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To Worry or Not: Accounting for the Diverging Trends in National and Local Market Concentration in the United States

Anubha Agarwal
Assistant Professor
Economics Area
Indian Institute of Management Bangalore
anubha.agarwal@iimb.ac.in

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To Worry or Not: Accounting for the Diverging Trends in National and Local Market Concentration in the United States*

Anubha Agarwal†

Indian Institute of Management Bangalore

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Abstract

Recent evidence suggests that local product market concentration may be declining even though there is uncontroversial evidence for the increase in national product market concentration in the United States. Using a standard model incorporating endogenous entry and markups to allow for multi-market firms, this paper reconciles this divergence in trends in national versus local concentration and explores its implications for consumer welfare. I show that the fall in local concentration is driven by a fall in market-entry costs that encourages entry into multiple geographic markets (fall in local concentration), but this is coupled with a decline in the number of firms in the economy (increase in national concentration). Calibration of the baseline model leads to striking results. A reduction of 10% in market-entry costs leads to: (1) An increase of 4.38% in the number of firms in a market and a decrease of 0.01% in the number of firms in the economy (2) An increase of 2.36% in aggregate consumption and real wages. Hence, increasing national market concentration in the United States may not be a cause of worry as it is associated with better outcomes for consumers.

Keywords: national market concentration, local market concentration, local markets
JEL Classification: E20, L10, L40

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†Indian Institute of Management Bangalore, Email: anubha.agarwal@iimb.ac.in

1 Introduction

The recent decades have seen a rise in market concentration,[1] average markups, and R&D spending at the aggregate level, coupled with declines in outcomes such as labor share, business dynamism and productivity growth in the United States.[2] Policymakers, academicians and the public in general have taken cognizance of these trends as they affect many important economic outcomes. Economists and policy makers tend to be worried about increasing levels of market concentration as it implies a decline in beneficial competitive forces and is associated with subsequent declines in measures of investments in R&D and growth. It has been documented that this trend is associated with many other trends such as increasing markups and market power (Gutiérrez and Philippon (2017); De Loecker and Eeckhout (2018); Hall (2018)), the rising profits of big firms (Barkai (2020)), declining labor market dynamism and firm entry (Decker, Haltiwanger, Jarmin, and Miranda (2017)), and declining wages and declining labor share (e.g. Dorn, Katz, Patterson, Van Reenen, et al. (2017)).

Even though some papers have questioned the interpretation of these studies in indicating the rise in market power,[3] and others have questioned the empirical validity of some documented trends,[4] the rise in national market concentration has been widely accepted as the main foundation for guiding economic policy regarding restricting monopolisation. For example, the United States Department of Justice and other

[1] Gutiérrez and Philippon (2017) find that this increase in market concentration in the United States has been most pronounced in non-manufacturing sectors. Barkai (2020) and Autor, Dorn, Katz, Patterson, and Van Reenen (2020) find that the national sales share of top firms has been rising since 1997 and also explain the decline in the labor share over the same period.

[2] See Akcigit and Ates (2019) for a detailed discussion of these trends.

[3] See Syverson (2019) and Hopenhayn, Neira, and Singhania (2018).

[4] For example, the evidence regarding markups is mixed. De Loecker, Eeckhout, and Unger (2020) find evidence for rising markups since the 1980s among publicly traded firms, but Traina (2018) shows that the evidence on markups depends on the measurement of variable costs. Edmond, Midrigan, and Xu (2020) show that aggregate markups have increased only modestly when weighted by costs rather than sales. Hall (2018) finds constant markups at the sectoral level using KLEMS productivity data. Similarly, Loukas and Brent (2018) find generally constant markups over time when also accounting for selling, general, and administrative expenses.

agencies evaluate the effects of mergers on market concentration using measures such as the Herfindahl-Hirschman Index[5] since it is widely accepted that increasing market concentration can be harmful for growth in an economy. In this context, evidence showing a fall in market concentration assumes importance as it speaks directly to the consequences for changes in productivity, R&D investment, growth, and other market outcomes.

Recently documented evidence shows that average local concentration (measured at disaggregated levels of geography like CBSA, county, ZIP code) has been falling in the United States, even though national concentration has been increasing. It is important to understand the underlying mechanisms and causes of this divergence since a fall in local concentration can have very different policy implications than what has been focused on based on evidence of an increase in national concentration. In this paper, I explain this divergence in trends in 'local' and 'national' concentration using a model with multi-market firms facing barriers to market entry in addition to barriers to firm formation. I also undertake a quantitative exercise to analyze the impact of a change in market-entry costs on consumer welfare. My benchmark results show that a 10% reduction in the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market leads to an increase of 4.375% in the number of firms in a geographic market (a measure of local concentration) and a decrease of 0.012% in the number of firms in the economy (a measure of national concentration). It also implies an increase of 2.360% in aggregate consumption and real wages. These results are robust to changing some of the parameter values in the model. My analysis is intended to encourage policy makers to rethink the design of antitrust policy that is aimed at restricting the expansion of 'big' firms with the objective of arresting the increase in national concentration. Other outcomes of interest, such as local concentration and the effects on the consumers' welfare should

[5] Details can be found here.

be taken into account to design better policy that is aimed at affecting outcomes related to market concentration. Increasing national concentration may not be a cause of worry (as it is often made to be) if it is accompanied by firms expanding into newer geographic markets and increasing the availability of varieties and aggregate consumption of the final consumer.

This paper is organized as follows. I discuss the motivation for this study in section 2 and provide a literature review in section 3. The baseline model, along with the equilibrium conditions and related analysis is described in section 4. Section 5 describes my quantitative exercises in which I calibrate my model to the United States and show the results of changing market-entry costs on national and local concentration, and consumer welfare. Some robustness checks of my quantitative results are described in section 6. I discuss the next steps and potential extensions to the baseline model in section 7 and conclude in section 8.

2 Motivation

Rossi-Hansberg, Sarte, and Trachter (2021) document that the positive trend in national product market concentration between 1990-2014 becomes a negative trend using measures of average local market concentration.[6] They measure concentration using the Herfindahl-Hirschman index (HHI), but their findings hold for a variety of statistics. The more geographically disaggregated the measure of concentration (measured at the CBSA, County, or ZIP code levels), the more striking its downward trend over the past decades. This difference is relevant as they show that local concentration is falling across SIC 8 industries that together account for 78% of employment and 72% of sales. Their analysis also shows that these diverging trends are always large but stronger in services, retail trade, and FIRE (Finance, Insurance and Real

[6] Some details in this section are taken from their paper for explanatory purposes.

Estate) relative to wholesale trade and manufacturing.

2.1 Data

To show these trends, they use The National Establishment Time Series (NETS) dataset covering the universe of firms and their plants in the United States in the period 1990-2014. The dataset includes sales and employment numbers of all plants at different levels of industrial and geographic disaggregation down to the SIC 8 product code. Since their analysis focuses on the relationship between market concentration and the geographic expansion of enterprises, they do not take into consideration industries like Mining, Agriculture, Forestry, and Fishing, Construction, and Transportation and Public Utilities since they are intrinsically tied to specific locations due to weather or endowments of natural resources. They also exclude from the analysis government establishments (including establishments belonging to enterprises whose headquarters are associated with a public administration SIC code), and establishments associated with central banking, education and non-profit institutions.

2.2 Measures of Concentration

Rossi-Hansberg et al. (2021) consider many different levels of industrial and geographic disaggregation such as an SIC 8-ZIP code pair. Establishments in their dataset are indexed by industry, i, location, l, and year, t. Industries are defined by an SIC 8 code and locations are defined by a latitude-longitude pair. They denote collections of industries into broader classifications (for example, SIC 4 or divisions) by d and collections of locations into broader geographies (ZIP codes, CBSAs, Counties, States, or the entire United States) by g.

Let $S^{I,G}_{e,i,l,t}$ denote the nominal sales of enterprise e in industry i at location l in year t, and $S^{I,G}_{e,i,g,t} = \sum_{l \in g} S^{I,G}_{e,i,l,t}$ its sales in the broader geography g (the sum of all its establishments' sales across all latitude-longitude pairs l in geography g.) The index I is the industrial level of aggregation (SIC 2, 4, 6 or 8), and index G indicates the geographic level of aggregation (zip code, CBSA, county or the entire United States) used to define a location l. Then $S^{I,G}_{e,i,g,t}$ is the enterprise's share of all sales in industry i located in geography g at date t for levels of aggregation G and I.

The main measure of market concentration for the analysis is the Herfindahl-Hirschman Index (HHI) defined as

$$C^{I,G}_{i,g,t} = \sum_{e} (s^{I,G}_{e,i,g,t})^2 \tag{1}$$

where $C^{I,G}_{i,g,t} \in [1/N^{I,G}_{i,g,t}, 1]$ is the sales concentration and $N^{I,G}_{i,g,t}$ is the number of enterprises in industry i and geography g at time t.[7]

2.3 Trends

Figure 1[8] shows a weighted average of the change in market concentration $\Delta C^{I,G}_{t}$ across all industry pairs (i, g) for different geographic levels G, such that

$$\Delta C^{I,G}_{t} = \sum_{i,g} w^{I,G}_{i,g,t} \Delta C^{I,G}_{i,g,t} \tag{2}$$

where weights $w^{I,G}_{i,g,t}$ are the employment shares of industry-geography (i, g) in aggregate employment in year t, and $\Delta C^{I,G}_{i,g,t}$ denotes the change in market concentration between year t and the first year for which sales in industry-geography pair (i, g) are observed. We see that the downward trend in market concentration is more pronounced for more geographically disaggregated measures of concentration. Figure 2 below shows a weighted average of the change in concentration across all industry-geography pairs (i, g) within a particular division d (like manufacturing, services etc.)

[7] The analysis undertaken in Rossi-Hansberg et al. (2021) finds similar results when using alternative measures of concentration such as sales share of the top firm or the 'adjusted' Herfindahl index. The online-only appendix supplement to their paper has more details.

[8] These figures are taken from Rossi-Hansberg et al. (2021).

for particular geographies.

$$\Delta C^{I,G}_{t,d} = \sum_{i \in d,g} w^{I,G}_{i,g,t,d} \Delta C^{I,G}_{i,g,t} \tag{3}$$

The figure shows that market concentration has increased at the national level and fallen at the ZIP code level across all divisions (the trends are more stark for retail trade).

Figure 1: Diverging economy-wide national and local concentration trends

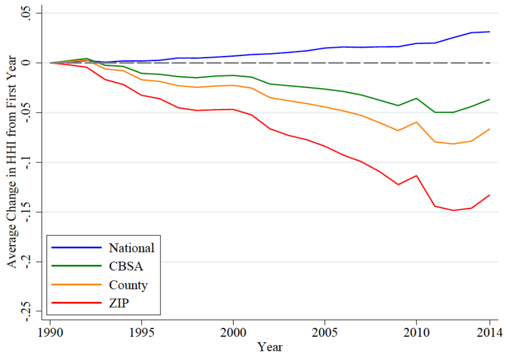


Figure 2: Diverging division-level national and local concentration trends

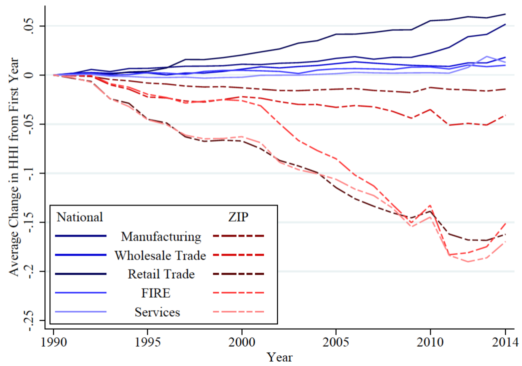


Figure 3 shows the divergence between national and local concentration at the ZIP code level for different degrees of industrial aggregation. It is clear that this divergence is more pronounced at the SIC 8 level, though it is also present at lower levels of industrial aggregation. Figure 4 focuses on employment rather than sales and still finds divergence in trends in national versus local concentration in the United States.

