large population characteristic of our model is, however, crucial in ensuring that this mechanism actually works. The very description of the incentive compatibility constraints in Proposition 3.4 or in (31) rely on the idea that when a single agent misrepresents type, it does not affect the social state μ^* or $\tilde{\mu}^*$. This stems from the measure zero characteristic of agents, an assumption that is not met in finite player models. Hence, a similar VCG mechanism will not succeed in implementing efficiency with minimum inequality in finite player models. In the concluding section, we present some conjectures about how our results may relate to the finite player case.

5 Rawlsian Outcome

Our focus in this paper has been on minimizing inequality at efficiency subject to incentive compatibility and budget balance (Theorem 4.1). As a byproduct of minimizing inequality, Proposition 3.4 also shows that our solution maximizes the lowest payoff at efficiency, again subject to incentive compatibility and budget balance. This suggests a connection between our problem and the problem of implementing the Rawlsian social choice function. The Rawlsian social choice function would seek to implement an outcome that maximizes the lowest payoff. In our model, if incentive compatibility is not a concern, then the Rawlsian outcome is simply the efficient state μ^* and the payment vector that ensures perfect equality.[20] But as we argued, such perfect equality at efficiency is not incentive compatible. Instead, by Proposition 3.4, if we restrict ourselves to the efficient state μ^*, then the best feasible solution to the Rawlsian problem is the transfer vector t^* or equivalently, the net payment vector β^* as defined in (28).

But suppose we consider the Rawlsian outcome over the set of all social states μ. This would be a social state and a vector of payments subject to incentive compatibility and budget balance that maximizes the minimum payoff. Would that outcome be the same as minimizing inequality at the efficient state? In this section, we present an example where this is not the case. Thus, the Rawlsian outcome is not necessarily the one that minimizes inequality at efficiency. The example is as follows.

Example 5.1 *Consider the model described in Section 2 with strategy set $\mathcal{S} = (0, \infty)$. Let the set of populations or types be $\mathcal{P} = \{1, 2, 3\}$ and the type distribution be $(m_1, m_2, m_3) = (0.2, 0.3, 0.5)$. Suppose the type specific cost functions are $c_p(x) = k_p x^2$ where $\{k_1, k_2, k_3\} = \{1, 2, 3\}$ and the production function is $\pi(\alpha) = 10\sqrt{\alpha}$. The average product is, therefore, $\frac{10\sqrt{\alpha}}{\alpha} = \frac{10}{\sqrt{\alpha}}$. Hence, given a social state μ with aggregate strategy $A(\mu) = \alpha$, the payoff of a type $p \in \{1, 2, 3\}$ agent in the tragedy of the commons F defined by (2) is*

$$F_{x,p}(\mu) = \frac{10x}{\sqrt{\alpha}} - k_p x^2. \tag{35}$$

The marginal product in Example 5.1 is $\pi'(\alpha) = \frac{5}{\sqrt{\alpha}}$. Therefore, the externality (7) an agent playing strategy x generates is $\frac{-5x}{\sqrt{\alpha}}$. Hence, from (35), we obtain the externality adjusted payoff (8) in our example,

$$\hat{F}_{x,p}(\mu) = \frac{5x}{\sqrt{\alpha}} - k_p x^2. \tag{36}$$

Proposition 2.4 then yields the efficient state of Example 5.1, which is $\mu^* = (m_1\delta_{\alpha_1^*}, m_2\delta_{\alpha_2^*}, m_3\delta_{\alpha_3^*})$ where

$$(\alpha_1^*, \alpha_2^*, \alpha_3^*) = (2.2956, 1.1478, 0.7652). \tag{37}$$

The aggregate strategy level at the efficient state is $\alpha^* = \sum_{p\in\mathcal{P}} m_p \alpha_p^* = 1.186$. Applying (13), we calculate $T(\mu^*) = \frac{5\sum_p m_p \alpha_p^*}{\sqrt{\alpha^*}} = 5\sqrt{\alpha^*} = 5.4453$.

[20] See the discussion preceding Section 3.2.

First, we illustrate inequality minimization at the efficient state. Let $t = (t_1, t_2, t_3)$ be a transfer vector. The payoff (19) takes the form

$$\begin{aligned}\phi_p(q, \mu^*, t) &= \alpha_q^* MP(\alpha^*) - k_p(\alpha_q^*)^2 + t_q \\ &= \frac{5\alpha_q^*}{\sqrt{1.186}} - k_p(\alpha_q^*)^2 + t_q \\ &= 4.5911\alpha_q^* - k_p(\alpha_q^*)^2 + t_q. \end{aligned} \tag{38}$$

Applying Proposition 3.4, we obtain the constraints

$$4.5911\alpha_1^* - (\alpha_1^*)^2 + t_1 = 4.5911\alpha_2^* - (\alpha_2^*)^2 + t_2, \tag{39}$$

$$4.5911\alpha_2^* - 2(\alpha_2^*)^2 + t_2 = 4.5911\alpha_3^* - 2(\alpha_3^*)^2 + t_3, \tag{40}$$

$$0.2t_1 + 0.3t_2 + 0.5t_3 = 5.4453, \tag{41}$$

with (39) and (40) being the IC constraints for types 1 and 2 respectively, and (41) being the budget balance condition. Solving these equations, we obtain the desired transfer vector t^* to be

$$(t_1^*, t_2^*, t_3^*) = (4.245, 5.5624, 5.8551). \tag{42}$$

Applying (28), we obtain the associated net payment vector

$$(\beta_1^*, \beta_2^*, \beta_3^*) = (-6.2943, 0.2927, 2.342). \tag{43}$$

Thus, the net payment vector (43) minimizes inequality at the efficient state (37) subject to incentive compatibility and budget balance. By (38), the resulting payoffs at the efficient state for the three types are

$$(\phi_1(1, \mu^*, t^*), \phi_2(2, \mu^*, t^*), \phi_3(3, \mu^*, t^*)) = (9.5146, 8.1972, 7.6117). \tag{44}$$