Figure 3: Diverging economy-wide trends in sales concentration

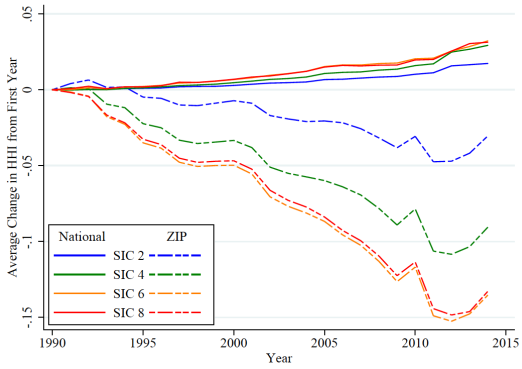
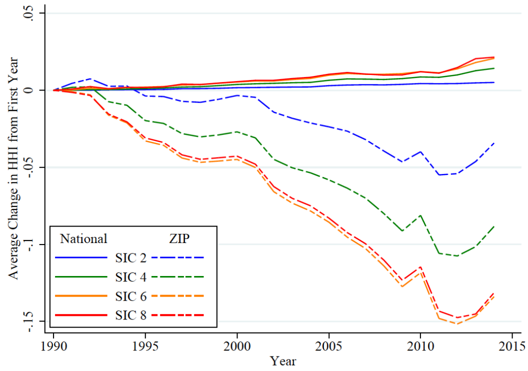


Figure 4: Diverging economy-wide trends in employment concentration



These facts point to the importance of firms expanding into new local markets, which is accompanied by a fall in local concentration. This in turn is suggestive of more competitive markets, not less competitive markets, as has been documented for the United States at the national level. The rising trend in national concentration is not necessarily a cause of concern in itself if it is accompanied by a fall in local concentration. Understanding the economic mechanisms underlying this divergence in trends is key to assessing the implications for economic growth and social welfare, since trends in concentration inevitably lead to linkages with firm pricing and innovation, and this can then be used to formulate optimal policy responses.

In this paper, I reconcile the differences in the empirical trends in local versus national concentration by extending a standard model of endogenous entry and markups to allow for multi-market firms which face barriers to entering a geographical market in addition to firm startup costs. This allows me to explain how more number of firms might enter a geographic market hence leading to increasing competition at local levels, and at the same time lead to consolidation of firms at the national level, thus leading to increasing national concentration. The aim of this paper is not to argue that national concentration is not increasing; it is, in fact, to stress the importance of concentration defined at disaggregated geographic levels as it is more relevant for issues related to firm pricing, availability of product varieties, and other strategic behavior at the firm level.

3 Literature Review

There is a vast amount of literature on the causes and effects of market concentration and market power. For example, Covarrubias, Gutiérrez, and Philippon (2020) study the trends in market shares in industries over the past 40 years in the United States. They find evidence of efficient concentration driven by intangible investment, tougher price competition and increasing productivity of market leaders during the 1990s, but evidence of inefficient concentration associated with higher prices, lower investment and lower productivity growth after 2000. Aghion, Bergeaud, Boppart, Klenow, and Li (2019) explain the rise in concentration and profits through falling firm-level costs of spanning multiple markets due to accelerating IT advances. Gutiérrez and Philippon (2017) study the potential drivers of rising concentration and show that declining competition has been a cause of under-investment in the business sector in the United States since the early 2000s. They provide some evidence that the increase in concentration can be explained by increasing regulations. Bessen (2020) explores the role of proprietary information technology systems (IT) in increasing industry concentration by raising the productivity of top firms relative to others. Olmstead-Rumsey (2019) finds that declining innovativeness of market laggards can account for about 40% of the rise in market concentration between the 1990s and the 2000s and the entire productivity slowdown in the United States.

Grullon, Larkin, and Michaely (2019) show that firms in industries with largest increases in product market concentration show higher profit margins and more profitable M&A deals, but find no evidence for increase in operational efficiency in those industries. They also find that lax enforcement of antitrust regulations and increasing technological barriers to entry appear to be important factors behind this trend. Barkai (2020) shows that the decline in competition plays a significant role in the decline in the labor and capital share, and a large increase in the share of pure profits offsets the declines in the shares of both labor and capital. Hence, at the aggregate level, there has been a persistent increase in sales concentration in the United States over the past two decades, which coincided with the decline in the labor share. Kehrig and Vincent (2018) find that there has been a major reallocation of value added toward the lower end of the labor share distribution since the 1980s due to units whose labor share fell as they grew in size. The negative co-movement between market concentration and labor share is often seen as evidence in favor of rising monopoly power (Barkai (2020)). Cavenaile, Celik, and Tian (2021) argue that the dynamic effects of increasing market concentration on innovation and productivity growth have been crucial in understanding the transformation of the United States in the recent decades and the notable increase in concentration and markups is not necessarily detrimental for welfare. This paper contributes to this literature on the causes and effects of market concentration by showing how a change in market-entry costs causes a divergence in trends in concentration and affects consumer welfare.

Another strand of literature relates to the role of concentration in affecting labor and firm level outcomes. Rinz (2018) uses data from the Longitudinal Business Database (LBD) to document trends in local industrial concentration from 1976 through 2015 and finds that unlike the trends in national concentration, local industrial concentration has been falling across its distribution. The author also estimates its effects on earnings outcomes within and across demographic groups. The estimates indicate that increased local concentration reduces earnings and increases inequality, but concentration actually moved in the other direction, and the magnitude of these effects has been modest relative to broader trends. Berger, Herkenhoff, and Mongey (2019) develop a general equilibrium, oligopsony model of the labor market that reproduces quasi-experimental evidence on imperfect productivity-wage pass-through, strategic behavior of dominant employers and the local labor market impact of mergers. They find that declining local concentration added 4 percentage points to labor's share of income between 1977 and 2013 in the United States. Hershbein, Macaluso, and Yeh (2018) study the time-series and cross-sectional properties of concentration in job creation, employment and vacancy flows in local labor markets in the United States. Among other things, they find that local labor market concentration decreased over time (dropping by at least 25% since 1976). They also undertake a statistical decomposition exercise which implies that the covariance between a local labor market's size and its concentration level decreased over time.

Jia (2008) uses a structural model to study competition by Walmart and other discount retail stores to show that the profits of previously available retailers decrease when 'Walmart comes to town'.[9] This is consistent with the finding in Rossi-Hansberg et al. (2021) that firms entering new markets lower concentration by taking market share away from local competitors. Syverson (2008) uses the ready-mixed concrete industry as an example of an industry with a local market. The expansion of Cemex, the top firm by sales in 2014 in the ready-mixed concrete industry, led to a fall in local concentration and an expansion in the local number of establishments in the industry. This paper contributes to this literature by documenting quantitative effects on labor market and firm outcomes which are linked to outcomes for national and local concentration.

The main motivation for this paper is the paper by Rossi-Hansberg et al. (2021) which documents the diverging trends in national and local market concentration in the United States. Carlson and Mitchener (2009) show how smaller incumbent banks responded to the entry of a large branch bank by adjusting their operations in a manner consistent with increased efficiency using the experience of California in the

[9] Holmes (2011) also studies the (geographic) expansion strategy of Walmart.

1920s and 1930s. They find that unit banks exposed to this competition were more likely to survive the Great Depression than banks not exposed to it, hence pointing towards a benefit of bank branch expansion at the local level which came with national level consolidation. Hence, there is evidence of how the easing of restrictions on branch banking resulted in more bank branches and more competition between branches, but fewer banks. This highlights the relevance of understanding the potential benefits of increasing national concentration if it is coupled with a fall in local concentration. To my knowledge, no study has explained the diverging trends in national and local market concentration in the United States using a model incorporating features of multi-market entry, firm-formation costs and market-entry costs. This paper fills this gap in the literature by developing such a model and calibrating it to the United States to show the quantitative effects of changing market-entry costs on outcomes related to market concentration and consumer welfare.

4 Model

I use a standard model of free entry and endogenous markups and allow for multi-market firms.[10] Firms face two kind of barriers - barriers to starting a firm (firm-formation cost) and barriers to entering a geographic market (market-entry cost). I consider a static economy framework where a representative final good firm uses imperfectly substitutable inputs from each of a continuum of geographic markets[11] of measure one to produce a final consumption good (the numeraire). There is a stand-in household having a continuum of members of measure one, each supplying one unit of labor.

[10]This model is based on the model used in Bento (2020), in which he obtains the empirically documented negative relationship between barriers to competition and firm-level innovation.

[11]This is similar to considering a continuum of product markets, as is common in the literature.

4.1 Baseline Framework

The market for final output is perfectly competitive. A representative firm uses inputs from all firms entering different geographic markets to produce output according to the production function below:

$$Y = \exp\left(\frac{\sigma}{\sigma-1}\int_0^1 \ln\left[\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}\right] dm\right) \tag{4}$$

where m indexes a continuum of geographic markets of measure one, y_{nm} is the quantity demanded of variety n from market m, N_m is the number of varieties/firms in market m,[12] and σ is the constant elasticity of substitution between varieties within a geographic market. I assume that the elasticity of substitution between geographic markets is equal to one.[13] The representative household/consumer in this economy consumes the final good produced by the representative final good firm, hence $Y = C$ is also the measure of consumer welfare/utility in this model.

In the baseline model, firms play a two-stage game given fixed productivity 'A'. In the first stage, they make the market entry decision, i.e. choose the number of geographic markets to enter 'g' and in the second stage, they make their labor demand decision, i.e. choose how much labor to hire for production in the given market 'l'. Firms that choose to produce and enter any market also pay a fixed cost of becoming a producer (firm-formation cost). Market-entry costs are costs like costs of market research, investment in differentiating one's product, capital requirements, customer switching costs, access to distribution channels and costs of obtaining licenses to operate in the geography, whereas firm-formation costs include costs associated with regulatory clearances, clearing government-mandated requirements and costs associated with capacity building. In addition, the firms also pay costs of innovation

[12] Each firm will choose to introduce only one product/variety per market (hence the equivalence between varieties and firms in a geographic market) in order to avoid cannibalizing its own profits.
[13] The results hold as long as this elasticity is lower than σ.

or R&D. Hence, a firm can choose to become a producer by paying $(c_F + c_g g^{\gamma+1})w$, where c_F is firm-formation cost, c_g is market-entry cost, g is the number of geographic markets entered by a firm and γ is the scaling parameter for market-entry cost such that $c_F > 0, c_g > 0$ and $\gamma > 0$.[14] L_p is the total labor used in production (as opposed to labor absorbed due to firm-formation costs and market-entry costs). Once market entry and labor demand decisions are made, firms produce output in each market using labor according to the following production function:

$$Y = Al \tag{5}$$

Firms enter till the value of forming a firm $V = g\pi - (c_F + c_g g^{\gamma+1})w$ is driven to zero.