We now consider the Rawlsian social choice function. Thus, instead of minimizing inequality at the efficient state, the planner wishes to implement an outcome that maximizes the lowest payoff across all social states in F. To explore this possibility, it suffices to consider states in monomorphic population states. This would ensure that within each population, payoffs would be equal. Thus, let $\mu = (m_1\delta_{\alpha_1}, m_2\delta_{\alpha_2}, m_3\delta_{\alpha_3})$ be such a social state in Example 5.1. The aggregate strategy is, therefore, $\alpha = \sum_{p=1}^{3} m_p\alpha_p$. Further, let $t = (t_1, t_2, t_3)$ be a transfer vector. Analogous to (19), denote as

$$\begin{aligned}\phi_p(q, \mu, t) &= \alpha_q AP(\alpha) - c_p(\alpha_q) + \alpha_q(MP(\alpha) - AP(\alpha)) + t_q \\ &= \alpha_q MP(\alpha) - c_p(\alpha_q) + t_q \\ &= \frac{5\alpha_q}{\sqrt{\alpha}} - k_p\alpha_q^2 + t_q, \end{aligned} \tag{45}$$

the post redistribution payoff of a type p agent who reports type to be q at the social state

μ and the transfer vector t. Unlike (38), (45) is defined at all social states of the form $\mu = (m_1\delta_{\alpha_1}, m_2\delta_{\alpha_2}, m_3\delta_{\alpha_3})$. Thus, an agent claiming to be of type q is assigned the strategy α_q and the net payment

$$\beta_p^R = t_p^R - \alpha_q(AP(\alpha) - MP(\alpha)) \tag{46}$$

Analogous to (13), we define

$$T(\mu) = \sum_{p=1}^{3} m_p\alpha_p(AP(\alpha) - MP(\alpha)) = 5\sqrt{\alpha} = 5\sqrt{0.2\alpha_1 + 0.3\alpha_2 + 0.5\alpha_3}. \tag{47}$$

Maximizing the minimum payoff means we solve

$$\begin{aligned} &\max_{(\mu,t)} \left[\min\left\{\phi_1(1,\mu,t), \phi_2(2,\mu,t), \phi_3(3,\mu,t)\right\}\right] \\ &\quad \text{subject to } \mu = (m_1\delta_{\alpha_1}, m_2\delta_{\alpha_2}, m_3\delta_{\alpha_3}) \\ &\quad \phi_p(p,\mu,t) \geq \phi_p(q,\mu,t) \text{ for all } p, q \in \mathcal{P} \\ &\quad m_1t_1 + m_2t_2 + m_3t_3 = 5\sqrt{0.2\alpha_1 + 0.3\alpha_2 + 0.5\alpha_3}. \end{aligned} \tag{48}$$

The constraints $\phi_p(p,\mu,t) \geq \phi_p(q,\mu,t)$ are the IC constraints, with $\phi_p(q,\mu,t)$ being as defined in (45). The last constraint in (48) is the budget balance condition arising from (47). It is equivalent to $\sum_{p\in\mathcal{P}} m_p\beta_p^R = 0$. We solve this maximization exercise numerically to obtain

$$\left(\alpha_1^R, \alpha_2^R, \alpha_3^R\right) = (2.4264, 0.9099, 0.6066), \tag{49}$$

$$\left(t_1^R, t_2^R, t_3^R\right) = (3.0358, 5.3356, 5.8876). \tag{50}$$

Applying (46) gives us the net payment vector

$$(\beta_1^R, \beta_2^R, \beta_3^R) = (-8.7392, 0.9199, 2.9438). \tag{51}$$

Thus, the social state at the Rawlsian outcome is $\mu^R = (m_1\delta_{\alpha_1^R}, m_2\delta_{\alpha_2^R}, m_3\delta_{\alpha_3^R})$. The aggregate strategy at this outcome is $\alpha^R = \sum_{p\in\mathcal{P}} m_p\alpha_p^R = 1.0616$. Inserting (49) and (50) in (45) and using $\{k_1, k_2, k_3\} = \{1, 2, 3\}$ from Example 5.1, we obtain the type specific payoffs at the Rawlsian outcome to be

$$\left(\phi_1(1,\mu^R,t^R), \phi_2(2,\mu^R,t^R), \phi_3(3,\mu^R,t^R)\right) = (8.9233, 8.0953, 7.7274). \tag{52}$$

Hence, the Rawlsian outcome in Example 5.1 subject to budget balance and incentive compatibility involves each type p agent playing α_p^R as characterized in (49) and receiving a net payment of (51). The resulting social state μ^R is clearly different from the efficient state μ^* characterized in (37). Example 5.1, therefore, illustrates that once we impose the budget balance and incentive compatibility conditions, the Rawlsian solution is different from the solution that minimizes inequality at the efficient state. This is further shown by the fact that the minimum payoff $\phi_3(3, \mu^R, t^R)$ in (52) is strictly higher than the minimum payoff $\phi_3(3, \mu^*, t^*)$ in (44).

Our calculation of the Rawlsian outcome has required, as in Sections 2 and 3, the knowledge of the type distribution m. But suppose the planner doesn't know m but wants to implement the Rawlsian outcome. A dominant strategy implementation mechanism like in Section 4 will work. Briefly, the planner asks agents to report types. Suppose the reported type distribution is $\tilde{m}$. The planner solves (48) but with respect to the distribution $\tilde{m}$. That generates strategies $\tilde{\alpha}_q^R$ and net transfers $\tilde{\beta}_q^R$ as in (49) and (51). The planner assigns the vector $(\tilde{\alpha}_q^R, \tilde{\beta}_q^R)$ to an agent who reports type q. As in Theorem 4.1, truthful revelation of type will be weakly dominant.

6 Conclusion

We have considered the implementation of efficiency with minimum inequality in a large population model of negative externalities. Agents are of different types which are distinguished by cost functions that are private information. Total output is a function of aggregate strategy which is shared among agents according to individual strategy. The model is, therefore, equivalent to a tragedy of the commons. Imposition of externality equivalent taxes restores efficiency in the model. The planner would like to redistribute the tax revenue as transfers so as to reduce inequality, as measured by the Gini coefficient of payoffs, at the efficient state while being subject to incentive compatibility and budget balance.

We first characterize the inequality minimizing vector of type-specific transfers. This transfer minus the externality tax generates the inequality minimizing net payment vector. We then describe a mechanism that would enable the planner to implement both the efficient state and the inequality minimizing net payment vector in dominant strategies. The planner asks agents to report their types and calculates the efficient state and inequality minimizing net payments based on reported types. Due to the large population characteristic of the model, it then becomes weakly dominant for all agents to report the type truthfully thereby implementing the desired objective of the planner.

Finally, while minimizing inequality at efficiency also ensures maximization of minimum payoff at the efficient state, it is not equivalent to implementing the Rawlsian social choice function. There may exist other states which are not efficient but where, through appropriate transfers, it is possible to further improve the welfare of the most disadvantaged agents in an incentive compatible manner while satisfying budget balance.

An important research question that arises is a more general characterization of the Rawlsian social state in a large population model. In the present paper, we have only provided a counterexample because that suffices to show that the inequality minimizing efficient outcome is not the Rawlsian outcome. But independent of efficiency, the Rawlsian outcome is interesting on its own and a more rigorous analysis of this outcome is worth exploring.

We conclude by emphasizing that, as demonstrated in the literature, our results cannot be attained in a finite player setting (Green and Laffont [16]). Nevertheless, we conjecture that as the number of agents increases, results in the finite case would approach our results. In particular, we expect strategies and transfers in the finite case to converge to the "efficiency with minimum inequality" outcomes we have characterized in this paper. We do not expect budget balance to hold. There may be a budget surplus or deficit with the planner for any fixed finite number of agents. But as that number increases, the present paper suggests that the surplus or deficit will go to zero. We leave the formal analysis of this convergence problem as a task for future research.

A Appendix

A.1 Appendix to Section 2

Proof of Proposition 2.3: In the discussion preceding Proposition 2.3, we have argued that $b_p(\alpha)$ is the unique best response of type p agents to any state μ such that $A(\mu) = \alpha$. The additional assumption that $c_p'(0) = 0$ then implies that for every $p \in \mathcal{P}$, this unique best response in (2) satisfies

$$AP(\alpha) = c_p'(b_p(\alpha)), \tag{53}$$

with $b_p(\alpha) \in (0, \infty)$ for all $\alpha \in (0, \infty)$.

Proposition 3.1 in Lahkar [20] shows that in large population aggregative games such as (2), all Nash equilibria can be characterized as solutions to (3). Due to our assumptions that $AP(\alpha)$ is strictly declining and c_p is strictly convex, we conclude from (53) that $b_p(\alpha)$ is strictly declining for all p. Hence, (3) has a unique solution, which we denote as α^N. By Proposition 3.1 in Lahkar [20], we then obtain the unique Nash equilibrium μ^N as defined in (4) where all type p agents play $b_p(\alpha^N)$. The aggregate strategy level at μ^N is, therefore, $\alpha^N = \sum_{p\in\mathcal{P}} m_p \alpha_p^N$ and condition (5) follows from (53). ■

Proof of Proposition 2.4: We first establish that (10) has a unique solution. The assumptions of our model imply that the unique best response $\hat{b}_p(\alpha)$ in $\hat{F}$ is characterized by

$$MP(\alpha) = c_p'\left(\hat{b}_p(\alpha)\right). \tag{54}$$

Due to the strict concavity of π, $MP(\alpha)$ is strictly declining. Hence, by a similar argument as in Proposition 2.3, $\hat{b}_p(\alpha)$ is strictly declining. This establishes uniqueness of the solution to (10). The argument in the proof of Proposition 2.3 then implies that μ^* is the unique Nash equilibrium of $\hat{F}$. The remaining conclusions follow from the discussion preceding Proposition 2.4. Proposition 5.6 in Lahkar and Mukherjee [22] shows this Nash equilibrium of $\hat{F}$ is also the efficient state of F defined by (2). The conclusion $\alpha_p^* < \alpha_p^N$ follows from (5), (11), the strict convexity of c_p and the fact that $MP(\alpha) < AP(\alpha)$. Condition (11) follows from (54). ■

Proof of Corollary 2.5: Since μ^* is the unique Nash equilibrium of the game $\hat{F}$ characterized by (8) and every agent has a unique best response to every state, α_p^* is the unique best response to μ^*

for a type p agent. Therefore,

$$\alpha_p^* MP(\alpha^*) - c_p(\alpha_p^*) > \alpha_q^* MP(\alpha^*) - c_p(\alpha_q^*). \tag{55}$$

By Observation 2.2, as $p < q$, $c_p(\alpha_q^*) < c_q(\alpha_q^*)$. Therefore,

$$\alpha_q^* MP(\alpha^*) - c_p(\alpha_q^*) > \alpha_q^* MP(\alpha^*) - c_q(\alpha_q^*). \tag{56}$$

Combining (55) and (56) and using (12), we obtain $\hat{F}_{\alpha_p^*,p}(\alpha^*) > \hat{F}_{\alpha_q^*,q}(\alpha^*)$. For the relationship between α_p^* and α_q^*, note from (11) that $c_p'(\alpha_p^*) = c_q'(\alpha_q^*) = MP(\alpha^*)$. By Assumption 2.1, if $p < q$, then $c_p'(\alpha_p^*) < c_q'(\alpha_p^*)$. The strict convexity of cost functions then imply that $\alpha_p^* > \alpha_q^*$. ■