4.2 Equilibrium

Given firm productivity (A), the decentralized equilibrium can be defined as a set of prices {P, p, w}: final good's price ($P = 1$, normalized), a common price for each variety (p), wage (w); and quantities {l, y, π, g, N, $\frac{N}{g}$, Y}: labor demand of each firm (l), output of each firm ($y = Al$), operating profits per variety/firm (π), number of geographic markets entered by a firm (g), number of firms/varieties per geographic market (N), number of firms in the whole economy $\left(\frac{N}{g}\right)$, and aggregate output (Y) produced by the final good producer such that:

- Given prices, the final good producer chooses y to maximize profit.
- Given prices and other decisions of rival firms, each firm chooses l and g to maximize its value of forming a firm V.
- Free Entry holds: V = gπ - $(c_F + c_g g^{\gamma+1})$w = 0

[14] All costs are specified in units of labor.

- Total labor demand: $L_d = Nl$
- Labor market clears: $L_p + \frac{N}{g}(c_F + c_g g^{\gamma+1})w = 1, \;\; L_p = L_d$

The first order condition of the final-good firm's optimization problem gives the following inverse demand function for variety n in geographic market m:

$$p_{nm} = Y \frac{y_{nm}^{-\frac{1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} \tag{6}$$

Operating profits for firm n in market m:

$$\pi_{nm} = p_{nm} y_{nm} - \frac{w y_{nm}}{A_n} = \frac{(\mu_{nm} A_n - 1) y_{nm} w}{A_n} \tag{7}$$

where $\mu_{nm} = p_{nm}/w$ and $\mu_{nm} A_n$ is firm n's markup in market m. Suppressing subscripts (since markets are identical in this case) and taking μ as given, the value of forming a firm is:

$$V = g.\pi_A - (c_F + c_g g^{\gamma+1}).w \tag{8}$$

As discussed above, each firm chooses how many geographic markets to enter (g) and labor demand (l) to maximize V. Hence I obtain the following first order conditions:[15]

$$labor\ demand : l = \left[\left(\frac{\sigma - 1}{\sigma} \right) \frac{Y}{w} \frac{(N-1)}{N^2} \right] \tag{9}$$

$$markets\ per\ firm : c_g g^{\gamma+1} = \frac{g(\mu A - 1) y}{(\gamma + 1) A} \tag{10}$$

In addition, I use the following free entry condition (since firms enter till the value of forming a firm is driven to zero):[16]

$$c_F = \frac{\gamma g \pi}{(\gamma + 1) w} = \frac{\gamma}{(\gamma + 1)} \frac{g y (\mu A - 1)}{A} \tag{11}$$

[15] See Appendix 9.1 for the derivations.
[16] See Appendix 9.2 for the derivation.

If the aggregate quantity of labor used in production is L_p, the total number of firms in the economy is $\frac{N}{g}$ and total labor available in the economy is one, then,

$$L_p = 1 - \frac{N}{g}(c_F + c_g g^{\gamma+1}) \tag{12}$$

We also know that all firms hire the same labor in equilibrium:

$$y = Al = A\frac{L_p}{N} \tag{13}$$

Using the previous equations[17]

$$L_p = \frac{1}{\mu A} = \frac{1}{MU} \tag{14}$$

Using the previous equations, I obtain the following equations that characterize the equilibrium values of the number of geographic markets entered per firm g, and number of firms in the economy $\frac{N}{g}$:[18]

$$c_g g^{\gamma+1} = \frac{gy(MU-1)}{A(\gamma+1)} = \frac{c_F}{\gamma} \tag{15}$$

$$\frac{N}{g} = \frac{\gamma}{\gamma+1}\frac{(MU-1)}{MUc_F} \tag{16}$$

Also, aggregate output (or consumption) can be expressed as:

$$Y = C = TFP.L_p = N^{\frac{1}{\sigma-1}}A.L_p = N^{\frac{\sigma}{\sigma-1}}y \tag{17}$$

[17] See Appendix 9.3 for the derivation.
[18] See Appendix 9.4 for the derivations.

4.3 Analysis

Given the expressions for the number of geographic markets entered by a representative firm and the total number of firms in the economy, I analyze how barriers to market entry and firm formation affect the total number of firms in the economy (a measure of national concentration) and the number of firms in a geographic market (which in turn determines local concentration).

4.3.1 Market-entry Costs

The impact of lower barriers to market entry (c_g) depends on how the equilibrium markup (MU) depends on the number of competitors in the market (N).

Case 1: Firms take the size of each market as given (monopolistic competition), and the markup is $MU = \frac{\sigma}{\sigma-1}$, which is independent of N. Then from equation 16, a fall in c_g does not affect the number of firms $\frac{N}{g}$. From equation 15, the adjustment to a lower c_g is through an increase in the number of geographic markets entered per firm (since $\frac{N}{g}$ remains constant, this increase in g implies a proportional increase in N). This decreases the market share of each firm, offsetting the higher profits per firm due to more markets entered, and the number of firms in the economy do not change.

Case 2: Firms take into account how their decisions affect the size of a market (Cournot competition), and the markup is $MU = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{N}{N-1}\right)$,[19] which is decreasing in the number of competitors per market N. Using equations 15 and 16, we can show that a fall in c_g leads to an increase in the number of varieties/firms in a geographic market (N), a fall in markups (MU), an even higher number of markets per firm (g), and fewer firms $\left(\frac{N}{g}\right)$.

Proposition: In the case of Cournot competition, a fall in costs of market entry (c_g)

[19] See Appendix 9.5 for the derivation.

is associated with a fall in the number of firms in the economy $\left(\frac{N}{g}\right)$ (increase in national concentration) and an increase in the number of varieties/firms in a geographic market (N) (fall in local concentration).

Proof: I show that a fall in c_g must imply an increase in N. Let us assume that it does not. Assume first that a change in c_g does not affect N. Then, since MU is a function of σ and N, it does not change. From equation 16, this implies that g does not change (N does not change and $\left(\frac{N}{g}\right)$ does not change since the right hand side of the equation does not change). But if g does not change, equation 15 cannot hold because the right hand side is fixed and c_g on the left hand side changes. Hence we have shown that a change in c_g must affect N. Now suppose that a fall in c_g lowers N. Since MU is decreasing in N, this must increase MU. From equation 16, g must fall (because $\left(\frac{N}{g}\right)$ is increasing in MU, so an increase in MU implies an increase in $\left(\frac{N}{g}\right)$ and since N is falling, g must fall too). Combining equations 15 and 16, we get

$$c_g g^\gamma = \frac{(MU-1)}{N.MU.(1+\gamma)} \tag{18}$$

If c_g, g, and N are falling and MU is increasing as above, then equation 18 cannot hold and we get a contradiction. Hence, a fall in c_g must imply an increase in N.

Since MU is decreasing in N, an increase in N due to fall in c_g leads to lower markups. From equations 15 and 16, this is associated with a higher g and a lower $\left(\frac{N}{g}\right)$ (since $\left(\frac{N}{g}\right)$ is increasing in MU). The intuition behind this result is that an increase in N in a geographic market increases the price elasticity of demand. As demand is more sensitive to the price charged by a firm, the incentive to increase the price falls. This in turn decreases the value of creating a firm, so free entry leads to lesser number of firms in equilibrium. Hence, a fall in market-entry costs leads to an increase in the number of varieties/firms in a particular geographic market (fall in local concentration), while leading to a fall in the number of firms in the economy (increase in national concentration).

4.3.2 Firm-formation Costs

The qualitative effects of changing the cost of firm formation (c_F) do not depend on how markups are related to the number of firms (N). Equations 15 and 16 show that the number of markets entered per firm (g) are increasing in c_F, and the number of firms $\left(\frac{N}{g}\right)$ in the economy is decreasing in c_F. We can refer to equation 18 to see how the number of varieties/firms (N) is affected. Given that g moves in the same direction as c_F, the number of competitors per market (N) is decreasing in c_F (so that $\frac{N}{g}$ is falling in c_F). Hence, a fall in c_F would reduce the number of markets entered by a firm (g), increase the number of firms in the economy $\left(\frac{N}{g}\right)$, increase the number of firms in a market (N), and reduce markups (MU).

In general, barriers to starting firms discourage startups, resulting in higher markups and higher market shares within each geographic market. Hence, firm formation barriers decrease the number of competitors per market (N), and reduce the number of firms in the economy $\left(\frac{N}{g}\right)$.

As discussed above, a change in barriers to entry in terms of market-entry costs can explain the increase in national concentration and the fall in local concentration in the United States. A fall in market-entry costs implies an increase in the number of firms in a local geographic market (a fall in local concentration) but a fall in the number of firms on the national level (an increase in national concentration).

5 Quantitative Analysis

5.1 Calibration

The values of four parameters must be determined: σ, γ, c_g and c_F. I calibrate the first two parameters externally, using values largely accepted in the literature. σ is set to 10 from Atkeson and Burstein (2008) and γ is set to 2 using estimates from Cavenaile, Celik, and Tian (2019), who estimate the cost convexity parameters in their model to be between 2 and 4 (their estimates correspond to estimates for $(1{+}\gamma)$, hence I consider the mean value of $(1{+}\gamma)$ to be 3, and set γ to 2 in my baseline calibration). In the model used by Cavenaile et al. (2019), the 'cost convexity' parameters reflect the costs in units of the final good to generate a certain Poisson rate of success in R&D by a firm. I interpret these as indicative of market-entry costs that depend on the number of markets a firm has already entered in my model. I calibrate the other parameters for the United States using the following equations from the model as described below (values of all parameters and variables are in their respective units):

1. $l = \frac{y}{A} = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{Y}{w}\right)\left(\frac{N-1}{N^2}\right)$
2. $c_F = \frac{\gamma}{\gamma+1}\frac{gy(\mu A-1)}{A}$
3. $c_g g^{\gamma+1} = \frac{g(\mu A-1)y}{(\gamma+1)A} = \frac{c_F}{\gamma}$
4. $\frac{N}{g} = \frac{\gamma}{\gamma+1}\frac{(\mu A-1)}{\mu A c_F}$
5. $L_p = \frac{1}{\mu A} = \frac{1}{MU}$
6. $MU = \mu A = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{N}{N-1}\right)$

Using the labor demand and labor market clearing conditions, I get $Y = w$.[20] I use the estimates of sales-weighted average markups in the United States from De Loecker

[20] See Appendix 9.6 for the derivation.

et al. (2020) who claim that the average markup charged was 61% over marginal cost in 2016. Hence, I set MU to 1.61 for my baseline calibration. From equation 5 above, I get $L_p = 0.6211$. Using the baseline values of σ and MU, I use equation 6 above to get $N = 3.2272$. I calibrate g, which is the number of geographic markets entered by a firm using the latest information on the total number of firms and the total number of establishments in the United States. Since the model implies that each firm will produce only one variety in one market (to avoid cannibalizing its own profits), it will set up only one establishment in each geographic market. Hence, g can be calibrated by dividing the total number of establishments in the United States (=25.924671 million from the National Establishments Time-Series or NETs data) by the total number of (employer) firms in the United States (=6.075937 million from the United States Census Bureau).[21] Hence, I get $g = 4.2668$. Next, using equations 2, 3 and 4 above, I obtain $c_F = 0.3340$ and $c_g = 0.0021$.[22] The values of the calibrated parameters are summarized in Table 1.