A.2 Appendix to Section 3

Proof of Lemma 3.1: We start with type $n-1$ and proceed in reverse order. For type $n-1$, (25) doesn't apply but suppose (24) holds. Thus,

$$\begin{aligned} &\phi_{n-1}(n-1, \mu^*, t) = \phi_{n-1}(n, \mu^*, t) \\ \Rightarrow &\alpha_{n-1}^* MP(\alpha^*) - c_{n-1}(\alpha_{n-1}^*) + t_{n-1} = \alpha_n^* MP(\alpha^*) - c_{n-1}(\alpha_n^*) + t_n. \end{aligned} \tag{57}$$

Now consider type $n-2$. Suppose (24) holds. Hence, $\phi_{n-2}(n-2, \mu^*, t) = \phi_{n-2}(n-1, \mu^*, t)$ or

$$\alpha_{n-2}^* MP(\alpha^*) - c_{n-2}(\alpha_{n-2}^*) + t_{n-2} = \alpha_{n-1}^* MP(\alpha^*) - c_{n-2}(\alpha_{n-1}^*) + t_{n-1}. \tag{58}$$

Then, to show (25), we need to show $\phi_{n-2}(n-2, \mu^*, t) > \phi_{n-2}(n, \mu^*, t)$. For this, we can use (58) and show $\phi_{n-2}(n-1, \mu^*) > \phi_{n-2}(n, \mu^*)$ or

$$\alpha_{n-1}^* MP(\alpha^*) - c_{n-2}(\alpha_{n-1}^*) + t_{n-1} > \alpha_n^* MP(\alpha^*) - c_{n-2}(\alpha_n^*) + t_n \tag{59}$$

Notice that we can derive (59) from (57) by adding $c_{n-1}(\alpha_{n-1}^*) - c_{n-2}(\alpha_{n-1}^*)$ and $c_{n-1}(\alpha_n^*) - c_{n-2}(\alpha_n^*)$ to the LHS and RHS of (57) respectively. But $\alpha_{n-1}^* > \alpha_n^*$ (Corollary 2.5). Hence, Assumption 2.1, $c_{n-1}(\alpha_{n-1}^*) - c_{n-2}(\alpha_{n-1}^*) > c_{n-1}(\alpha_n^*) - c_{n-2}(\alpha_n^*)$. But then, this establishes (59).

For type $n-3$, we need to argue that if $\phi_{n-3}(n-3, \mu^*, t) = \phi_{n-3}(n-2, \mu^*, t)$, then $\phi_{n-3}(n-3, \mu^*, t) > \phi_{n-3}(n-1, \mu^*, t^)$ and $\phi_{n-3}(n-3, \mu^*, t) > \phi_{n-3}(n, \mu^*, t)$. To show $\phi_{n-3}(n-3, \mu^*, t) > \phi_{n-3}(n-1, \mu^*, t)$, we proceed as before for type $n-2$ and show $\phi_{n-3}(n-2, \mu^*, t) > \phi_{n-3}(n-1, \mu^*, t)$. This follows from (58) if we add $c_{n-2}(\alpha_{n-2}^*) - c_{n-3}(\alpha_{n-2}^*)$ and $c_{n-2}(\alpha_{n-1}^*) - c_{n-3}(\alpha_{n-1}^*)$ to the LHS and RHS of (58) respectively and then note that because $\alpha_{n-2}^* > \alpha_{n-1}^*$, $c_{n-2}(\alpha_{n-2}^*) - c_{n-3}(\alpha_{n-2}^*) > c_{n-2}(\alpha_{n-1}^*) - c_{n-3}(\alpha_{n-1}^*)$.

To show $\phi_{n-3}(n-3, \mu^*, t) > \phi_{n-3}(n, \mu^*, t)$, the above argument means it suffices to show $\phi_{n-3}(n-1, \mu^*, t) > \phi_{n-3}(n, \mu^*, t)$. For that, we use (59), which we have established. Note that

$\phi_{n-3}(n-1, \mu^*, t) > \phi_{n-3}(n, \mu^*, t)$ is equivalent to

$$\alpha_{n-1}^* MP(\alpha^*) - c_{n-3}(\alpha_{n-1}^*) + t_{n-1} > \alpha_n^* MP(\alpha^*) - c_{n-3}(\alpha_n^*) + t_n \tag{60}$$

We can obtain (60) by adding $c_{n-2}(\alpha_{n-1}^*) - c_{n-3}(\alpha_{n-1}^*)$ and $c_{n-2}(\alpha_n^*) - c_{n-3}(\alpha_n^*)$ to the LHS and RHS of (59) respectively. The desired conclusion then follows by noting that because $\alpha_{n-1}^* > \alpha_n^*$, $c_{n-2}(\alpha_{n-1}^*) - c_{n-3}(\alpha_{n-1}^*) > c_{n-2}(\alpha_n^*) - c_{n-3}(\alpha_n^*)$.

For the remaining types $p \in \{1, 2, \cdots, n-4\}$, we can proceed similarly through an inductive argument. For each type p, we use the arguments established for type $p+1$ and prove the claim. This would establish the lemma. ■

Proof of Lemma 3.2: By (24), we have $\phi_p(p, \mu^*, t) = \phi_p(p+1, \mu^*, t)$ for all $p \in \{1, 2, \cdots, n-1\}$. Moreover, according to our assumption we have $c_p(\alpha_{p+1}^*) < c_{p+1}(\alpha_{p+1}^*)$. So we have

$$\begin{aligned}
\phi_p(p, \mu^*, t) &= \alpha_p^* MP(\alpha^*) - c_p\left(\alpha_p^*\right) + t_p \\
&= \alpha_{p+1}^* MP(\alpha^*) - c_p\left(\alpha_{p+1}^*\right) + t_{p+1} \\
&> \alpha_{p+1}^* MP(\alpha^*) - c_{p+1}\left(\alpha_{p+1}^*\right) + t_{p+1} = \phi_{p+1}(p+1, \mu^*, t)
\end{aligned}$$

This establishes part 1. For part 2, again from (24), we have $\alpha_p^* MP(\alpha^*) - c_p\left(\alpha_p^*\right) + t_p = \alpha_{p+1}^* MP(\alpha^*) - c_p\left(\alpha_{p+1}^*\right) + t_{p+1}$. Rearrangement gives us $t_{p+1} = t_p + c_{p+1}\left(\alpha_{p+1}^*\right) - c_p\left(\alpha_p^*\right) - MP(\alpha^*)\left[\alpha_{p+1}^* - \alpha_p^*\right]$. From (11) and Proposition 2.4, we know that $MP(\alpha^*) = c_p'(\alpha_p^*)$. Thus we can write $t_{p+1} = t_p + c_{p+1}\left(\alpha_{p+1}^*\right) - c_p\left(\alpha_p^*\right) - c_p'(\alpha_p^*)[\alpha_{p+1}^* - \alpha_p^*]$. The strict convexity of $c(\cdot)$ implies $c_p\left(\alpha_{p+1}^*\right) - c_p\left(\alpha_p^*\right) - c_p'(\alpha_p^*)[\alpha_{p+1}^* - \alpha_p^*] > 0$. But by Observation 2.2, $c_{p+1}\left(\alpha_{p+1}^*\right) > c_p\left(\alpha_{p+1}^*\right)$.

Hence, $c_{p+1}\left(\alpha_{p+1}^*\right) - c_p\left(\alpha_p^*\right) - c_p'(\alpha_p^*)[\alpha_{p+1}^* - \alpha_p^*] > 0$, which gives us the desired result that $t_{p+1} > t_p$ for any $p = 1, 2, \ldots, n-1$. ■

Proof of Lemma 3.3: Recall from (19) that if the transfer vector is t, then $\phi_p(p, \mu^*, t) = \alpha_p^* MP(\alpha^*) - c_p\left(\alpha_p^*\right) + t_p$ and $\phi_p(q, \mu^*, t) = \alpha_q^* MP(\alpha^*) - c_p\left(\alpha_q^*\right) + t_q$. The fact that α_p^* is the unique best response to μ^* for a type p agent in the game $\hat{F}$ defined by (8) implies $\alpha_p^* MP(\alpha^*) - c_p\left(\alpha_p^*\right) > \alpha_q^* MP(\alpha^*) - c_p\left(\alpha_q^*\right)$.

Moreover, as $p > q$, $t_p > t_q$ by Lemma 3.2(2). Combining these arguments, we obtain $\alpha_p^* MP(\alpha^*) - c_p\left(\alpha_p^*\right) + t_p > \alpha_q^* MP(\alpha^*) - c_p\left(\alpha_q^*\right) + t_q$, or $\phi_q(q, \mu^*, t) > \phi_q(p, \mu^*, t)$.

Thus, (24) ensures that agents of type p do not have the incentive to claim to be of types $p+1$, $p+2$ etc (Lemma 3.1). That same condition also implies $t_1 < t_2 < \cdots < t_n$, i.e. Lemma 3.2(2), which then implies the present result that such agents will also not claim to be $q < p$. Therefore, if (24) is satisfied, no agent has any incentive to misrepresent type. ■

Proof of Proposition 3.4: Consider an arbitrary transfer scheme $\hat{t} \neq t^*$ that satisfies the IC constraints (20) and the budget balance condition. By (19), the payoff of an agent of type p who claims to be of type q under the transfer scheme $\hat{t}$ is $\phi_p(q, \mu^*, \hat{t}) = \alpha_q^* MP(\alpha^*) - c_p\left(\alpha_q^*\right) + \hat{t}_q$. It is easy to see that only difference between payoffs under $\hat{t}$ and t^* is the transfers, i.e. $\phi_p(p, \mu^*, \hat{t}) > (=) < \phi_p(p, \mu^*, t^*)$ if and only if $\hat{t}_p > (=) < t^*_p$.

First we make a few observations about the relation between t^* and $\hat{t}$. Since $\hat{t}$ is incentive compatible, it also satisfies the inequality (20). Thus, $\phi_p(p, \mu^*, \hat{t}) \geq \phi_p(p+1, \mu^*, \hat{t})$ or $\alpha^*_p MP(\alpha^*) - c_p(\alpha^*_p) + \hat{t}_p \geq \alpha^*_{p+1} MP(\alpha^*) - c_p(\alpha^*_{p+1}) + \hat{t}_{p+1}$ holds for all $p \in \{1, 2, \cdots, n-1\}$. To simplify notation, we denote $\lambda_{s,t} = \left[\alpha^*_s MP(\alpha^*) - c_s(\alpha^*_s)\right] - \left[\alpha^*_t MP(\alpha^*) - c_s(\alpha^*_t)\right]$. The notation $\lambda_{s,t}$ represents the difference in the payoff of a type s agent at the efficient state when he announces his type truthfully and the payoff that type s agent would receive when he announces some other type t. Now using this notation we can then rewrite the preceding inequality $\phi_p(p, \mu^*, \hat{t}) \geq \phi_p(p+1, \mu^*, \hat{t})$ as $\hat{t}_{p+1} \leq \hat{t}_p + \lambda_{p,p+1}$.[21] We now divide our proof into smaller claims.

(i) **Claim 1**: Suppose $\hat{t}_k > t^*_k$ for some $k \in \mathcal{P}$ then $\hat{t}_p > t^*_p$ for all $p \in \{1, 2, \cdots, k-1\}$.

Proof: From the above arguments we know that $\hat{t}_k \leq \hat{t}_{k-1} + \lambda_{k-1,k}$. Let $\hat{t}_k = t^*_k + \epsilon_k$ where $\epsilon_k > 0$. This implies, $t^*_k + \epsilon_k \leq \hat{t}_{k-1} + \lambda_{k-1,k}$. Now substitute the value of t^*_k in terms of t^*_{k-1}. We get $t^*_{k-1} + \lambda_{k-1,k} + \epsilon_k \leq \hat{t}_{k-1} + \lambda_{k-1,k} \Rightarrow t^*_{k-1} < \hat{t}_{k-1}$. We can apply this argument inductively to obtain the desired result.