Table 1: Baseline Model Parameters

Parameter	Description	Value	Source
σ	Elasticity of substitution between varieties	10	Atkeson and Burstein (2008)
γ	Scaling parameter for market-entry cost	2	Cavenaile et al. (2019)
c_F	firm-formation cost	0.3340	Internally Calibrated
c_g	market-entry cost	0.0021	Internally Calibrated

Having obtained the values of all the parameters (σ, γ, c_g and c_F) and the values of MU, g, L_p and N, I calculate the values of $\frac{N}{g}$, l, y, Y, p, w and π for the baseline model. Using the values of N and g obtained earlier, I get $\frac{N}{g} = 0.75635$. Using equation 1 for labor demand, I get $l = 0.1925$. I normalize $A = 1$, so $y = 0.1925$ and using equation 17, I get $Y = C = 0.7075$. From equation 6, I get $p = 1.139$ (the price of the final good is normalized to 1, i.e. $P = 1$). Using the values of p and MU, I get $w = 0.7075$ (the same can be obtained using $Y = w$ as shown above). Then

[21] See here for the data.
[22] See Appendix 9.7 for details.

using equation 7, $\pi = 0.0831$. These initial values of all the variables are shown in the column named 'Initial Value' in Table 2.

5.2 Analysis

Recent advances in Information, Communication, and Transportation technologies and the intensified use of the internet and social media have led to decreases in market-entry costs. It is much easier to obtain information, establish, and nourish business relationships with customers in new markets. Collecting reliable and useful information about buyers, competitors, and overall market conditions has made it easier to set up shop in new geographic markets.[23] Hence, I consider a comparative statics exercise to analyze the effect of market-entry costs on local and national concentration in my baseline model. Using the initial values of variables and parameters obtained above, the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market is 14.0514% $\left(\frac{c_g g^{\gamma+1} w}{gy} = 0.140514\right)$. For the purpose of analysis of the effect of market-entry costs on concentration, I consider a 10% (or 1.41 percentage points) reduction in the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market,[24] i.e. $\frac{c_g g^{\gamma+1} w}{gy} = 0.126463$. The value of c_g that corresponds to this number is 0.00189, which I treat as the 'final' value of this parameter. Then I analyze the effect of this change in market-entry costs on all variables of interest, especially those corresponding to measures of national and local concentration and consumer welfare as follows. I use equation 15 to calculate the number of geographic markets entered by a firm (g) and equation 18 to calculate the number of firms in a geographic market (N). Once I obtain g and N, I can calculate $\left(\frac{N}{g}\right)$. I find the new level of markups (MU) charged by the firm using the relationship between MU, σ and N. The total

[23] For example, Jiang (2021) shows that information and communication technology can increase firms' geographic coverage by reducing internal communication costs.

[24] I consider a reduction of 10% for explaining the main mechanisms of my model as accurate data on market-entry costs in geographic markets in the United States is hard to observe.

labor used in production L_p can then be found using the reciprocal of the markups. Labor demand (l) and firm output (y) in each geographic market can be found using L_p and N. Then $Y = C$ can be found using equation 17 and p can be found using equation 6. Wage (w) can be found using MU and p (or using $Y = w$) and finally profits can be found once we know the price, wage and output. The values of all these endogenous variables that are consistent with the 'new' value c_g are shown in the column named 'Final Value' in Table 2.

5.3 Results

The column named 'Change' in Table 2 records the increase or decrease in the respective endogenous variable. All the variables of interest change in the expected direction as predicted by the theoretical model. A 10% reduction in the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market (or a 10% reduction in c_g) leads to an increase of 4.375% in the number of firms in a geographic market 'N' (fall in local concentration) and a 0.012% decrease in the number of firms in the economy (increase in national concentration). The average firm charges 1.845% lower markups in a market and earns 5.054% lower profits in a geographic market, but also enters 4.387% more markets. There is an increase of 1.884% in the total labor used in production versus the labor used for firm formation and market-entry costs due to the fall in market-entry costs. Even though the labor demand and output produced in a geographic market fall by 2.390%, the aggregate output produced in the economy (which is also the total consumption) increases by 2.360%. Both prices and wages in a market increase, but real wages ($w/P = w/1$) increase by 2.360% ($Y = C = w$).

Hence, there is a divergence in the trends in national and local market concentration - national market concentration increases but local market concentration falls. This is accompanied by an increase in welfare of the consumers as there is an increase in aggregate consumption and real wages. Thus, an increase in national concentration may not necessarily be a cause of worry as on average, firms enter more markets and there are more firms in each geographic market when market-entry costs fall. This allows them to charge lower markups in a particular geographic market, and this benefits the consumers as they can enjoy more consumption along with earning higher real wages.

Table 2: Baseline Results ($\sigma = 10, \gamma = 2$, MU=1.61)

Variable	Description	Initial Value	Final Value	Change
MU	Markup charged by a firm	1.61	1.5803	$\downarrow$ 1.845%
g	Number of geographic markets entered by a firm	4.2668	4.4540	$\uparrow$ 4.387%
L_p	Labor used for production	0.6211	0.6328	$\uparrow$ 1.884%
N	Number of firms in a geographic market	3.2272	3.3684	$\uparrow$ 4.375%
N/g	Total Number of firms in the economy	0.75635	0.75626	$\downarrow$ 0.012%
l	Labor demand by a firm	0.1925	0.1879	$\downarrow$ 2.390%
y	Output produced by a firm	0.1925	0.1879	$\downarrow$ 2.390%
$Y = C$	Aggregate Output in the economy	0.7075	0.7242	$\uparrow$ 2.360%
p	Price of output in each geographic market	1.1390	1.1440	$\uparrow$ 0.439%
w	Wage in each geographic market	0.7075	0.7242	$\uparrow$ 2.360%
π	Profit of a firm in each geographic market	0.0831	0.0789	$\downarrow$ 5.054%

6 Robustness of Quantitative Results

The results obtained above use the benchmark calibrated values of $\sigma = 10$, $\gamma = 2$ and $MU = 1.61$. It is important to check whether the above results hold for other values of the parameters used in the literature as well. In this section, I carry out similar exercises as documented above for different values of the parameters. In most cases, I change one of the parameter values at a time while keeping the others constant at the baseline levels.

6.1 Elasticity of Substitution between Varieties (σ)

Broda and Weinstein (2010) analyze the prices and quantities of all products purchased by the representative household in their study describing the extent of product creation and destruction in the United States. They estimate that the median elasticity of substitution across-brand modules within a product group is 7.5, hence I use this value of $\sigma = 7.5$ as a robustness check of my benchmark results. Table 3 shows the calibrated parameters for this case (this c_g corresponds to a value of 15.60% for the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market). The 'Initial Value' column in Table 4 shows the initial values of all the variables corresponding to this calibration. Then I consider a 10% change in the costs of market entry as a proportion of the output produced in a market, i.e. the firm now spends 14.041% of its output in a geographic market on market-entry costs and the 'new' c_g is 0.0018. The re-calculated values of all the endogenous variables are shown in the 'Final Value' column of Table 4.

Table 3: Model Parameters with $\sigma = 7.5$

Parameter	Description	Value	Source
σ	Elasticity of substitution between varieties	7.5	Broda and Weinstein (2010)
γ	Scaling parameter for market-entry cost	2	Cavenaile et al. (2019)
c_F	firm-formation cost	0.3054	Internally Calibrated
c_g	market-entry cost	0.0020	Internally Calibrated

As expected, when the market-entry costs decrease, there is an increase in the number of firms in a geographic market (fall in local concentration), an increase in the number of geographic markets entered by a firm, and a fall in the number of firms in the economy (increase in national concentration). A 10% fall in market-entry costs leads to an increase of 2.964% in the number of firms in a market and a decrease of 0.012% in the number of firms in the economy. This is accompanied by a 1.591% increase in aggregate consumption and real wages, putting the emphasis on consumers being better off even though market concentration is increasing at the national level.

Table 4: Results with $\sigma = 7.5, \gamma = 2$, MU=1.61

Variable	Description	Initial Value	Final Value	Change
MU	Markup charged by a firm	1.61	1.5919	$\downarrow$ 1.124%
g	Number of geographic markets entered by a firm	4.2668	4.3940	$\uparrow$ 2.981%
L_p	Labor used for production	0.6211	0.6282	$\uparrow$ 1.143%
N	Number of firms in a geographic market	3.5295	3.6341	$\uparrow$ 2.964%
N/g	Total Number of firms in the economy	0.8272	0.8271	$\downarrow$ 0.012%
l	Labor demand by a firm	0.1760	0.1729	$\downarrow$ 1.761%
y	Output produced by a firm	0.1760	0.1729	$\downarrow$ 1.761%
$Y = C$	Aggregate Output in the economy	0.7541	0.7661	$\uparrow$ 1.591%
p	Price of output in each geographic market	1.2141	1.2193	$\uparrow$ 0.428%
w	Wage in each market	0.7541	0.7661	$\uparrow$ 1.591%
π	Profit of a firm in each geographic market	0.0810	0.0784	$\downarrow$ 3.210%

6.2 Scaling Parameter for market-entry cost (γ)

Since the literature considers $(1+\gamma)$ to be between 2 and 4 and hence γ to be between 1 and 3, I also consider $\gamma = 1$ as a robustness check of my baseline results. Table 5 shows the model parameters when $\gamma = 1$, and Table 6 shows the initial values and final values of the variables after the market-entry cost is reduced by 10%. The initial c_g corresponds to a value of 21.64% for the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market which is then reduced to 19.48%, i.e. $c_g = 0.01242$. I obtain the expected results as the number of firms in a geographic market increases by 5.24%, signifying a fall in local concentration and the number of firms in the economy decreases by 0.013%, signifying an increase in national concentration. Consumer welfare rises as aggregate consumption and real wages increase by 2.813%. All other variables also have the expected changes, hence these results are consistent with the benchmark results.[25]

Table 5: Model Parameters with $\gamma = 1$

Parameter	Description	Value	Source
σ	Elasticity of substitution between varieties	10	Atkeson and Burstein (2008)
γ	Scaling parameter for market-entry cost	1	Cavenaile et al. (2019)
c_F	firm-formation cost	0.2505	Internally Calibrated
c_g	market-entry cost	0.0138	Internally Calibrated

[25] I also do this exercise using $\gamma = 3$ and obtain qualitatively similar patters except for the number of firms in the economy. See Appendix 9.8 for details.

Table 6: Results with $\sigma = 10, \gamma = 1$, MU=1.61

Variable	Description	Initial Value	Final Value	Change
MU	Markup charged by a firm	1.61	1.5748	$\downarrow$ 2.186%
g	Number of geographic markets entered by a firm	4.2668	4.4910	$\uparrow$ 5.255%
L_p	Labor used for production	0.6211	0.6350	$\uparrow$ 2.238%
N	Number of firms in a geographic market	3.2272	3.3963	$\uparrow$ 5.240%
N/g	Total Number of firms in the economy	0.75635	0.75625	$\downarrow$ 0.013%
l	Labor demand by a firm	0.1925	0.1870	$\downarrow$ 2.857%
y	Output produced by a firm	0.1925	0.1870	$\downarrow$ 2.857%
$Y = C$	Aggregate Output in the economy	0.7075	0.7274	$\uparrow$ 2.813%
p	Price of output in each geographic market	1.1390	1.1453	$\uparrow$ 0.553%
w	Wage in each market	0.7075	0.7274	$\uparrow$ 2.813%
π	Profit of a firm in each geographic market	0.0831	0.0781	$\downarrow$ 6.017%

6.3 Markup (MU)

Edmond et al. (2020) estimate cost-weighted markups for the United States and find that their average is lower at about 1.25 instead of 1.61 for sales-weighted markups. I consider this cost-weighted average markup and re-calibrate my model, keeping all other parameters at their baseline levels. With this change, the calibrated values of c_F and c_g are 0.0632 and 0.0004 respectively. After a change of 10% in market-entry costs, 'new' c_g is 0.00027. The number of firms entering a particular market increases by 4.178%, the number of firms in the economy increases slightly by 0.019%, and consumer welfare increases by 0.960%. Since the lower level of markups reduces the firms' incentive to enter more geographic markets, they would need to face a lower elasticity of substitution between varieties in a market to offset this reduction in the incentive to enter more markets in response to a fall in market-entry costs. Hence, I redo this exercise by setting the markups to 1.25 and setting $\sigma = 7.5$. The initial values of the parameters are shown in Table 7 and the results for this exercise are shown in Table 8 below. The 'new' c_g is 0.00027 in this case.

The number of firms entering a particular market increases by 1.363% and the number of firms in the economy decreases by 0.089%. The impact on consumer welfare is in the expected direction - aggregate consumption and real wages increase by 0.329%.

Table 7: Model Parameters with MU = 1.25 and $\sigma = 7.5$

Parameter	Description	Value	Source
σ	Elasticity of substitution between varieties	7.5	Broda and Weinstein (2010)
γ	Scaling parameter for market-entry cost	2	Cavenaile et al. (2019)
c_F	firm-formation cost	0.0438	Internally Calibrated
c_g	market-entry cost	0.0003	Internally Calibrated

Table 8: Results with $\sigma = 7.5, \gamma = 2$, MU=1.25

Variable	Description	Initial Value	Final Value	Change
MU	Markup charged by a firm	1.25	1.2486	$\downarrow$ 0.112%
g	Number of geographic markets entered by a firm	4.2668	4.3287	$\uparrow$ 1.451%
L_p	Labor used for production	0.8	0.8009	$\uparrow$ 0.113%
N	Number of firms in a geographic market	13.0	13.1772	$\uparrow$ 1.363%
N/g	Total Number of firms in the economy	3.0468	3.0441	$\downarrow$ 0.089%
l	Labor demand by a firm	0.0615	0.0608	$\downarrow$ 1.138%
y	Output produced by a firm	0.0615	0.0608	$\downarrow$ 1.138%
$Y = C$	Aggregate output in the economy	1.1870	1.1909	$\uparrow$ 0.329%
p	Price of output in each geographic market	1.4838	1.4864	$\uparrow$ 0.175%
w	Wage in each market	1.1870	1.1909	$\uparrow$ 0.329%
π	Profit of a firm in each geographic market	0.0183	0.0180	$\downarrow$ 1.639%

As described above, my results on the effect of market-entry costs on the divergence in trends between national and local market concentration are robust to changing the values of the underlying parameters of the model.

7 Next Steps

Though the benchmark model is successful in explaining the divergence in trends in national and local market concentration, it abstracts from firm heterogeneity. All the firms face exogenously fixed productivity levels and the same market-entry costs in the benchmark model. Some additional insights can be gained when some firm heterogeneity is accounted for as a large dispersion in firm productivity exists (Syverson (2011)) and productivity dispersion has increased dramatically over time in the United States (Barth, Bryson, Davis, and Freeman (2016); Andrews, Criscuolo, and Gal (2016)). Also, Reenen (2018) finds that the differences between firms in relation to their relative productivities and sales have increased in the United States.

He suggests that attributing the falling labor share of GDP, sluggish productivity growth, and declining business dynamism to increasing sales concentration is premature. Many industries have the 'winner take most/all' feature due to globalization and new technologies rather than a general increase in market concentration. Hence, I propose the following next steps in view of this literature and to consider the role of 'superstar firms' in accounting for the divergence in trends in national and local market concentration in the United States.

The firm level productivity (A) can be endogenized, i.e. firms do not face an exogenously given productivity level, but choose the level of productivity that comes with a cost of productivity investment. In this case, the welfare consequences could change further, as a lower cost of market entry would also impact innovation and TFP, but all firms would choose the same productivity level and I would not be able to capture the role of superstars without introducing further heterogeneity. To this end, I plan to introduce ex-ante heterogeneity in the market-entry cost (c_g) or in firm-level productivity (A). Another extension would be to model endogenous firm productivity with ex-ante heterogeneity in the cost of productivity investment, as well as endogenous firm productivity with ex-ante heterogeneity in market-entry cost. I also plan to look for relevant micro-data to further analyze the trends in national and local concentration. Disaggregated data would also be useful in calibrating some parameters that are externally calibrated in this version and estimating richer models that can shed more light on the channels impacting the divergence in trends in national and local market concentration.

7.1 Extension: Heterogeneous Firm Productivities

In this section, I describe the model with endogenous firm productivities to account for the role of productivity dispersion in firms within the United States.[26]

[26]This part is a work in progress, hence I do not have full results for this part yet.

7.1.1 Model Environment

I build on the baseline model described above by allowing firms to choose their productivity levels in addition to choosing the number of markets entered and their labor demand.[27] Each market is populated by an endogenous number of firms who interact strategically and play a three-stage game. In the first stage, they choose the level of productivity A_n (n indexes the firm), and the total cost of productivity investment is given by $c_A A_n^\theta w$, where $c_A > 0$ is the cost of productivity investment and θ is a scaling parameter such that $\theta > 1$. In the second stage, the firms choose how many markets to enter g_n and in the third stage they choose their labor demand l_n. The other elements of the model stay the same - firms face barriers to starting the firm and barriers to entering a geographic market. Each firm produces its own variety in market m using a linear production technology:

$$y_{nm} = A_{nm} l_{nm} \tag{19}$$

where A_{nm} is the productivity of firm n in market m and l_{nm} is labor used by firm n in market m. Total production in market m is:

$$y_m = \left(\sum_{n=1}^{N_m} (y_{nm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{20}$$

where y_{nm} is the production of firm n in market m, N_m is the number of firms in market m and σ is the elasticity of substitution between varieties in a market.

A representative final good firm uses the inputs from firms in a continuum of geographic markets (of measure one) to produce a final consumption good as in the

[27] I borrow some elements of this extended model setup from Cavenaile et al. (2021)

benchmark model:

$$Y = \exp\left(\frac{\sigma}{\sigma-1}\int_0^1 \ln\left[\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}\right]dm\right) \tag{21}$$

where m indexes a continuum of geographic markets of measure one, y_{nm} is the quantity demanded of variety n from market m, N_m is the number of varieties/firms in market m and σ is the constant elasticity of substitution between varieties in a geographic market as before.

There is a household having a continuum of members (of measure one), each supplying one unit of labor who consumes the final good produced by the representative final good firm. Firms enter a market till the value of forming a firm $V = g_n\pi_n(A_n, \{A_{-n}\}, N) - (c_F + c_g g_n^{\gamma+1} + c_A A_n^{\theta})w$ is driven to 0.

7.1.2 Solving the Model

The model can be solved using backward induction.

Stage 3: Given $A_n, \{A_{-n}\}$ and N, Firm n chooses l_n (or y_n) to maximize profits:

$$\max_{y_{nm}} \; p_{nm}y_{nm} - wl_{nm} \tag{22}$$

This gives the following best response function for the firm:[28]

$$y_{nm} = \left[\frac{\sigma-1}{\sigma}A_n\frac{Y}{w}\frac{\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma} \tag{23}$$

[28]See Appendix 9.9 for the derivations in this subsection.

which is the same as:

$$y_{nm} = \frac{\sigma - 1}{\sigma} A_n \frac{Y}{w} \frac{\sum_{l \neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \tag{24}$$

The relative production between two firms in a geographic market can then be expressed as:

$$\implies \left(\frac{y_{nm}}{y_{km}}\right)^{\frac{1}{\sigma}} = \frac{A_n}{A_k} \frac{\sum_{l \neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l \neq k} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} \tag{25}$$

For each geographic market m, this is a system of N_m equations and N_m unknown output ratios which can be solved given the relative productivities of the firms. Once I have the output ratios, I can solve for profits of the firms as given by:

$$\pi_{nm} = \frac{Y}{\left(\sum_{l=1}^{N_m} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \left[\frac{\sigma + \sum_{l \neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sigma}\right] \tag{26}$$

Stage 2: Given $A_n, \{A_{-n}\}$, N and the solution of stage 3 of the model, the firms choose the number of markets to enter g. They solve the following problem:

$$\max_{g_n} \; V_n = g_n \pi_n(A_n, \{A_{-n}\}, N) - (c_F + c_g g_n^{\gamma+1} + c_A A_n^{\theta}) w \tag{27}$$

$$\implies \pi_n(A_n, \{A_{-n}\}, N) = (\gamma + 1) c_g g_n^{\gamma} w \tag{28}$$

Stage 1: Given $\{A_{-n}\}$, N and the solutions of stages 2 and 3 of the model, the firms choose their productivity levels A_n by solving the following problem:

$$\max_{A_n} \; V_n = \hat{g_n}(A_n, \{A_{-n}\}, N) \pi_n(A_n, \{A_{-n}\}, N) - (c_F + c_g \hat{g_n}(A_n, \{A_{-n}\}, N)^{\gamma+1} + c_A A_n^{\theta}) w \tag{29}$$

where $\hat{g_n}(A_n, \{A_{-n}\}, N)$ is the solution obtained in stage 2 of the model.

$$\implies \hat{g_n}\frac{\delta\pi_n(A_n, \{A_{-n}\}, N)}{\delta A_n} = \theta c_A A_n^{\theta-1} w \tag{30}$$

This can be solved using an algorithm that gives the optimal A_n given $\{A_{-n}\}$. I plan to start by assuming 2 different productivity levels to keep the model tractable and solvable. The Nash Equilibrium can be found by imposing symmetry such that $A_n = A_{-n} = A^*$. Solving this model numerically and deriving quantitative results (as in the case of the benchmark model) would be the next steps in this project.

8 Conclusion

This paper develops a model to explain the divergence in trends related to national and local product market concentration in the United States. The model features endogenous entry and markups where firms face barriers to firm formation and barriers to market entry in geographic markets and even the simple baseline model is able to show how lower barriers to market entry encourage entry into more local markets (leading to falling local concentration), while limiting the number of firms in the national market (leading to an increase in national concentration). The model is calibrated using data for the United States, and the quantitative effects of falling market-entry costs on the number of firms in a geographic market, the number of markets entered by a firm, the total number of firms in the economy and outcomes related to consumer welfare such as aggregate consumption and real wages are analyzed.

The quantitative exercises deliver all the results that are consistent with the model - A 10% reduction in the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market leads to an increase of 4.375%

in the number of firms in a market, an increase of 4.387% in the number of geographic markets entered by a firm, and a decrease of 0.012% in the number of firms in the economy. It also leads to an increase of 2.360% in aggregate consumption and real wages. These results are robust to changing the values of different parameters. For example, a reduction in the parameter for the elasticity of substitution between varieties in a market is associated with the change in market-entry costs leading to an increase of 2.964% in the number of firms in a geographic market, an increase of 2.981% in the number of geographic markets entered by a firm, and a decrease of 0.012% in the number of total firms in the economy. This change is also accompanied by an increase of 1.591% in aggregate consumption and real wages. When the scaling parameter for market-entry cost is reduced, a 10% reduction in market-entry cost leads to an increase of 5.240% in the number of firms in a geographic market, an increase of 5.255% in the number of geographic markets entered by a firm, and a decrease of 0.013% in the number of firms in the economy. It also leads to an increase of 2.813% in total consumption and real wages.