(ii) **Claim 2**: Suppose $\hat{t}_k < t^*_k$ for some $k \in \mathcal{P}$ then $\hat{t}_p < t^*_p$ for all $p \in \{k+1, k+2, \ldots, n\}$.

Proof: We know that $\hat{t}_{k+1} \leq \hat{t}_k + \lambda_{k,k+1}$. Let $\hat{t}_k = t^*_k - \delta_k$ where $\delta_k > 0$. This implies, $\hat{t}_{k+1} \leq t^*_k - \delta_k + \lambda_{k,k+1}$. Now substitute the value of t^*_k in terms of t^*_{k+1}. We get $\hat{t}_{k+1} \leq t^*_{k+1} - \lambda_{k,k+1} - \delta_k + \lambda_{k,k+1} \Rightarrow \hat{t}_{k+1} < t^*_{k+1}$. We can apply this argument inductively to obtain the desired result.

(iii) **Claim 3**: If $\hat{t} \neq t^*$ then agents can be partitioned into at most three sets L, M and R where $L = \{1, 2, \ldots, l\}$, $M = \{l+1, l+2, \ldots, r-1\}$[22] and $R = \{r, r+1, \ldots, n\}$ for some $1 \leq l < r \leq n$ such that $\hat{t}_p > t^*_p$ for all $p \in L$, $\hat{t}_p = t^*_p$ for all $p \in M$ and $\hat{t}_p < t^*_p$ for all $p \in R$. The set L and R are always non-empty.

Proof: Define $\tilde{l} = \text{Max}\{p \in \mathcal{P} \mid \hat{t}_p > t^*_p\}$. By definition of $\tilde{l}$, there exists no $p > \tilde{l}$ such that $\hat{t}_p > t^*_p$. Moreover, according to Claim 1, for all $p = 1, 2, \ldots, \tilde{l} - 1$ we have $\hat{t}_p > t^*_p$. Hence, $l = \tilde{l}$ and the set $L = \{1, 2, \ldots, \tilde{l}\}$. Similarly, define $\tilde{r} = \text{Min}\{p \in \mathcal{P} \mid \hat{t}_p < t^*_p\}$. By definition of $\tilde{r}$ there exists no $p < \tilde{r}$ such that $\hat{t}_p < t^*_p$. Moreover, according to Claim 2, for all $p = \tilde{r}+1, \tilde{r}+2, \ldots, n$ we have $\hat{t}_p < t^*_p$. Hence, $r = \tilde{r}$ and the set $R = \{\tilde{r}, \tilde{r}+1, \ldots, n\}$. It is easy to see that $\tilde{l} < \tilde{R}$. Define set $M = \mathcal{P} \setminus L \cup R$. It is obvious that if $p \in M$ then $\hat{t}_p = t^*_p$ and $\tilde{l} < p < \tilde{r}$. Thus we obtain $M = \{\tilde{l}+1, \tilde{l}+2, \ldots, \tilde{r}-1\}$.

Now we argue that both sets L and R are non-empty. Without loss of generality, let $L = \emptyset$. This implies that $\hat{t}_p \leq t^*_p$ for all $p \in \mathcal{P}$ and strict inequity must hold for at least some $q \in \mathcal{P}$ otherwise $\hat{t} \equiv t^*$ which contradicts our hypothesis that $\hat{t} \neq t^*$. But this will imply that

[21] The t^* transfer vector satisfies this relation with equality, i.e. $t^*_{p+1} = t^*_p + \lambda_{p,p+1}$ for all $p \in \{1, 2, \ldots, n-1\}$.

[22] Set M could be empty and in that case we have $r = l + 1$.

$\sum_{p=1}^{n} \hat{t}_p < \sum_{p=1}^{n} t_p^* = T(\mu^*)$ which contradicts our hypothesis that transfer $\hat{t}$ is budget balanced. Hence set L is non-empty. Virtually a similar argument can establish that if set $R = \emptyset$ then it implies $\sum_{p=1}^{n} \hat{t}_p > \sum_{p=1}^{n} t_p^* = T(\mu^*)$. This again contradicts our hypothesis. So, set R is also non-empty. This completes the proof.

(iv) **Claim 4**: Transfer t^* minimizes the Gini coefficient.

Proof: To simplify the exposition, let us denote the equilibrium payoff for an agent of type p, associated with transfer t, as $\phi_p(t)$, i.e., $\phi_p(t) = \phi_p(p, \mu^*, t)$. Using this notation and Lemma 3.2(1), we can rank the payoffs in descending order as $\phi_1(t) > \phi_2(t) > \ldots > \phi_n(t)$. The Gini coefficient, as defined in (18), can then be reformulated in terms of these ordered payoffs as follows:[23]

$$\mathcal{G}(\mu^*, t) = 1 - \frac{1}{\Gamma_1(t)} \sum_{p=n}^{1} m_p \Big(\Gamma_{p+1}(t) + \Gamma_p(t) \Big), \text{ where } \Gamma_p = \sum_{q=n}^{p} m_q \phi_q \text{ and } \Gamma_{n+1} = 0 \quad (61)$$

The term $\Gamma_p(\cdot)$ represents the aggregate payoff of all groups from type p till n. This reformulated expression (61) characterizes the Gini coefficient $\mathcal{G}(\mu^*, t)$ for any arbitrary transfer t in terms of descending payoffs. This formulation is more tractable to demonstrate that t^* minimizes it.

The key to the proof lies in the ranking of these aggregate payoffs $\Gamma_p(t)$. We claim that for the payoff vectors $\phi(t^*)$ and $\phi(\hat{t})$, associated with the optimal transfer t^* and any arbitrary transfer $\hat{t}$ respectively, the following strict inequalities hold:

$$\underbrace{\Gamma_p(t^*)}_{\sum_{q=n}^{p} m_q \phi_q(t^*)} > \underbrace{\Gamma_p(\hat{t})}_{\sum_{q=n}^{p} m_q \phi_q(\hat{t})} \quad \text{for all } p > 1, \quad (62)$$

with equality at $p = 1$, since $\Gamma_1(\cdot)$ equals the mean of the distribution: $\Gamma_1(t^*) = \bar{F}(\mu^*) = \Gamma_1(\hat{t})$.

From the equilibrium payoff definition $\phi_p(t) = \alpha_p^* MP(\alpha^*) - c_p(\alpha_p^*) + t_p$, it follows that any differences between equilibrium payoffs $\phi_p(t^*)$ and $\phi_p(\hat{t})$ arise solely from differences in transfers, i.e., $\phi_p(t^*) - \phi_p(\hat{t}) \equiv t_p^* - \hat{t}_p$ for all $p \in \mathcal{P}$. Thus, the ordering of these two transfers, as established in Claim 3, translates directly into a corresponding ordering of these payoffs.

Now the strict inequality between aggregate payoffs (62), should hold if we extend the implications of Claim 3. Recall the partition of the type set $\mathcal{P} = L \cup M \cup R$ as described in Claim 3.

[23] See Gastwirth [12] and Dorfman [9]. Additionally, we wish to clarify that, in standard definitions of the Gini coefficient, summations typically proceed from 1 to p (or n, as applicable), reflecting the convention of ordering income levels of different groups in ascending order. In our model, however, we reverse this order to align with the ranking of payoffs, which are arranged in descending order with respect to the agents' types.

– For high-cost agents, $p \in R = \{r, r+1, \dots, n\}$, $t_p^* > \hat{t}_p$, implying $\phi_p(t^*) > \phi_p(\hat{t})$. This establishes $\Gamma_p(t^*) > \Gamma_p(\hat{t})$ for all $p \in R$.

– For intermediate types, $p \in M = \{l+1, l+2, \dots, r-1\}$, $t_p^* = \hat{t}_p$, implying $\phi_p(t^*) = \phi_p(\hat{t})$. This again maintains the previous dominance: $\Gamma_p(t^*) > \Gamma_p(\hat{t})$ for all $p \in M \cup R$.

The final step is to assume that this claim is false and hence there exists a $1 < p' \in L$ such that $\Gamma_{p'}(t^*) \leq \Gamma_{p'}(\hat{t})$. But for this part of the partition set $p \in L$, we have $t_p^* < \hat{t}_p$, implying $\phi_p(t^*) < \phi_p(\hat{t})$ for all $p \in \{1, 2, \dots p'\} \subset L$. Consequently, $\sum_{q=p'-1}^{1} m_q \phi_q(t^*) < \sum_{q=p'-1}^{1} m_q \phi_q(\hat{t})$. Combining this inequality with the hypothesis results into

$$\Gamma_1(t^*) = \sum_{q=p'-1}^{1} m_q \phi_q(t^*) + \overbrace{\sum_{q=n}^{p'} m_q \phi_q(t^*)}^{\Gamma_{p'}(t^*)} < \overbrace{\sum_{q=n}^{p'} m_q \phi_q(\hat{t})}^{\Gamma_{p'}(\hat{t})} + \sum_{q=p'-1}^{1} m_q \phi_q(\hat{t}) = \Gamma_1(\hat{t}).$$

But this contradicts the fact that $\Gamma_1(t^*) = \bar{F}(\mu^*) = \Gamma_1(\hat{t})$. Hence, $\Gamma_p(t^*) > \Gamma_p(\hat{t})$ must hold for all $p > 1$.

Finally having shown the inequality (62) implies $\Gamma_{p+1}(t^*) + \Gamma_p(t^*) > \Gamma_{p+1}(\hat{t}) + \Gamma_p(\hat{t})$ for all p, while $\Gamma_1(t^*) = \Gamma_1(\hat{t})$. These inequalities establish that $\mathcal{G}(\mu^*, t^*) < \mathcal{G}(\mu^*, \hat{t})$, proving that t^* minimizes the Gini coefficient. ■

The order $t_1^* < t_2^* < \cdots < t_n^*$ follows from Lemma 3.2(2). The payoff $\phi_p(p, \mu^*, t^*)$ in (26) follows from (16) and the notation introduced in (19). The order $\phi_1(1, \mu^*, t^*) > \phi_2(2, \mu^*, t^*) > \cdots > \phi_n(n, \mu^*, t^*)$ follows from Lemma 3.2(1). Hence, individual rationality will be satisfied if $\phi_n(n, \mu^*, t^*) > 0$. To see why this holds, we now establish another claim.

(v) **Claim 5**: The transfer vector t^* maximizes the lowest post-redistribution payoff, i.e. the payoff of type n agents.

Proof: By part 1 of Lemma 3.2, type n has the lowest post redistribution payoff under any incentive compatible transfer vector t. Now consider vectors t^* and $\hat{t}$, both satisfying incentive compatibility and budget balance. Suppose the claim is not true and transfer vector $\hat{t}$ can do better. This is possible only if $\hat{t}_n > t_n^*$. But Claim 1 then implies that $\hat{t}_p > t_p^*$ for all $p \geq 1$. This implies that $\sum_{p=1}^{n} \hat{t}_p > \sum_{p=1}^{n} t_p^* = T(\mu^*)$, which means transfer $\hat{t}$ is not budget balanced. We, therefore, arrive at a contradiction. Hence, the claim is true.