These robustness checks lend credibility to my benchmark results and hence I show that a fall in market-entry costs can explain the divergence in trends in national and local market concentration in the United States. The quantitative exercises in this paper show that increasing national market concentration may not be a cause of worry as it is associated with better outcomes for consumers. Even though past studies have highlighted reasons for us to worry about increasing national market concentration, I show that the positive effects on consumers that come in the form of more varieties, more consumption, and higher real wages may be strong enough to counteract some of these forces. This analysis, together with future research in this area, is intended to contribute to an informed reassessment of policies regarding product market concentration.

References

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., & Li, H. (2019). *A theory of falling growth and rising rents* (Tech. Rep.). National Bureau of Economic Research.

Akcigit, U., & Ates, S. T. (2019). *What happened to us business dynamism?* (Tech. Rep.). National Bureau of Economic Research.

Andrews, D., Criscuolo, C., & Gal, P. (2016). The global productivity slowdown, technology divergence and public policy: a firm level perspective. *Brookings Institution Hutchins Center Working Paper*, *24*.

Atkeson, A., & Burstein, A. (2008). Pricing-to-market, trade costs, and international relative prices. *The American Economic Review*, *98*(5), 1998–2031. Retrieved from http://www.jstor.org/stable/29730160

Autor, D., Dorn, D., Katz, L. F., Patterson, C., & Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, *135*(2), 645–709.

Barkai, S. (2020). Declining labor and capital shares. *The Journal of Finance*, *75*(5), 2421–2463.

Barth, E., Bryson, A., Davis, J. C., & Freeman, R. (2016). It's where you work: Increases in the dispersion of earnings across establishments and individuals in the united states. *Journal of Labor Economics*, *34*(S2), S67–S97.

Bento, P. (2020). Competition, innovation, and the number of firms. *Review of Economic Dynamics*, *37*, 275–298.

Berger, D. W., Herkenhoff, K. F., & Mongey, S. (2019). *Labor market power* (Tech. Rep.). National Bureau of Economic Research.

Bessen, J. (2020). Industry concentration and information technology. *The Journal of Law and Economics*, *63*(3), 531-555. Retrieved from https://doi.org/10.1086/708936 doi: 10.1086/708936

Broda, C., & Weinstein, D. E. (2010). Product creation and destruction: Evidence and price implications. *American Economic Review*, *100*(3), 691–723.

Carlson, M., & Mitchener, K. J. (2009). Branch banking as a device for discipline: Competition and bank survivorship during the great depression. *Journal of Political Economy*, *117*(2), 165-210. Retrieved from https://doi.org/10.1086/599015 doi: 10.1086/599015

Cavenaile, L., Celik, M. A., & Tian, X. (2019). Are markups too high? competition, strategic innovation, and industry dynamics. *Competition, Strategic Innovation, and Industry Dynamics (August 1, 2019)*.

Cavenaile, L., Celik, M. A., & Tian, X. (2021). The dynamic effects of antitrust policy on growth and welfare. *Journal of Monetary Economics*.

Covarrubias, M., Gutiérrez, G., & Philippon, T. (2020). From good to bad concentration? us industries over the past 30 years. *NBER Macroeconomics Annual*, *34*, 1-46. Retrieved from https://doi.org/10.1086/707169 doi: 10.1086/707169

Decker, R. A., Haltiwanger, J., Jarmin, R. S., & Miranda, J. (2017). Declining dynamism, allocative efficiency, and the productivity slowdown. *American Economic Review*, *107*(5), 322–26.

De Loecker, J., & Eeckhout, J. (2018). *Global market power* (Tech. Rep.). National Bureau of Economic Research.

De Loecker, J., Eeckhout, J., & Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, *135*(2), 561–644.

Dorn, D., Katz, L. F., Patterson, C., Van Reenen, J., et al. (2017). Concentrating on the fall of the labor share. *American Economic Review*, *107*(5), 180–85.

Edmond, C., Midrigan, V., & Xu, D. Y. (2020). *How costly are markups?* (Tech. Rep.). National Bureau of Economic Research.

Grullon, G., Larkin, Y., & Michaely, R. (2019). Are us industries becoming more concentrated? *Review of Finance*, *23*(4), 697–743.

Gutiérrez, G., & Philippon, T. (2017). *Declining competition and investment in the us* (Tech. Rep.). National Bureau of Economic Research.

Hall, R. E. (2018). *New evidence on the markup of prices over marginal costs and the role of mega-firms in the us economy* (Tech. Rep.). National Bureau of Economic Research.

Hershbein, B., Macaluso, C., & Yeh, C. (2018). *Concentration in us local labor markets: evidence from vacancy and employment data* (Tech. Rep.). Working paper.

Holmes, T. J. (2011). The diffusion of wal-mart and economies of density. *Econometrica*, *79*(1), 253–302.

Hopenhayn, H., Neira, J., & Singhania, R. (2018). *From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share* (Tech. Rep.). National Bureau of Economic Research.

Jia, P. (2008). What happens when wal-mart comes to town: An empirical analysis of the discount retailing industry. *Econometrica*, *76*(6), 1263–1316.

Jiang, X. (2021). Information and communication technology and firm geographic expansion.

Kehrig, M., & Vincent, N. (2018, November). *The micro-level anatomy of the labor share decline* (Working Paper No. 25275). National Bureau of Economic Research. Retrieved from http://www.nber.org/papers/w25275 doi: 10.3386/w25275

Loukas, K., & Brent, N. (2018). Accounting for factorless income.

Olmstead-Rumsey, J. (2019). Market concentration and the productivity slowdown.

Reenen, J. V. (2018). *Increasing differences between firms: market power and the macro-economy* (Tech. Rep.).

Rinz, K. (2018). Labor market concentration, earnings inequality, and earnings mobility. *Center for Administrative Records Research and Applications Working Paper*, *10*.

Rossi-Hansberg, E., Sarte, P.-D., & Trachter, N. (2021). Diverging trends in national and local concentration. *NBER Macroeconomics Annual*, *35*(1), 115–150.

Syverson, C. (2008). Markets: Ready-mixed concrete. *Journal of Economic Perspectives*, *22*(1), 217–234.

Syverson, C. (2011). What determines productivity? *Journal of Economic literature*, *49*(2), 326–65.

Syverson, C. (2019). Macroeconomics and market power: Context, implications, and open questions. *Journal of Economic Perspectives*, *33*(3), 23–43.

Traina, J. (2018). Is aggregate market power increasing? production trends using financial statements. *Production Trends Using Financial Statements (February 8, 2018)*.

9 Appendices

9.1 First Order Conditions

From equation 6, we know that

$$p_{nm} = Y \frac{y_{nm}^{-\frac{1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} \tag{31}$$

Since $A_1 = A_2 = ... = A_n = A$, we also know that

$$\pi_{nm} = p_{nm} y_{nm} - \frac{w y_{nm}}{A} = Y \frac{y_{nm}^{-\frac{1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} y_{nm} - \frac{w y_{nm}}{A} \tag{32}$$

$$\implies \pi_{nm} = Y \frac{y_{nm}^{\frac{\sigma-1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} - \frac{w y_{nm}}{A} \tag{33}$$

The firm's problem is:

$$\max_{y_{nm}} \quad g\pi_{nm} - (c_F + c_g g^{\gamma+1}).w \tag{34}$$

$$\implies \max_{y_{nm}} \quad gY \frac{(y_{nm})^{\frac{\sigma-1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} - gwl - (c_F + c_g g^{\gamma+1}).w \tag{35}$$

In a particular market m, the first order condition with respect to y_n is:

$$\frac{Y[\sum_{n=1}^{N} y_n^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma} y_n^{-\frac{1}{\sigma}} - y_n^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma} y_n^{-\frac{1}{\sigma}}]}{\left[\sum_{n'=1}^{N} (y_{n'})^{\frac{\sigma-1}{\sigma}}\right]^2} - \frac{w}{A} = 0 \tag{36}$$

$$\implies \frac{Y[\frac{\sigma-1}{\sigma} y_n^{-\frac{1}{\sigma}} \sum_{k \neq n} y_k^{\frac{\sigma-1}{\sigma}}]}{\left[\sum_{k=1}^{N} (y_k)^{\frac{\sigma-1}{\sigma}}\right]^2} = \frac{w}{A} \tag{37}$$

$$\implies y_n^{\frac{1}{\sigma}} = \frac{\frac{Y}{w} A \frac{\sigma-1}{\sigma} \sum_{k \neq n} y_k^{\frac{\sigma-1}{\sigma}}}{\left[\sum_{k=1}^{N} (y_k)^{\frac{\sigma-1}{\sigma}}\right]^2} \tag{38}$$

$$\implies y_n = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} \frac{\sum_{k \neq n} y_k^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{k=1}^{N} y_k^{\frac{\sigma-1}{\sigma}} \right)^2} \right]^{\sigma} \tag{39}$$

Imposing $y_n = y_k = y$ in equilibrium,

$$y^{\frac{1}{\sigma}} = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} \frac{\sum_{k \neq n} y^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{k=1}^{N} y^{\frac{\sigma-1}{\sigma}} \right)^2} \right] \tag{40}$$

$$\implies y^{\frac{1}{\sigma}} = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} \frac{(N-1) y^{\frac{\sigma-1}{\sigma}}}{\left(N y^{\frac{\sigma-1}{\sigma}} \right)^2} \right] \tag{41}$$

$$\implies y^{\frac{1}{\sigma}} = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} \frac{(N-1) y^{\frac{\sigma-1}{\sigma}}}{N^2 \left(y^{\frac{\sigma-1}{\sigma}} \right)^2} \right] \tag{42}$$

$$\implies y^{\frac{1}{\sigma}} = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} \frac{(N-1)}{N^2 \left(y^{\frac{\sigma-1}{\sigma}} \right)} \right] \tag{43}$$

$$\implies N^2 \left(y^{\frac{\sigma-1}{\sigma}} \right) y^{\frac{1}{\sigma}} = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} (N-1) \right] \tag{44}$$

$$\implies N^2 y = \left[\frac{\sigma - 1}{\sigma} A \frac{Y}{w} (N-1) \right] \tag{45}$$

$$\implies y = \left[\left(\frac{\sigma - 1}{\sigma} \right) \frac{AY}{w} \frac{(N-1)}{N^2} \right] \tag{46}$$

Since $l = \frac{y}{A}$,

$$\implies l = \left[\left(\frac{\sigma - 1}{\sigma} \right) \frac{Y}{w} \frac{(N-1)}{N^2} \right] \tag{47}$$

The firm's problem is:

$$\implies \max_{g} \quad g\pi - (c_F + c_g g^{\gamma+1}).w \tag{48}$$

F.O.C. w.r.t. g:

$$\pi - c_g(\gamma + 1) g^{\gamma} w = 0 \tag{49}$$

$$\implies c_g g^{\gamma+1} = \frac{\pi g}{w(\gamma+1)} \tag{50}$$

Since $\pi = \frac{(\mu A-1)yw}{A}$, we have

$$c_g g^{\gamma+1} = \frac{\pi g}{w(\gamma+1)} = \frac{\frac{(\mu A-1)yw}{A} g}{w(\gamma+1)} = \frac{(\mu A-1)gy}{A(\gamma+1)} \tag{51}$$

Hence,

$$c_g g^{\gamma+1} = \frac{g(\mu A-1)y}{(\gamma+1)A} \tag{52}$$

Go back to section 4.2.

9.2 Free Entry Condition

The value of forming a firm is driven to zero due to free entry:

$$g\pi - (c_F + c_g g^{\gamma+1}).w = 0 \tag{53}$$

$$\implies c_F = \frac{g\pi}{w} - c_g g^{\gamma+1} \tag{54}$$

Since $c_g g^{\gamma+1} = \frac{\pi g}{w(\gamma+1)}$ from equation 50,

$$\implies c_F = \frac{g\pi}{w} - \frac{\pi g}{w(\gamma+1)} = \frac{(\gamma+1)g\pi - g\pi}{(\gamma+1)w} = \frac{\gamma g\pi}{(\gamma+1)w} \tag{55}$$

Since $\pi = \frac{(\mu A-1)wy}{A}$,

$$\implies c_F = \frac{\gamma}{\gamma+1}\frac{g}{w}\frac{(\mu A-1)wy}{A} = \frac{\gamma}{(\gamma+1)}\frac{gy(\mu A-1)}{A} \tag{56}$$

Hence, $c_F = \frac{\gamma}{(\gamma+1)}\frac{gy(MU-1)}{A}$

Go back to section 4.2.

9.3 Equation for L_p

We know that:

$$c_g g^{\gamma+1} = \frac{g(\mu A - 1)y}{(\gamma+1)A} \tag{57}$$

$$c_F = \frac{\gamma}{(\gamma+1)} \frac{gy(\mu A - 1)}{A} \tag{58}$$

Also,

$$L_p = 1 - \frac{N}{g}(c_F + c_g g^{\gamma+1}) \tag{59}$$

$$\implies L_p = 1 - \frac{N}{g}\left(\frac{\gamma}{(\gamma+1)} \frac{gy(\mu A - 1)}{A} + \frac{g(\mu A - 1)y}{(\gamma+1)A}\right) \tag{60}$$

$$\implies L_p = 1 - \frac{N}{g}\left(\frac{(\gamma+1)gy(\mu A - 1)}{A(\gamma+1)}\right) \tag{61}$$

$$\implies L_p = 1 - \frac{N}{g}\left(\frac{gy(\mu A - 1)}{A}\right) \tag{62}$$

$$\implies L_p = 1 - \frac{Ny}{A}(\mu A - 1) \tag{63}$$

Since $y = Al = A\frac{L_p}{N}$, $L_p = \frac{Ny}{A}$. Hence,

$$L_p = 1 - L_p(\mu A - 1) \tag{64}$$

$$\implies L_p + L_p(\mu A - 1) = 1 \tag{65}$$

$$\implies L_p(1 + \mu A - 1) = 1 \tag{66}$$

$$\implies L_p = \frac{1}{\mu A} = \frac{1}{MU} \tag{67}$$

Go back to section 4.2

9.4 Equilibrium Conditions

We know that

$$c_g g^{\gamma+1} = \frac{g(\mu A - 1)y}{(\gamma+1)A} = \frac{g(MU - 1)y}{(\gamma+1)A} \tag{68}$$

$$c_F = \frac{\gamma}{\gamma+1}\frac{gy(\mu A - 1)}{A} = \frac{\gamma}{\gamma+1}\frac{gy(MU - 1)}{A} \tag{69}$$

From the equations above,

$$c_g g^{\gamma+1} = \frac{c_F}{\gamma} \tag{70}$$

We also know that

$$y = Al = \frac{AL_p}{N} \tag{71}$$

$$\implies N = \frac{AL_p}{y} = \frac{A}{yMU} = \frac{A}{y\mu A} \quad \left(L_p = \frac{1}{MU} = \frac{1}{\mu A}\right) \tag{72}$$

From equation 69, $c_F = \frac{\gamma}{\gamma+1}\frac{gy(\mu A-1)}{A}$

$$\implies \frac{A}{y} = \frac{\gamma}{\gamma+1}\frac{g(\mu A - 1)}{c_F} \tag{73}$$

Substituting this in equation 72, we get

$$N = \frac{\gamma}{\gamma+1}\frac{g(\mu A - 1)}{c_F \mu A} \tag{74}$$

$$\implies \frac{N}{g} = \frac{\gamma}{\gamma+1}\frac{(MU - 1)}{c_F \mu A} = \frac{\gamma}{\gamma+1}\frac{(MU - 1)}{MU c_F} \tag{75}$$

Go back to section 4.2.

9.5 Markup in the case of Cournot competition

We want to show the following: $MU = \left(\frac{\sigma}{\sigma-1}\right) . \left(\frac{N}{N-1}\right)$, i.e.

$\mu A = \frac{p}{w}.A = \left(\frac{\sigma}{\sigma-1}\right) . \left(\frac{N}{N-1}\right)$

From the inverse demand function for variety n in market m,

$$p_{nm} = Y \frac{y_{nm}^{-\frac{1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} \tag{76}$$

where $Y = \exp\left(\frac{\sigma}{\sigma-1}\int_0^1 \ln\left[\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}\right] dm\right)$

Operating profits

$$\pi_{nm} = p_{nm} y_{nm} - \frac{w y_{nm}}{A} \tag{77}$$

Replacing p_{nm} from equation 76,

$$\pi_{nm} = Y \frac{y_{nm}^{-\frac{1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} y_{nm} - \frac{w y_{nm}}{A} \tag{78}$$

$$\pi_{nm} = Y \frac{y_{nm}^{\frac{\sigma-1}{\sigma}}}{\sum_{n'=1}^{N_m} (y_{n'm})^{\frac{\sigma-1}{\sigma}}} - \frac{w y_{nm}}{A} \tag{79}$$

Dropping subscripts ($y = y'$ in this case),

$$\pi = Y \frac{y^{\frac{\sigma-1}{\sigma}}}{\sum (y)^{\frac{\sigma-1}{\sigma}}} - \frac{wy}{A} \tag{80}$$

Fully differentiating with respect to 'y', the first order condition is:

$$\frac{Y[(\sum y^{\frac{\sigma-1}{\sigma}})\frac{\sigma-1}{\sigma} y^{-\frac{1}{\sigma}} - y^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma} y^{-\frac{1}{\sigma}}]}{(\sum y^{\frac{\sigma-1}{\sigma}})^2} - \frac{w}{A} = 0 \tag{81}$$

$$\frac{Y[(N y^{\frac{\sigma-1}{\sigma}})\frac{\sigma-1}{\sigma} y^{-\frac{1}{\sigma}} - y^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma} y^{-\frac{1}{\sigma}}]}{(N y^{\frac{\sigma-1}{\sigma}})^2} - \frac{w}{A} = 0 \tag{82}$$

$$\frac{Y.y^{\frac{\sigma-1}{\sigma}}.\frac{\sigma-1}{\sigma}.y^{-\frac{1}{\sigma}}.(N-1)}{N^2.(y^{\frac{\sigma-1}{\sigma}})^2} - \frac{w}{A} = 0 \tag{83}$$

$$Y.\left(\frac{N-1}{N^2}\right).\left(\frac{\sigma-1}{\sigma}\right).\frac{y^{-\frac{1}{\sigma}}}{y^{\frac{\sigma-1}{\sigma}}} - \frac{w}{A} = 0 \tag{84}$$

$$Y.\left(\frac{N-1}{N^2}\right).\left(\frac{\sigma-1}{\sigma}\right).y^{-1} - \frac{w}{A} = 0 \tag{85}$$

$$\frac{Y}{Ny} \cdot \left(\frac{N-1}{N}\right) \cdot \left(\frac{\sigma-1}{\sigma}\right) - \frac{w}{A} = 0 \tag{86}$$

From equation 76,

$$p = \frac{Y y^{-\frac{1}{\sigma}}}{N y^{\frac{\sigma-1}{\sigma}}} = \frac{Y}{Ny} \tag{87}$$

$$\implies Y = Npy \tag{88}$$

Substituting this in equation 86,

$$\frac{pNy}{Ny} \cdot \left(\frac{N-1}{N}\right) \cdot \left(\frac{\sigma-1}{\sigma}\right) - \frac{w}{A} = 0 \tag{89}$$

$$\implies p. \left(\frac{N-1}{N}\right) \cdot \left(\frac{\sigma-1}{\sigma}\right) = \frac{w}{A} \tag{90}$$

$$\implies \frac{p}{w}.A = \left(\frac{N}{N-1}\right) \cdot \left(\frac{\sigma}{\sigma-1}\right) \tag{91}$$

$$\implies MU = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{N}{N-1}\right) \tag{92}$$

Go back to section 4.3.1.

9.6 Showing $Y = w$

We know that $L_d = Nl$ and $L_p + \frac{N}{g}(c_F + c_g g^{\gamma+1} w) = 1$. From equation 9 above, we know that $l = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{N-1}{N^2}\right)\left(\frac{Y}{w}\right)$ and from equation 14, we know that $L_p = \frac{1}{MU}$.

$$L_d = Nl = L_p = \frac{1}{MU} \tag{93}$$

$$\implies N. \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{N-1}{N^2}\right)\left(\frac{Y}{w}\right) = \frac{1}{MU} \tag{94}$$

$$\implies \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{N-1}{N}\right)\left(\frac{Y}{w}\right) = \frac{1}{MU} \tag{95}$$

We also know that $MU = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{N}{N-1}\right)$. So,

$$\implies \frac{1}{MU}\frac{Y}{w} = \frac{1}{MU} \tag{96}$$

$$\implies Y = w \tag{97}$$

Go back to section 5.1.

9.7 Values of c_F and c_g

We know that

$$c_g g^{\gamma+1} = \frac{gy(MU-1)}{A(\gamma+1)} \tag{98}$$

$$\implies (MU-1) = \frac{c_g g^{\gamma+1} A(\gamma+1)}{gy} \tag{99}$$

We also know that $\frac{N}{g} = \left(\frac{\gamma}{\gamma+1}\right)\frac{(MU-1)}{MU.c_F}$. Substituting the expression for (MU-1) from above, we get

$$\frac{N}{g} = \left(\frac{\gamma}{\gamma+1}\right)\frac{1}{MU.c_F}\frac{c_g g^{\gamma+1} A(\gamma+1)}{gy} \tag{100}$$

$$\implies N = \frac{\gamma}{MU.c_F}\frac{c_g g^{\gamma+1} A}{y} \tag{101}$$

$$\implies c_g = \frac{NyMU.c_F}{\gamma g^{\gamma+1} A} \tag{102}$$

From equation 98,

$$c_g = \frac{gy(MU-1)}{A(\gamma+1)g^{\gamma+1}} \tag{103}$$

Equating equations 102 and 103,

$$\frac{NyMU.c_F}{\gamma g^{\gamma+1} A} = \frac{gy(MU-1)}{A(\gamma+1)g^{\gamma+1}} \tag{104}$$

$$\implies \frac{N.MU.c_F}{\gamma} = \frac{g(MU-1)}{(\gamma+1)} \tag{105}$$

I get the value for c_F by putting in the values for N, MU, γ, g. Also, I get the value of c_g using: $c_g g^{\gamma+1} = \frac{c_F}{\gamma}$, $\implies c_g = \frac{c_F}{\gamma g^{\gamma+1}}$.

Go back to section 5.1.