Note that the equal redistribution transfer scheme $t_p = T(\mu^*)$, for all $p \in \{1, 2, \cdots, n\}$ also satisfies incentive compatibility and budget balance. Hence, by Claim 5, $\phi_n(n, \mu^*, t^*) > \phi_n(n, \mu^*, T(\mu^*))$. But by (16), $\phi_n(n, \mu^*, T(\mu^*)) = \alpha_p^* MP(\alpha^*) - c_p(\alpha_p^*) + T(\mu^*) > 0$. Hence, individual rationality is satisfied. ■

A.3 Appendix to Section 4

Proof of Theorem 4.1: A single agent cannot influence the type distribution $\tilde{m}$ and, hence, the aggregate strategy level $\tilde{\alpha}^*$ or the aggregate tax $T(\tilde{\mu}^*)$. Consider an agent p. Given $\tilde{\alpha}^*$, $(\tilde{\alpha}_p^*, \tilde{t}_p^*)$ satisfy (31). We now apply arguments akin to Lemmas 3.1–3.3 and Proposition 3.4 but with respect to the reported type distribution $\tilde{m}$. For all $\tilde{m}$, it is weakly incentive compatible for type p to reveal type truthfully and, if fact, strictly so if $p = n$. Hence, $\tilde{m} = m$ and $(\tilde{\mu}^*, \tilde{t}^*) = (\mu^*, t^*)$, but in that case, the net payment vector $\tilde{\beta}^* = \tilde{\beta}^*$ by (28) and (32). Hence, (μ^*, β^*) gets implemented. The conclusions about budget balance and individual rationality follow from Proposition 3.4. The payoff $\phi_p(p, \mu^*, t^*)$ follows from (26) and the resulting Gini coefficient (34) follows from (18). ■

References

[1] Arrow K (1979) The Property Rights Doctrine and Demand Revelation Under Incomplete Information. in Economics and Human Welfare, ed. by M. Boskin. New York: Academic Press.

[2] Bandhu S, Lahkar R (2023) Evolutionary robustness of dominant strategy implementation. Economic Theory, 76(2), pp.685-721.

[3] Chakrabarti A, Lahkar R (2018) An Evolutionary Analysis of Growth and Fluctuations with Negative Externalities. Dyn. Games Appl. 2018:733–760.

[4] Chen YC, Li J (2018). Revisiting the foundations of dominant-strategy mechanisms. Journal of Economic Theory, 178, 294-317.

[5] Chung KS, Ely JC (2007). Foundations of dominant-strategy mechanisms. The Review of Economic Studies, 74(2), 447-476.

[6] Clarke E (1971) Multi-part pricing of public goods. Public Choice 11:17–23.

[7] Corchón L (1994) Comparative statics for aggregative games the strong concavity case. Math. Soc. Sci. 28:151–165.

[8] d'Aspremont C, Gérard-Varet L–A (1979) Incentives and incomplete information. J. Public Econ. 11: 25-45.

[9] Dorfman R (1979). A formula for the Gini coefficient. The review of economics and statistics, 146-149.

[10] Elliott M, Golub B, Jackson MO (2014) Financial networks and contagion. Am Econ Rev 104:3115—53

[11] Fleurbaey M (2010). Assessing risky social situations. Journal of Political Economy, 118(4), 649-680.

[12] Gastwirth JL (1972). The estimation of the Lorenz curve and Gini index. The review of economics and statistics, 306-316.

[13] Gini C (1912) Variabilità e Mutuabilità. Contributo allo Studio delle Distribuzioni e delle Relazioni Statistiche. [Fasc. I.]. Tipogr. di P. Cuppini.

[14] Gini C (1921) Measurement of inequality of incomes. The economic journal, 31(121), 124-125.

[15] Grant S, Kajii A, Polak B, Safra Z. (2010). Generalized utilitarianism and Harsanyi's impartial observer theorem. Econometrica, 78(6), 1939-1971.

[16] Green J, Laffont J-J (1979) Incentives in public decision-making. Elsevier North-Holland

[17] Groves T (1973) Incentives in teams. Econometrica 41:617–631.

[18] Holmström B, Myerson RB (1983). Efficient and durable decision rules with incomplete information. Econometrica: Journal of the Econometric Society, 1799-1819.

[19] Kalai E (1977). Proportional solutions to bargaining situations: interpersonal utility comparisons. Econometrica: Journal of the Econometric Society, 1623-1630.

[20] Lahkar R (2019) Elimination of non-individualistic preferences in large population aggregative games. J. Math. Econ. 84:150–165.

[21] Lahkar R, Mukherjee S (2019) Evolutionary Implementation in a Public Goods Game. J. Econ. Theory 181:423–460.

[22] Lahkar R, Mukherjee S (2020) Dominant Strategy Implementation in a Large Population Public Goods Game. Econ. Lett. 197:109616.

[23] Lahkar R, Mukherjee S (2021) Evolutionary Implementation in Aggregative Games. Math. Soc. Sci. 109:137–151.

[24] Mukherjee S, Muto N, & Sen A (2024). Implementation in undominated strategies with applications to auction design, public good provision and matching. Journal of Economic Theory, 216, 105783.

[25] Ohseto S (2006) Characterizations of Strategy-Proof and Fair Mechanisms for Allocating Indivisible Goods. Econ. Theory 29:111–121.

[26] Pápai S (2003) Groves Sealed Bid Auctions of Heterogeneous Objects with Fair Prices. Soc. Choice Welf. 20:371–385.

[27] Rawls J (1999) A theory of justice: Revised edition. Harvard university press.

[28] Ray D (1998) Development economics. Princeton University Press, Princeton.

[29] Sandholm WH (2001) Potential games with continuous player sets. J. Econ. Theory 97:81–108.

[30] Sandholm WH (2002) Evolutionary Implementation and Congestion Pricing. Rev. Econ. Stud 69:667–689.

[31] Sandholm WH (2005) Negative Externalities and Evolutionary Implementation. Rev. Econ. Stud 72:885–915.

[32] Sokoloff KL, Engerman SL (2000) Institutions, Factor Endowments, and Paths of Development in the New World. J. Econ. Perspect. 14:217–32.

[33] Sprumont Y (2013) Constrained-optimal Strategy-proof Assignment: Beyond the Groves mechanisms. J. Econ. Theory 148:1102–1121.

[34] Tadenuma K, Thomson W (1995). Games of Fair Division. Games Econ. Behav. 9:191–204.

[35] Vickrey W (1961) Counterspeculation, auctions, and competitive sealed tenders. J. Finance 16:8–37.