9.8 Robustness Check with $\gamma = 3$

Table 9 shows the calibrated parameters when $\gamma = 3$. The initial value of c_g corresponds to a value of 11.42% for the average expenditure on market-entry costs as a proportion of output produced by a firm in a geographic market. This is reduced by 10% and this fraction falls to 10.28% (or to the 'new' c_g of 0.00036). With this new market-entry cost, the new values and changes in all the endogenous variables are shown in Table 10. I obtain that the number of firms in a geographic market increases by 1.218% (fall in local concentration) but there is no change in the overall concentration as there is no change in the number of firms in the economy. This may be because the presence of higher costs of market entry due to the higher scaling parameter γ makes it more costly for firms to expand into newer markets, hence the rise in g and the rise in N is lower and comparable in value, which leads to the number of firms $\left(\frac{N}{g}\right)$ staying constant. The other benchmark results continue to hold - firms still charge lower markups and earn lower profits in a market and consumer welfare still rises.

Table 9: Model Parameters with $\gamma = 3$

Parameter	Description	Value	Source
σ	Elasticity of substitution between varieties	10	Atkeson and Burstein (2008)
γ	Scaling parameter for market-entry cost	3	Cavenaile et al. (2019)
c_F	firm-formation cost	0.3757	Internally Calibrated
c_g	market-entry cost	0.0004	Internally Calibrated

Table 10: Results with $\sigma = 10, \gamma = 3$, MU=1.61

Variable	Description	Initial Value	Final Value	Change
MU	Markup charged by a firm	1.61	1.6014	$\downarrow$ 0.534%
g	Number of geographic markets entered by a firm	4.266778	4.318713	$\uparrow$ 1.217%
L_p	Labor used for production	0.6211	0.6245	$\uparrow$ 0.547%
N	Number of firms in a geographic market	3.22717	3.26647	$\uparrow$ 1.218%
N/g	Total Number of firms in the economy	0.75635	0.75635	0%
l	Labor demand by a firm	0.1925	0.1912	$\downarrow$ 0.675%
y	Output produced by a firm	0.1925	0.1912	$\downarrow$ 0.675%
$Y = C$	Aggregate Output in the economy	0.7075	0.7123	$\uparrow$ 0.678%
p	Price of output in each geographic market	1.1390	1.1405	$\uparrow$ 0.132%
w	Wage in each market	0.7075	0.7123	$\uparrow$ 0.678%
π	Profit of a firm in each geographic market	0.0831	0.0819	$\downarrow$ 1.444%

Go back to section 6.2.

9.9 Stage 3: Choosing l or y

I first solve the final good producer's problem to get the inverse demand function, which is substituted in the variety producer's problem.

Final Good Producers: The final good is produced competitively. The representative final good producer chooses the quantity of each variety in each market which maximizes profit. The final good producer's problem is ($P = 1$):

$$\max_{\left[(y_{nm})_{n=1}^{N_m}\right]_{m=0}^{1}} \exp\left(\frac{\sigma}{\sigma-1}\int_0^1 \ln\left[\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}\right]dm\right) - \int_0^1\left(\sum_{n=1}^{N_m}p_{nm}y_{nm}\right)dm \tag{106}$$

$$\exp\left(\frac{\sigma}{\sigma-1}\int_0^1 \ln\left[\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}\right]dm\right)\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}}\right)\left(\frac{\sigma-1}{\sigma}\right)(y_{nm})^{-\frac{1}{\sigma}} = p_{nm} \tag{107}$$

$$\implies Y\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}}\right)\left(\frac{\sigma-1}{\sigma}\right)(y_{nm})^{-\frac{1}{\sigma}} = p_{nm} \tag{108}$$

$$\implies Y\left(\frac{1}{\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}}\right)(y_{nm})^{-\frac{1}{\sigma}} = p_{nm} \tag{109}$$

$$\implies p_{nm} = \frac{y_{nm}^{-\frac{1}{\sigma}}Y}{\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}} \tag{110}$$

$$\implies y_{nm}^{-\frac{1}{\sigma}} = \frac{p_{nm}\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}}{Y} \tag{111}$$

$$\implies (y_{nm})^{\frac{1}{\sigma}} = \frac{Y}{p_{nm}\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}} \tag{112}$$

$$\implies y_{nm} = \left[\frac{Y}{p_{nm}\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}}\right]^{\sigma} \tag{113}$$

$$\implies \frac{y_{nm}}{y_{lm}} = \left(\frac{p_{lm}}{p_{nm}}\right)^{\sigma} \tag{114}$$

Variety Producers: The producers solve the following problem (using p_{nm} from equation 110 above):

$$\max_{y_{nm}} p_{nm}y_{nm} - wl_{nm} = \max_{y_{nm}} \frac{y_{nm}^{-\frac{1}{\sigma}}Y}{\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}}y_{nm} - \frac{wy_{nm}}{A_{nm}} \tag{115}$$

$$= \max_{y_{nm}} \frac{y_{nm}^{\frac{\sigma-1}{\sigma}}Y}{\sum_{n=1}^{N_m}(y_{nm})^{\frac{\sigma-1}{\sigma}}} - \frac{wy_{nm}}{A_{nm}} \tag{116}$$

$$\implies \frac{Y[\sum_{n=1}^{N_m} y_{nm}^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma}y_{nm}^{-\frac{1}{\sigma}} - y_{nm}^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma}y_{nm}^{-\frac{1}{\sigma}}]}{\left(\sum_{n=1}^{N_m} y_{nm}^{\frac{\sigma-1}{\sigma}}\right)^2} - \frac{w}{A_{nm}} = 0 \tag{117}$$

$$\implies \frac{Y\frac{\sigma-1}{\sigma}y_{nm}^{-\frac{1}{\sigma}}[\sum_{n=1}^{N_m} y_{nm}^{\frac{\sigma-1}{\sigma}} - y_{nm}^{\frac{\sigma-1}{\sigma}}]}{\left(\sum_{n=1}^{N_m} y_{nm}^{\frac{\sigma-1}{\sigma}}\right)^2} - \frac{w}{A_{nm}} = 0 \tag{118}$$

$$\implies \frac{Y\frac{\sigma-1}{\sigma}y_{nm}^{-\frac{1}{\sigma}}[\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}]}{\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2} - \frac{w}{A_{nm}} = 0 \tag{119}$$

$$\implies \frac{Y\frac{\sigma-1}{\sigma}y_{nm}^{-\frac{1}{\sigma}}[\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}]}{\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2} = \frac{w}{A_{nm}} \tag{120}$$

$$\implies \frac{Y\frac{\sigma-1}{\sigma}[\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}]}{y_{nm}^{\frac{1}{\sigma}}\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2} = \frac{w}{A_{nm}} \tag{121}$$

$$\implies y_{nm}^{\frac{1}{\sigma}} = \frac{YA_n\frac{\sigma-1}{\sigma}[\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}]}{w\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2} \tag{122}$$

$$\implies y_{nm} = \left[\frac{\sigma-1}{\sigma}A_{nm}\frac{Y}{w}\frac{\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma} \tag{123}$$

$$\implies y_{nm} = \left[\frac{\sigma-1}{\sigma}A_{nm}\frac{Y}{w}\frac{\frac{1}{\left(y_{nm}^{\frac{\sigma-1}{\sigma}}\right)^2}\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}}{\frac{1}{\left(y_{nm}^{\frac{\sigma-1}{\sigma}}\right)^2}\left(\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma} \tag{124}$$

$$\implies y_{nm} = \left[\frac{\sigma-1}{\sigma}A_{nm}\frac{Y}{w}\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{y_{nm}^{\frac{\sigma-1}{\sigma}}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma} \tag{125}$$

$$\implies y_{nm} = \left[\frac{\sigma-1}{\sigma}A_{nm}\frac{Y}{w}\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma}\frac{1}{y_{nm}^{\sigma-1}} \tag{126}$$

$$\implies y_{nm}^{1+\sigma-1} = \left[\frac{\sigma-1}{\sigma}A_{nm}\frac{Y}{w}\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma} \tag{127}$$

$$\implies y_{nm}^{\sigma} = \left[\frac{\sigma-1}{\sigma}A_{nm}\frac{Y}{w}\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\right]^{\sigma} \tag{128}$$

$$\implies y_{nm} = \left(\frac{\sigma-1}{\sigma}\right)A_{nm}\left(\frac{Y}{w}\right)\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \tag{129}$$

Using equation 122, relative production between two firms in a geographic market can be written as:

$$\left(\frac{y_{nm}}{y_{km}}\right)^{\frac{1}{\sigma}} = \frac{A_{nm}}{A_{km}} \frac{\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq k} y_{lm}^{\frac{\sigma-1}{\sigma}}} \tag{130}$$

$$\implies \left(\frac{y_{nm}}{y_{km}}\right)^{\frac{1}{\sigma}} = \frac{A_{nm}}{A_{km}} \frac{\left(\frac{1}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} \sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}}{\left(\frac{1}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} \sum_{l\neq k} y_{lm}^{\frac{\sigma-1}{\sigma}}} \tag{131}$$

$$\implies \left(\frac{y_{nm}}{y_{km}}\right)^{\frac{1}{\sigma}} = \frac{A_{nm}}{A_{km}} \frac{\sum_{l\neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq k} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} \tag{132}$$

Now I derive an expression for p_{nm} after substituting the expression for $y_{nm}^{-\frac{1}{\sigma}}$:

$$\implies p_{nm} = \frac{\sigma}{\sigma-1} \frac{w}{A_{nm}} \frac{\sum_{l=1}^{N_m} y_{lm}^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l\neq n} y_{lm}^{\frac{\sigma-1}{\sigma}}\right)} \tag{133}$$

$$\implies p_{nm} = \frac{\sigma}{\sigma-1} \frac{w}{A_{nm}} \frac{\sum_{l=1}^{N_m} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} \tag{134}$$

We also know that

$$\pi_{nm} = \left(p_{nm} - \frac{w}{A_{nm}}\right) y_{nm} \tag{135}$$

substituting in p_{nm} from equation 134 above,

$$\implies \pi_{nm} = \left(\frac{\sigma}{\sigma-1} \frac{w}{A_{nm}} \frac{\sum_{l=1}^{N_m} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{w}{A_{nm}}\right) y_{nm} \tag{136}$$

$$\implies \pi_{nm} = \frac{\sigma}{\sigma-1} \frac{w}{A_{nm}} \left(\frac{\sum_{l=1}^{N_m} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq n} \left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{\sigma-1}{\sigma}\right) y_{nm} \tag{137}$$

Substituting the expression for y_{nm},

$$\implies \pi_{nm} = \frac{\sigma}{\sigma-1}\frac{w}{A_{nm}}\left(\frac{\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{\sigma-1}{\sigma}\right)\left(\frac{\sigma-1}{\sigma}\right)A_{nm}\left(\frac{Y}{w}\right)\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \tag{138}$$

$$\implies \pi_{nm} = Y\left(\frac{\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{\sigma-1}{\sigma}\right)\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \tag{139}$$

$$\implies \pi_{nm} = Y\left[\frac{\sigma\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} - (\sigma-1)\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sigma\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}\right]\frac{\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \tag{140}$$

$$\implies \pi_{nm} = Y\left[\frac{\sigma\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} - (\sigma-1)\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sigma\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\right] \tag{141}$$

$$\implies \pi_{nm} = \frac{Y}{\sigma\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\left[\sigma\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} - \sigma\sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{142}$$

$$\implies \pi_{nm} = \frac{Y}{\sigma\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\left[\sigma\left(\frac{y_{nm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{143}$$

$$\implies \pi_{nm} = \frac{Y}{\left(\sum_{l=1}^{N_m}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\left[\frac{\sigma + \sum_{l\neq n}\left(\frac{y_{lm}}{y_{nm}}\right)^{\frac{\sigma-1}{\sigma}}}{\sigma}\right] \tag{144}$$

Go back to section 7.1.2.